EP2200 Queuing Theory and Teletraffic Systems

Make-up exam, Monday, June 11th, 2012, 14:00-19:00, Q17.

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Allowed help: Calculator, Beta mathematical handbook or similar, provided sheets of queuing theory formulas and Laplace transforms. Erlang tables are not attached since they are not needed to solve the exam problems.

1.

You start a small music streaming service using your computer at home as server. You can stream one song at a time. Requests arriving to your server are waiting in a FIFO queue. You measure how long time it takes to send a song (that is, only transmission time), and find that it is exponentially distributed with a mean of 2 seconds.

a) You would like to evaluate the delay your users experience from the time they send their request, until they receive the entire song, modeling your system as an M/M/1 queue. Motivate the selection of the queuing model. (2p)

b) How many requests can arrive per second so that the average delay is less than 10 seconds? (2p)

c) How many requests can arrive per second so that the 90% of the requests experience less than 10 seconds delay? (3p)

d) Your service is becoming popular, and the request arrival rate is 0.5 arrivals per second. To ensure an average delay of 10 seconds, you decide to limit the queue length. Requests that arrive when the queue length reached this maximum are dropped. What should be the queue length limit to ensure the 10 seconds average delay and what is the probability of dropping a request? (Hint: Consider the Markov chain of the system). (3p)

2.

A small company bought a Matlab license for 3 users. The Matlab server is installed in a central computer, and 3 out of the 10 engineers of the company can access it simultaneously. Once started to use Matlab, the engineer works with it for an exponentially distributed time with an average of 15 minutes, and then gets back to other duties for one hour on average, again with exponentially distributed time. If 3 clients are already running when the engineer tries to connect, he is blocked and therefore he gets back to other duties for an hour on average.

a) All the engineers start with other duties in the morning. What is the expected time until the first connection to the Matlab server? Motivate your answer. (2p)

b) Give the Kendall notation of the system, and draw the state transition diagram. What is the probability that all 3 clients are running, if we assume that the system is in steady state? (3p)

c) What is the probability that an engineer starts to use Matlab when no one else is using it? What is the probability that an engineer cannot use Matlab when he wants to? (3p)

d) In average, how much time an engineer spends with other duties in 8 working hours? (2p)

3.

Consider a queuing network of two nodes. A source generates packets as a Poisson process with intensity 10 packets/s, and sends the generated packets immediately to node 1 with probability 0.4, and to node 2 with probability 0.6. Node 1 is an M/M/1 server with mean service rate of 12 packets/s. Node 2 is an $M/E_2/1/2$ server with mean service rate of 3 packets/s. Packets sent to node 2 are rejected if there is no queuing position left. The rejected packets leave the system without any delay.

a) Consider the packets that are sent to node 2 when node 2 is empty (not serving any packet). What are the mean and the second moment of the time such packets spend in the system? (2p)

b) Draw the state diagram of node 2, give the balance equations, and then calculate the state probability distribution in steady state. (3p)

c) Determine the average number of packets rejected per second at node 2. What is the probability that an arbitrary packet leaves the queuing network without service? (2p)

d) What is the average number of packets in the queuing network, and what is the average time a packet spends in the queuing network? (3p)

4.

A small bank office in Stockholm has only one counter providing service to customers. Two types of customers arrive with two independent Poisson processes of intensity 4 customers/hour for type I, and 2 customers/hour for type II. The service time of the customers is exponentially distributed with a mean of 6 minutes for type I, and with a mean of 12 minutes for type II. Customers need to wait in a FIFO queue with infinite size if the clerk working at the counter is busy.

a) What is the average service time of customers? Give the probability that an arbitrary customer finds the clerk busy. (2p)

b) Calculate the average number of customers waiting in the queue, and the average waiting time of an arbitrary customer. (2p)

Now consider the case when type II customers are served with preemptive priority.

c) What is the average waiting time of type II customers? What is the probability that there are less than 2 customers of type II in the bank? (3p)

d) Consider the same preemptive priority system. What is the average waiting time (that is, the time from arrival until the start of the first service attempt) for the type I customers? What is their average service time (that is, the time from the start of the first service attempt until the end of service)? (3p)

5. Answer the following short questions.

a) You would like to by a bike on Blocket, the Swedish second-hand web-shop. New advertisements appear according to a Poisson process, 10 per day in average. Bikes find new owner after an exponentially distributed time, 1 day in average. Define an appropriate queuing system. Give the average number of bikes on sale and give the probability that there is exactly one bike on Blocket that you could buy. (3p)

b) Two streams of packets are multiplexed at a router. Packets of both streams arrive according to a Poisson process. Prove, that the multiplexed arrivals give a Poisson process as well. (2p)

c) Consider an M/G/1 system with vacation. The arrival rate is two arrivals per time unit, and the service time is exactly 0.25 time units. The vacation time is exponentially distributed with mean of one time unit. Calculate the average waiting time. (2p)

d) Consider an open queuing network of two M/M/1 queues as shown on the figure. $\lambda=1$, $\mu=2$, $p=0.25$. What is the probability that the queuing network is empty? (3p)