(1)

a) M / M / 1 queue

- Arrival: can be considered as Poisson, since requests arrive from a large population and are not correlated.
- Service: given, Exponential
- # Servers: given, 1
- infinite buffer capacity: we assume that all requests can be registered

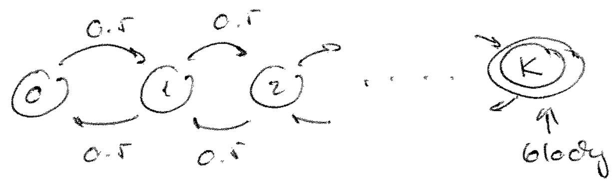
b) $T_{M/M/1} = \frac{1}{\mu - \lambda}$, $T = 10\,s$, $\mu = 0.5\ /s$ $\Rightarrow$ $\boxed{\lambda = 0.4\ 1/s}$

c) $F_T(t) = 1 - e^{-(\mu - \lambda)\cdot t}$, $t = 10\,s$, $F_T(10) = 0.9$

$0.9 = 1 - e^{-(0.5 - \lambda)\cdot 10}$ $\Rightarrow$ $\boxed{\lambda = 0.27\ 1/s}$

d) $\lambda = 0.5$, $\mu = 0.5$ $\Rightarrow$ $\rho = 1$

Consider an M / M / 1 / K system (K-1 requests can wait)



From balance equation: $p_0 = p_1 = \dots p_K = \frac{1}{K+1}$

$\Rightarrow P(\text{block}) = \frac{1}{K+1}$

$$N = \sum_{k=0}^{K} \frac{1}{K+1}\cdot k = \frac{1}{K+1}\cdot\frac{K\cdot(K+1)}{2} = \frac{K}{2}$$

$$T = \frac{N}{\lambda_{eff}} = \frac{K/2}{\lambda\cdot\left(1 - \frac{1}{K+1}\right)}$$ for $T = 10$, $\lambda = 0.5$

$$10 = \frac{\cancel{K}/2}{0.5\cdot \cancel{K}/K+1} = K + 1$$ $\Rightarrow$ $\boxed{K = 9}$ $\Rightarrow$ 8 requests can wait

$\boxed{P(\text{block}) = 0.1}$

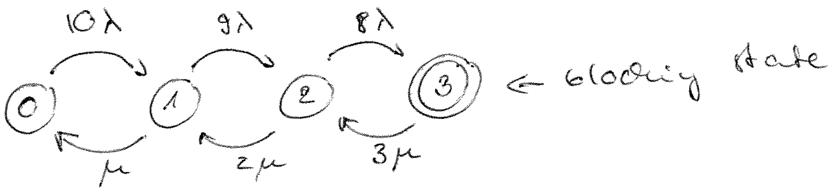
(2.)

Engset system

a) Time until the first connection = minimum of 10 Exp(1) distributed times = ~~Exp(10)~~ Exp(10 · 1) $\Rightarrow \bar{T}_1 = \frac{1}{10}$ hours = 6 minutes

b) M / M / 3 / 3 / 10 $\quad \lambda = 1$ (request / hour)

$\mu = 4$ (service / hour)



$p_0 \cdot 10 = p_1 \cdot 4$

$p_1 \cdot 9 = p_2 \cdot 8$

$p_2 \cdot 8 = p_3 \cdot 12$

$p_0 + p_1 + p_2 + p_3 = 1$

$\{p_0, p_1, p_2, p_3\} = \left\{ \frac{16}{131}, \frac{40}{131}, \frac{45}{131}, \frac{30}{131} \right\}$

$= \{0.12, 0.3, 0.34, 0.23\}$

P(all clients are running) = $p_3 = \frac{30}{131} \approx 0.23$

c) P(new request when the ~~state~~ system is in state ∅)

$$= \frac{10 \cdot p_0}{10p_0 + 9p_1 + 8p_2 + 7 \cdot p_3} = \frac{1.2}{1.2 + 2.7 + 2.72 + 1.61} = \frac{1.2}{8.23} = 0.14$$

$$P(\text{engineer blocked}) = \frac{7 \cdot p_3}{10p_0 + 9p_1 + 8p_2 + 7p_3} = \frac{1.61}{8.23} = 0.19$$

d) - ~~Average number of clients running: $\bar{N} = p_1 + 2p_2 + 3p_3 =$~~

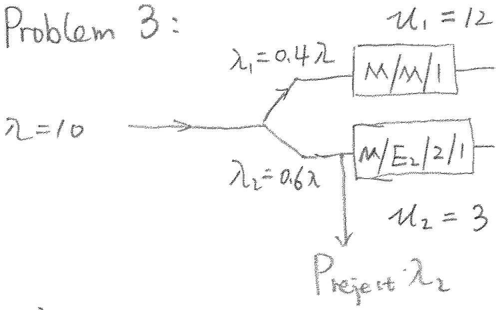
$\lambda_{eff} = p_0 \cdot 10\lambda + p_1 \cdot 9\lambda + p_2 \cdot 8\lambda = 6.62 \cdot \lambda = 6.62$

All time spent with Matlab = $\lambda_{eff} \, \bar{x} \cdot T = 6.62 \cdot 0.25 \cdot 8 = 13.24$

One engineer spends with Matlab = $\frac{\lambda_{eff} \cdot \bar{x} \cdot T}{10} = 1.3$ hour

One engineer spends with other duties = 6.7 hours.

Problem 3: unit: packets/s

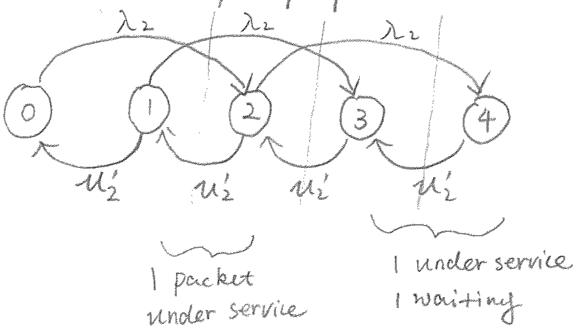


a) $T \sim \text{Erlang}(2, u_2')$

$$\overline{T} = \frac{2}{u_2'} = \frac{1}{u_2} = \frac{1}{3} \text{ Seconds} \qquad u_2' = 2u_2 = 6$$

$$\overline{T^2} = V[T] + \overline{T}^2 = \frac{2}{(u_2')^2} + \frac{1}{9} = \frac{1}{6} \text{ seconds}^2$$

b) node 2 $\sim M/E_2/2/1$



Balance equations:

$$\left.\begin{aligned}
P_0\lambda_2 &= P_1 u_2' \\
P_0\lambda_2 + P_1\lambda_2 &= P_2 u_2' \\
P_1\lambda_2 + P_2\lambda_2 &= P_3 u_2' \\
P_2\lambda_2 &= P_4 u_2' \\
\sum_{i=0}^{4} P_i &= 1
\end{aligned}\right\}$$

$$\Rightarrow P_0 = \frac{1}{9}, \quad P_1 = \frac{1}{9}, \quad P_2 = \frac{2}{9}, \quad P_3 = \frac{3}{9}, \quad P_4 = \frac{2}{9}$$

c) $$\lambda_{rejet} = \lambda_2(P_3 + P_4) = \lambda \times 0.6(P_3 + P_4) = 6 \cdot \frac{5}{9} = \frac{10}{3} \text{ packets/s}$$

$$P_{reject} = 0.6(P_3 + P_4) = \frac{3}{5} \cdot \frac{5}{9} = \frac{1}{3}$$

d) $$\overline{N_1} = \frac{\lambda_1}{u_1} / (1 - \frac{\lambda_1}{u_1}) = \frac{1/3}{2/3} = \frac{1}{2} \text{ packets}$$

$$\overline{N_2} = 0 \cdot P_0 + 1 \cdot (P_1 + P_2) + 2(P_3 + P_4) = \frac{3}{9} + \frac{10}{9} = \frac{13}{9} \text{ packets}$$

$$\overline{N} = \overline{N_1} + \overline{N_2} = \frac{1}{2} + \frac{13}{9} = \frac{9+26}{18} = \frac{35}{18} \text{ packets}$$

or:

Little:

$$\overline{T} = \frac{\overline{N}}{\lambda} = \frac{35/18}{10} = \frac{7}{36} \text{ s}$$

Little: $$\overline{T_1} = \frac{\overline{N_1}}{\lambda_1} = \frac{1}{8} \text{ s}$$

$$\overline{T_2} = \frac{\overline{N_2}}{\lambda_2(1 - P_b)} = \frac{\overline{N_2}}{\lambda_2(1 - P_3 - P_4)} = \frac{\frac{13}{9}}{6 \cdot \frac{4}{9}} = \frac{13}{24} \text{ s}$$

$$\overline{T} = \underbrace{0.4}_{\text{Prob. of served by node 1}} \overline{T_1} + \underbrace{0.6(1 - P_b)}_{\text{Prob. of served by node 2}} \cdot \overline{T_2} + \underbrace{0.6 P_b}_{\text{prob. of leaving without service}} \cdot 0 = \frac{1}{20} + \frac{13}{90} = \frac{7}{36} \text{ s}$$

Problem 4.

$\lambda_1 = 4$, $u_1 = \frac{60}{6} = 10$ unit: customer / hour

$\lambda_2 = 2$, $u_2 = \frac{60}{12} = 5$

a) $M/H_2/1$

$\bar{S_1} = \frac{1}{u_1} = \frac{1}{10}$ $\bar{S_2} = \frac{1}{u_2} = \frac{1}{5}$

$P_1 = \frac{\lambda_1}{\lambda_1 + \lambda_2} = \frac{2}{3}$, $P_2 = \frac{\lambda_2}{\lambda_1 + \lambda_2} = \frac{1}{3}$

$\bar{S} = P_1 \bar{S_1} + P_2 \bar{S_2} = \frac{1}{15} + \frac{1}{15} = \frac{2}{15}$ hour $= 8$ mins

$P_{busy} = \rho = (\lambda_1 + \lambda_2) \cdot \bar{S} = 6 \cdot \frac{2}{15} = \frac{4}{5}$

b) $\overline{S^2} = P_1 \overline{S_1^2} + P_2 \overline{S_2^2} = \frac{2}{3} \cdot \frac{2}{u_1^2} + \frac{1}{3} \frac{2}{u_2^2} = \frac{1}{25}$

$V[S] = \overline{S^2} - \bar{S}^2 = \frac{1}{25} - (\frac{2}{15})^2 = \frac{1}{45}$

$C_S^2 = \frac{V[S]}{\bar{S}^2} = \frac{5}{4}$ $\rho = \frac{4}{5}$

$\bar{N_q} = \frac{1 + C_S^2}{2} \cdot \frac{\rho^2}{1-\rho} = \frac{18}{5} = 3.6$ customers

$\bar{W} = \frac{\bar{N_q}}{(\lambda_1 + \lambda_2)} = \frac{3}{5}$ hours $= 36$ mins.

c) Preemptive resume priority: type II > type I

type II: $M/M/1$

$\bar{W_2} = \frac{\rho_2}{u_2 - \lambda_2} = \frac{\lambda_2/u_2}{u_2 - \lambda_2} = \frac{\frac{2}{5}}{5-2} = \frac{2}{15}$ hours $= 8$ mins

$P(N_2 < 2) = P_0 + P_1 = (1-\rho_2) + (1-\rho_2)\rho_2 = \frac{21}{25}$

d) $R_1 = \frac{1}{2}(\lambda_2 \overline{S_2^2} + \lambda_1 \overline{S_1^2}) = \frac{1}{2}(\lambda_2 \cdot \frac{2}{u_2^2} + \lambda_1 \cdot \frac{2}{u_1^2}) = \frac{3}{25}$

$\bar{W_1} = \frac{\bar{R_1}}{(1-\rho_2)(1-\rho_2-\rho_1)} = \frac{\frac{3}{25}}{(1-\frac{4}{10})(1-\frac{4}{10}-\frac{2}{5})} = 1$ hour

$\bar{S_1}' = \frac{\bar{S_1}}{1-\rho_1} = \frac{1/10}{1-4/10} = \frac{1}{6}$ hours $= 10$ mins

(5)

a) It is an $M/M/\infty/\infty$ system:
- all ad's are accepted
- all bikes are on sale.

$\lambda = 10$
$\bar{x} = 1 = T$ $\Big\}$ $N \overset{\text{Little}}{=} \lambda \cdot T = 10$

$$P_k = \frac{\rho^k}{k!} e^{-\rho} \quad (M/M/m/m \text{ as } m \to \infty)$$

$$P_1 = \rho \cdot e^{-\rho} = 10 \cdot e^{-10} = 4.5 \cdot 10^{-3}$$

b) Poisson arrival $\Leftrightarrow$ Exponential interarrival time

$P(\tau_1 < t) = e^{-\lambda_1 t}$, $P(\tau_2 < t) = e^{-\lambda_2 t}$

$P(\tau^* < t) = P(\tau_1 < t) P(\tau_2 < t) = e^{-\lambda_1 t} e^{-\lambda_2 t} = e^{-(\lambda_1 + \lambda_2) t} \Rightarrow$

the multiplexed process is Poisson $(\lambda_1 + \lambda_2)$

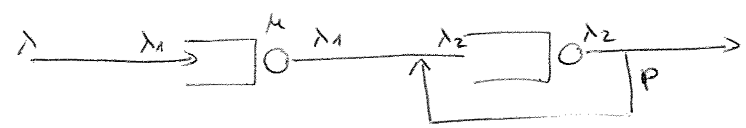
c) M/G/1 with vacation

$$\overline{W} = \frac{\lambda \overline{x^2}}{2(1-\rho)} + \frac{\overline{V^2}}{2\overline{V}}$$

$\lambda = 2$
$x = \frac{1}{4}$, deterministic $\Rightarrow$
$\overline{x^2} = x^2 = \frac{1}{16}$

$\overline{V} = 1$, exp $\Rightarrow$
$\overline{V^2} = 2$

$$\overline{W} = \frac{1 \cdot \frac{1}{16}}{2(1-\frac{2}{4})} + \frac{2}{2 \cdot 1} = \frac{9}{8}$$

d) 

$\lambda_1 = \lambda = 1$
$\lambda_2 = \lambda_1 + p \cdot \lambda_2 \Rightarrow$
$\lambda_2 = \frac{\lambda_1}{1-p} = \frac{1}{\frac{3}{4}} = \frac{4}{3} \Rightarrow$

$\Rightarrow \rho_1 = \lambda_1 / \mu = \frac{1}{2}$
$\rho_2 = \lambda_2 / \mu = \frac{2}{3}$

$P(\text{nw empty}) = P($~~queue~~ system 1 empty$)P($system 2 empty$) = (1-\rho_1)(1-\rho_2) = \frac{1}{6}$