

Constants and formulas

Space Physics EF2240

PHYSICAL CONSTANTS

c	speed of light	$3 \times 10^8 \text{ ms}^{-1}$
e	elementary charge	$1.6 \times 10^{-19} \text{ C}$
m_e	electron mass	$0.91 \times 10^{-30} \text{ kg}$
m_p	proton mass	$1.67 \times 10^{-27} \text{ kg}$
h	Planck's constant	$6.625 \times 10^{-34} \text{ Js}$
k_B	Boltzmann's constant	$1.38 \times 10^{-23} \text{ JK}^{-1}$
μ_0	permeability of free space	$4\pi \times 10^{-7} \text{ Hm}^{-1}$
ϵ_0	permittivity of free space	$8.854 \times 10^{-12} \text{ Fm}^{-1}$
R_E	Earth's radius	6378 km
a	Earth's magnetic dipole moment	$8 \times 10^{22} \text{ Am}^2$
σ_{SB}	Stefan-Boltzmann's constant	$5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$

p_d	dynamic pressure	ρv^2
p	thermal pressure	$n k_B T$
v_D	drift velocity	$\frac{\mathbf{F} \times \mathbf{B}}{qB^2}$
R_m	magnetic Reynolds number	$\mu_0 \sigma v_c l_c$
n	index of refraction for plasma	$\sqrt{1 - \frac{f_{pe}^2}{f^2}}$
α_{fl}	loss cone angle	$\arcsin \sqrt{B/B_{\max}}$

CONVERSIONS

1 eV	electron volt	$1.6 \times 10^{-19} \text{ J}$
1 AU	astronomical unit	$1.496 \times 10^{11} \text{ m}$
1 γ	Gamma	10^{-9} T
1 G	Gauss	10^{-4} T
1 LY	light year	$9.461 \times 10^{15} \text{ m}$

PLASMA PHYSICS

ω_p	plasma frequency	$\sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}$
ω_c	cyclotron-/gyro frequency	$\frac{eB}{m_e}$
r_L	gyro/Larmor radius	$\frac{mv_{\perp}}{eB}$
μ	magnetic moment	$\frac{mv_{\perp}^2}{2B}$
λ_D	Debye length	$\sqrt{\frac{\epsilon_0 k_B T_e}{n_e e^2}}$
v_A	Alfvén velocity	$\frac{B}{\sqrt{\mu_0 \rho}}$
p_B	magnetic pressure/energy density	$\frac{B^2}{2\mu_0}$

PLASMA ELECTRODYNAMICS

$j_{\parallel}$	parallel current density	$\sigma_{\parallel} E_{\parallel}$
j_P	Pedersen current density	$\sigma_P E_{\perp}$
j_H	Hall current density	$\sigma_H E_{\perp}$
$\sigma_{\parallel}$	parallel conductivity	$\sigma_{\parallel} = \sigma_e + \sigma_i$
σ_P	Pedersen conductivity	$\sigma_e \frac{1}{1 + \omega_{ge}^2 \tau_e^2} + \sigma_i \frac{1}{1 + \omega_{gi}^2 \tau_i^2}$
σ_H	Hall conductivity	$\sigma_e \frac{\omega_{ge} \tau_e}{1 + \omega_{ge}^2 \tau_e^2} - \sigma_i \frac{\omega_{gi} \tau_i}{1 + \omega_{gi}^2 \tau_i^2}$
σ_e	electron conductivity	$e^2 n \tau_e / m_e$
σ_i	ion conductivity	$e^2 n \tau_i / m_i$
Ampère's law for infinite current sheet		$j_z = -\frac{1}{\mu_0} \frac{\partial B_x}{\partial y}$

RELATIVISTIC FIELD TRANSFORMATIONS

Electric field ($\perp \mathbf{u}$)

$$\mathbf{E}' = \frac{\mathbf{E} + \mathbf{u} \times \mathbf{B}}{\sqrt{1 - u^2/c^2}}$$

Magnetic field ($\perp \mathbf{u}$)

$$\mathbf{B}' = \frac{\mathbf{B} - (\mathbf{u}/c^2) \times \mathbf{E}}{\sqrt{1 - u^2/c^2}}$$

The components parallel to $\mathbf{u}$ remain unchanged.

RELATIVISTIC MECHANICS

Gamma factor

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Relativistic momentum

$$\mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1 - v^2/c^2}} = \gamma m\mathbf{v}$$

Relativistic energy

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} = \gamma mc^2$$

Rest energy

$$E = mc^2$$

Relation between energy and momentum

$$E^2 = p^2 c^2 + m^2 c^4$$

OTHER

Dipole magnetic field

$$B_r = B_p \left(\frac{R_E}{r}\right)^3 \cos \theta$$

$$B_\theta = \frac{B_p}{2} \left(\frac{R_E}{r}\right)^3 \sin \theta$$

where B_p is the magnetic field strength at the poles

Dipole magnetic field, alternative formulation

$$B_r = \frac{\mu_0 a}{2\pi} \frac{1}{r^3} \cos \theta$$

$$B_\theta = \frac{\mu_0 a}{2\pi} \frac{1}{r^3} \sin \theta$$

where a is the magnetic dipole moment

Parker spiral

$$\frac{B_\theta}{B_r} = \tan \psi = \frac{\omega_{\text{sun}} r}{u_{\text{sw}}}$$

Energy for light quantum

$$E = hf$$

Wien's displacement law

$$\lambda = \frac{2.90 \times 10^{-3}}{T}$$

Stefan-Boltzmann's law

$$J = \sigma_{\text{SB}} T^4$$

Dispersion relation for electromagnetic waves in vacuum

$$c = f\lambda$$