EP2200 Queuing Theory and Teletraffic Systems

Make-up exam, Friday, June 10th, 2011, 8.00-13.00, Q31.

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Allowed help: Calculator, Beta mathematical handbook or similar, provided sheets of queuing theory formulas and Laplace transforms. Erlang tables are not attached since they are not needed to solve the exam problems.

1. Groups of customers belonging to a large population arrive at a Poisson fashion to a small restaurant with 4 tables with a rate of 4 groups per hour. Each table can accommodate exactly one group, for simplicity. The restaurant employs FCFS service policy.

The groups that manage to be seated stay in the restaurant for an exponential amount of time with an average of 1 hour. The groups that find the restaurant full decide to queue up in the line, if there is no more than one group waiting.

a) Draw the state diagram of the queuing system. Calculate the probability that an arbitrary arriving group of customers gets service immediately. (3p)

b) Calculate the probability that an arriving group of customers decides to leave without eating. What is the average number of customer groups leaving this way within an hour? (2p)

c) What is the average duration of the period of time that the system is fully blocked, i.e. arriving customers will leave the system without service? What is the average duration of the non-blocking period? (3p)

d) Assume now that all customer groups decide to enter the queue if the place is full and wait in the bar for an exponential amount of time with an average of 60 minutes. If there is still no place to be seated after this time, they pay the drinks and leave. Calculate the average of the total time an arbitrary group of customers spends in the restaurant (no matter if they eat or not). (2p)

2. 4 students are conducting literature study. The study consists of reading research papers which first need to be printed in paper. Each student reads a paper for an exponential amount of time with an average of 5 minutes. He then sends a new paper to the printer and waits until it is printed, to start reading again. The printing time of a paper is modeled as exponential with an average value of 1 min. The students share 2 printers. The documents wait in a common printing queue if both printers are busy.

a) Define the Markov chain of the system by defining the states and drawing the state transition diagram. Give the utilization of each printer (the percentage of time a printer is busy). (3p)

b) Calculate the probability that a random paper cannot be printed immediately at any of the two printers and it needs to wait. (2p)

c) Calculate the average waiting time of a paper (that has to wait). What is the probability that the paper will be in the system (queue + printer) for more than 5 minutes? (3p)

d) Now assume that the printers have different printing speeds. The printing time of a paper is exponential but with average values of 1 min and 2 min for printers 1 and 2, respectively. Students send the document to the fast printer first; if the printer is busy, the document is immediately forwarded to the slow printer, and if this printer is busy as well, the document waits in the common printing queue. Define the states and draw the state transmission diagram. (2p)

3. Consider a queuing system with Erlang-n ($n>1$) service time distribution, Poisson arrivals, a single server and one queuing position. The arrival intensity is λ, the average system time is x.

a) Prove that the coefficient of variation of the service time is less than 1. (2p)

b) Consider n=3, and model the system with a Markov-chain. Define the states and give the state transition intensities. Give a set of balance equations needed to calculate the state probabilities. (3p)

(corrected, it was 2p in the exam printout)

c) Identify the blocking states and give the probability that an arriving customer is blocked, as a function of the state probabilities. (2p)

d) Give the average number of customers in the system and the mean system time, as a function of the state probabilities. (3p)

Note that you do not need to calculate the state probabilities.

4. Consider a radio network, where one high priority and one low priority transmitter need to send their packets to their receivers via a wireless medium with capacity of 200 000 bit/s. The wireless medium can be used by only one transmitter at a time. Packets are generated at each transmitter according to a Poisson process and wait in a separate FIFO queue if the channel is busy. The high priority transmitter sends its packets with preemptive resume priority. The mean packet arrival rates at the high and at the low priority transmitter are 300 packet/s and 150 packet/s, respectively. Packets generated at the high priority transmitter have an exponentially distributed length with a mean of 400 bits. The low priority transmitter generates packets with a constant length of 200 bits.

a) Calculate the mean packet delay (transmission time + waiting time of a packet) for the high priority transmitter, and the mean packet delay for the low priority transmitter. (3p)

b) What is the probability density function of the packet waiting time for the high priority transmitter? Calculate the probability that it is larger than 1ms. (2p)

Assume now that the high priority transmitter activates a power save mode. In the power save mode, it turns off its antenna whenever its queue becomes empty. The antenna remains switched off for 10ms, and then the transmitter checks if there are new packets waiting. If the queue is not empty, the transmitter turns on its antenna; otherwise the antenna remains switched off for another 10ms, and so on.

c) What are the mean packet delay and the mean queue length of the high priority transmitter in the power save mode? (3p)

d) If the power consumption of the antenna is 0.001 watt (1 watt =1 Joule/s) in normal mode and 0 watt if turned off, how much energy in average can be saved within an hour? (2p)

5. Answer the following short questions.

a) You would like to model a service time distribution as hyper-Exponential with two stages. Based on your measurements, the average service time is 2 sec, the coefficient of variation is $c^2=1.5$. Give a possible set of parameters (α_i, μ_i), i=1,2. (3p)

b) Consider an M/G/1 queuing system, with arrival rate $\lambda=0.5$ and service time distribution given by $S^*(s)=4/(s+2)^2$. Calculate the mean waiting time. (2p)

c) Consider an M/G/1 system with vacation. The arrival rate is one arrival per 2 time units, the mean service time is 0.5 time units, the vacation time is fixed and is equal to 2 time units. Calculate the probability that more than 2 requests are waiting in the queue at the end of a single vacation period. (2p)

d) Consider an open queuing network of two M/M/1 queues as shown on the figure. $\lambda=1$, $\mu=2$, p=0.25. Calculate the average time the customers spend in the queuing network. (3p)