EP2200 Queuing Theory and Teletraffic Systems
Thursday, December 15th, 2011, 14.00-19.00, Q21,Q26.
Available teacher: Viktoria Fodor
Allowed help: Calculator, Beta mathematical handbook or similar, provided sheets of queuing theory formulas and Laplace transforms. Erlang tables are not attached since they are not needed to solve the exam problems.

1. Two types of jobs arrive at a shared computing resource. Type I jobs arrive according to a Poisson process with intensity 25 jobs/sec, and Type II jobs arrive according to a Poisson process with intensity 75 jobs/sec. The jobs require independent, exponentially distributed service times with an average of 1/120 sec. Jobs wait for service in an unlimited buffer.
a) Give the Kendall notation and the continuous time Markov chain of the system. Calculate the average system (waiting + service) time. (2p)
For the rest of the problem consider the case, when Type I jobs are served with high priority, according to the preemptive resume policy.
b) Calculate the average system and waiting times for Type I jobs. What is the probability that an arbitrary Type I job needs to wait for more than 1 sec? (2p)
c) What is the probability that the service of a Type II job is interrupted? (2p)
d) Draw the continuous time Markov chain of the system. Motivate your solution. (2p)
e) How many jobs are in the system on average? What is the average number of Type I and Type II jobs? (2p)

2. Consider the entrance queue of a night-club in Stockholm. Groups of people arrive at the queue in a Poisson fashion at an average rate of 60 groups per hour. The groups decide to join the queue after observing its length. For a queue length of k groups, an arbitrary arriving group decides to join the queue with a probability $p_k = 1 - \frac{k}{5}, k \leq 5$, otherwise it leaves immediately. The time intervals between two successive groups entering the club are independent exponentially distributed variables with rates depending on the queue length, i.e. $\mu_k = \frac{k}{2}\text{min}^{-1}, k = 1, 2, ...$
a) Draw the state diagram of the queue and compute the probability that an arbitrary arriving group of people finds 5 groups already waiting in the queue. (3p)
b) Calculate the probability that an arbitrary arriving group decides to wait in the queue. (2p)
b) For a group that decides to wait calculate the average time it stays in the queue. (2p)
c) Assume, now, that people are "impatient" due to the cold weather. Thus, an arriving group that decides to join the queue only waits for an exponentially distributed time with an average of 10 minutes, after which it leaves the queue, without entering the club. Re-calculate the average time of an arbitrary group in the queue of the club. (3p)

3 Consider an $M/E_2/1/2$ system. The average service time is 2 seconds, the arrival rate is 0.5 arrivals per second.
a) Give the Markov chain of the system, and calculate the state probability distribution in steady state. What is the probability that an arbitrary arriving customer is blocked? (3p)
b) Calculate the average number of customers in the system, the average system time and the average waiting time, considering all accepted customers. (2p)
c) Consider the customers that arrive when the server is busy. Calculate the average

waiting time of these customers, and give the distribution of the waiting time. (3p)
d) To simplify calculations, you model the system as an $M/M/1/2$ queue with the same average service time. Do you underestimate or overestimate the blocking probability of the customers this way? Can you guess how the blocking probability changes if you model the system as an $M/D/1/2$ queue? (2p)

4. Consider a source A and two destinations B and C. The source can send packets to only one destination at a time. The link capacity from A to B is 10 kbit/s, and the one from A to C is 20 kbit/s. Packets destined to B and C are generated at A with two independent Poisson processes of intensity 60 packets/s and 40 packets/s, respectively. All packets consist of a header with a constant length of 20 bits, and a payload, which has an exponentially distributed length with the average of 100 bits. Packets need to wait in a FIFO queue with infinite size if the source is busy.
a) Give the mean packet transmission time. What is the probability that the source is idle? (2p)
b) Calculate the mean number of packets waiting in the queue, and the mean packet delay (transmission time + waiting time) of an arbitrary packet. (3p)
Now assume the source starts an automatic maintenance when it becomes idle, during which it cannot transmit any packet. The length of every maintenance period is exponentially distributed with a mean of 2 ms.
c) What is the mean remaining maintenance time observed by an arbitrary packet when it arrives? What is the mean remaining maintenance time observed by packets arriving when the system is empty? (2p)
d) Calculate the mean delay (waiting time + transmission time) of an arbitrary packet, and of packets sent to B and C, respectively. (3p)

5. Solve the following short problems.
a) There are two secretaries helping the students at the STEX office. It takes an exponentially distributed time to help a student, with a mean of 2 minutes. You arrive, when a student is already waiting. What is your expected waiting time? What is the probability that you have to wait more than 3 minutes before you get help? (2p)
b) A small office has two coffee machines used by 10 employees. The time to make coffee varies according to an exponential distribution and it has an average of 2 minutes. Employees work for an exponentially distributed time with a mean of 1 hour, and then they go for a coffee. If both machines are being used, they go back to their rooms and continue to work for another 1 hour on average (with exponentially distributed time). What is the probability, that an employee goes back to his room without a cup of coffee? (2p)
c) Consider a queueing network of two M/M/1 queues. Jobs arrive to the first queue. After service, they return to the first queue with probability 0.2, and move to the second queue otherwise. After service at the second queue jobs leave the system. The arrival intensity is 1 job per second. The average service time is 0.5 second in both queues. Calculate the probability that the queuing network is empty. (2p)
d) The Christmas fair in Stockholm downtown is visited by lots of people. On average 10 people enter the area of the fair within one minute, the arrival process can be modelled as Poissonian. They stay for an exponentially distributed time, with an average of an hour. How many people are at the fair on average, and what is the probability that the area is empty at an arbitrary point of time? (2p)