Pattern Recognition 1: Introduction

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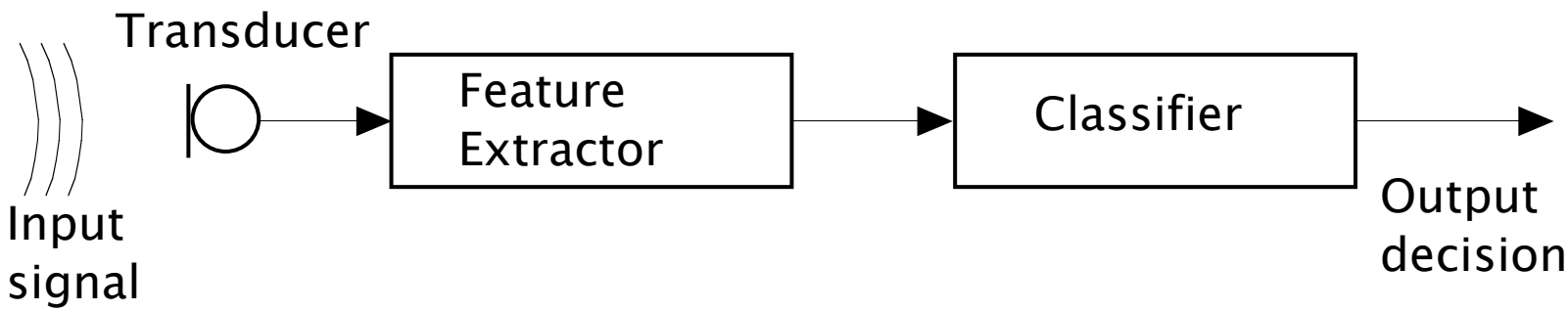
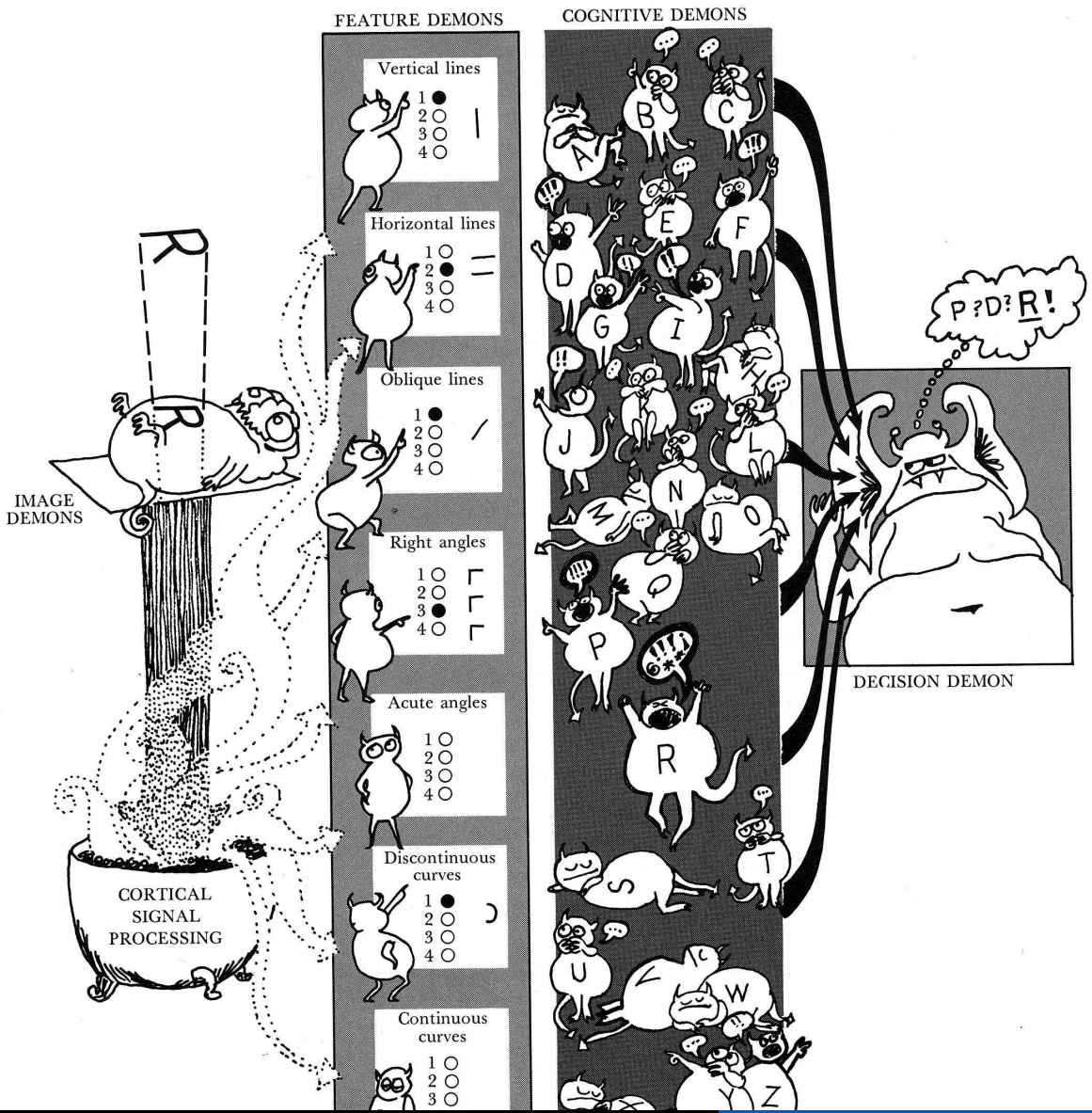
Aug 27, 2012

What is it?

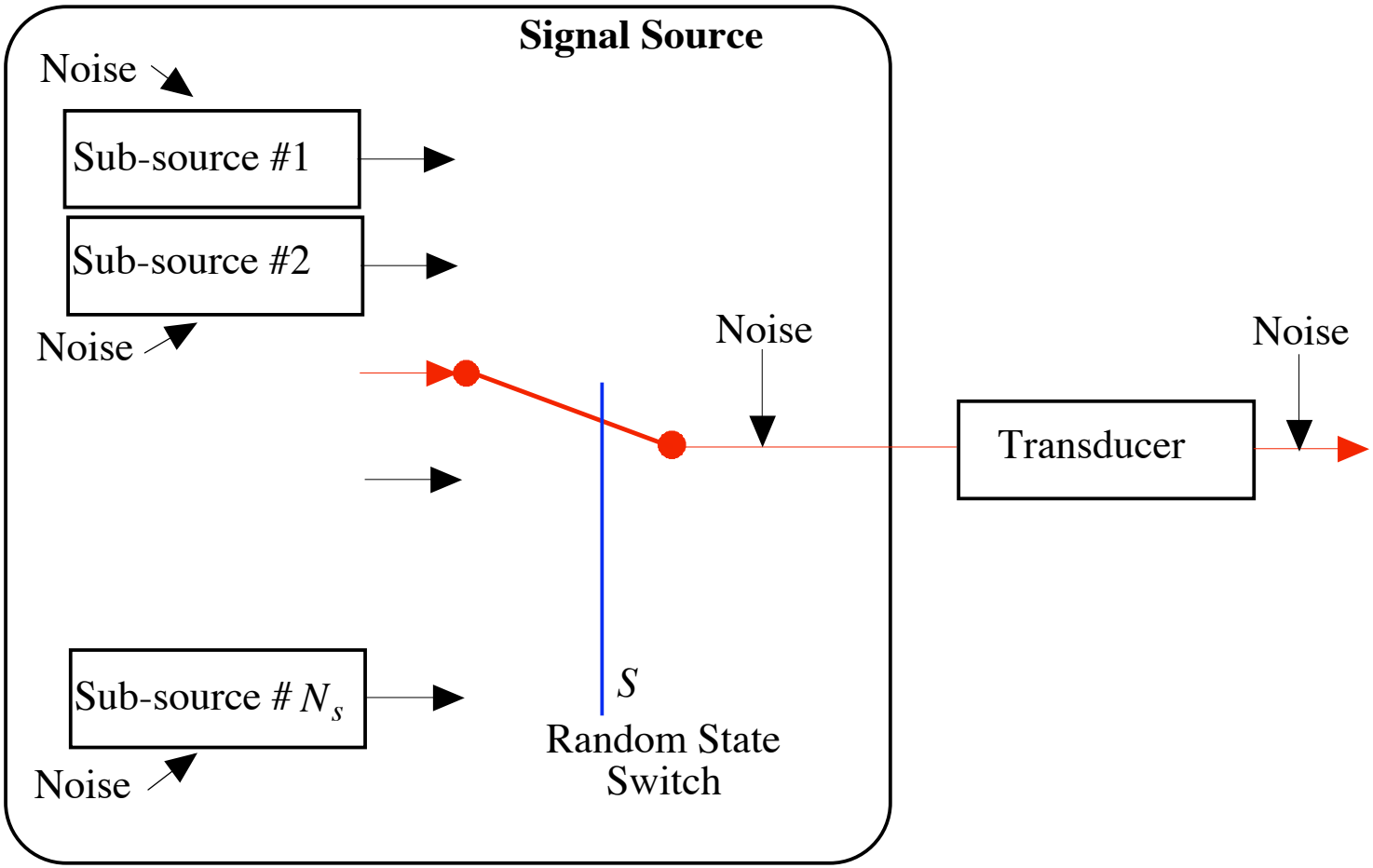
A Pattern-Classification System



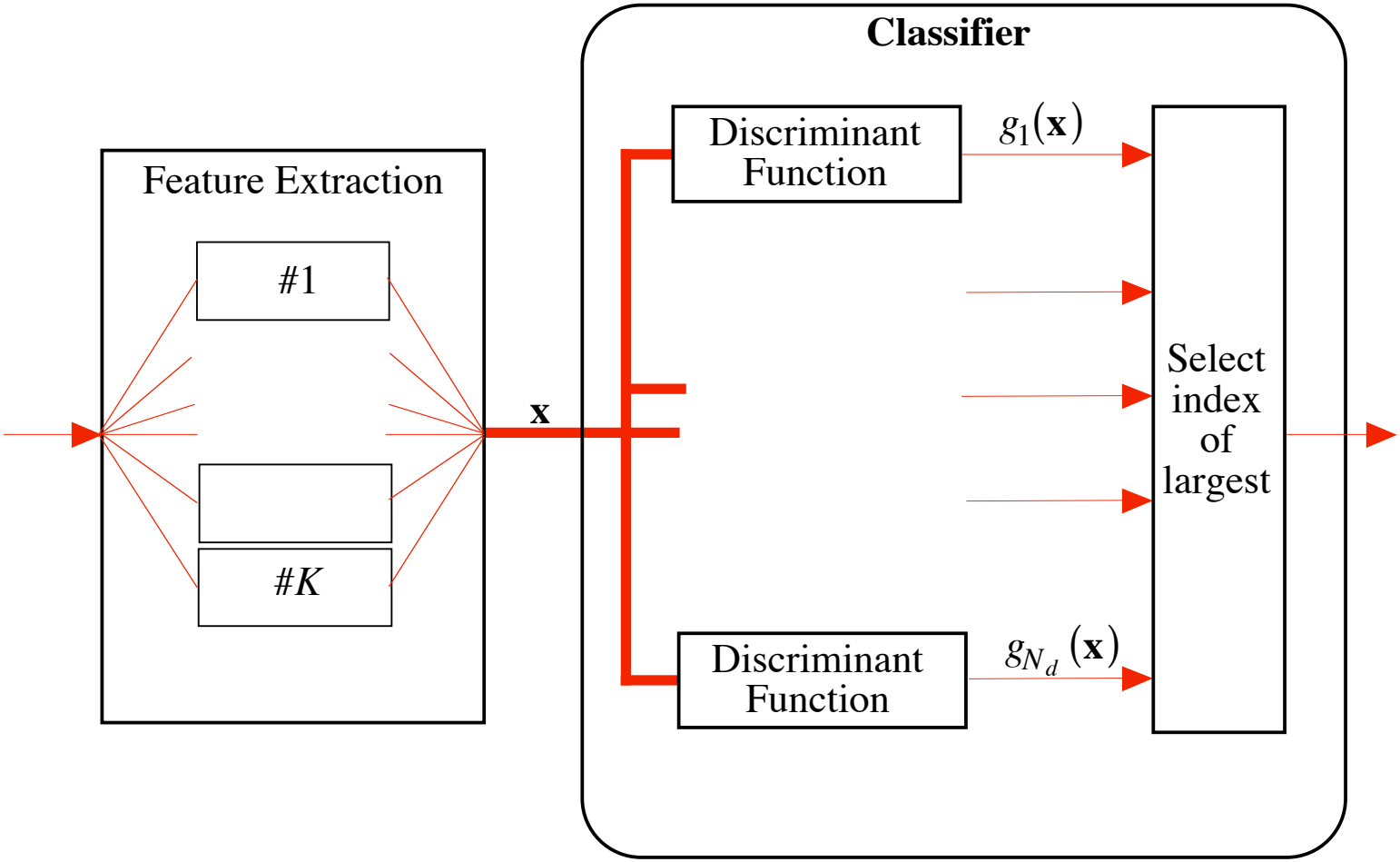
Human Pattern Classification – the Demon Model



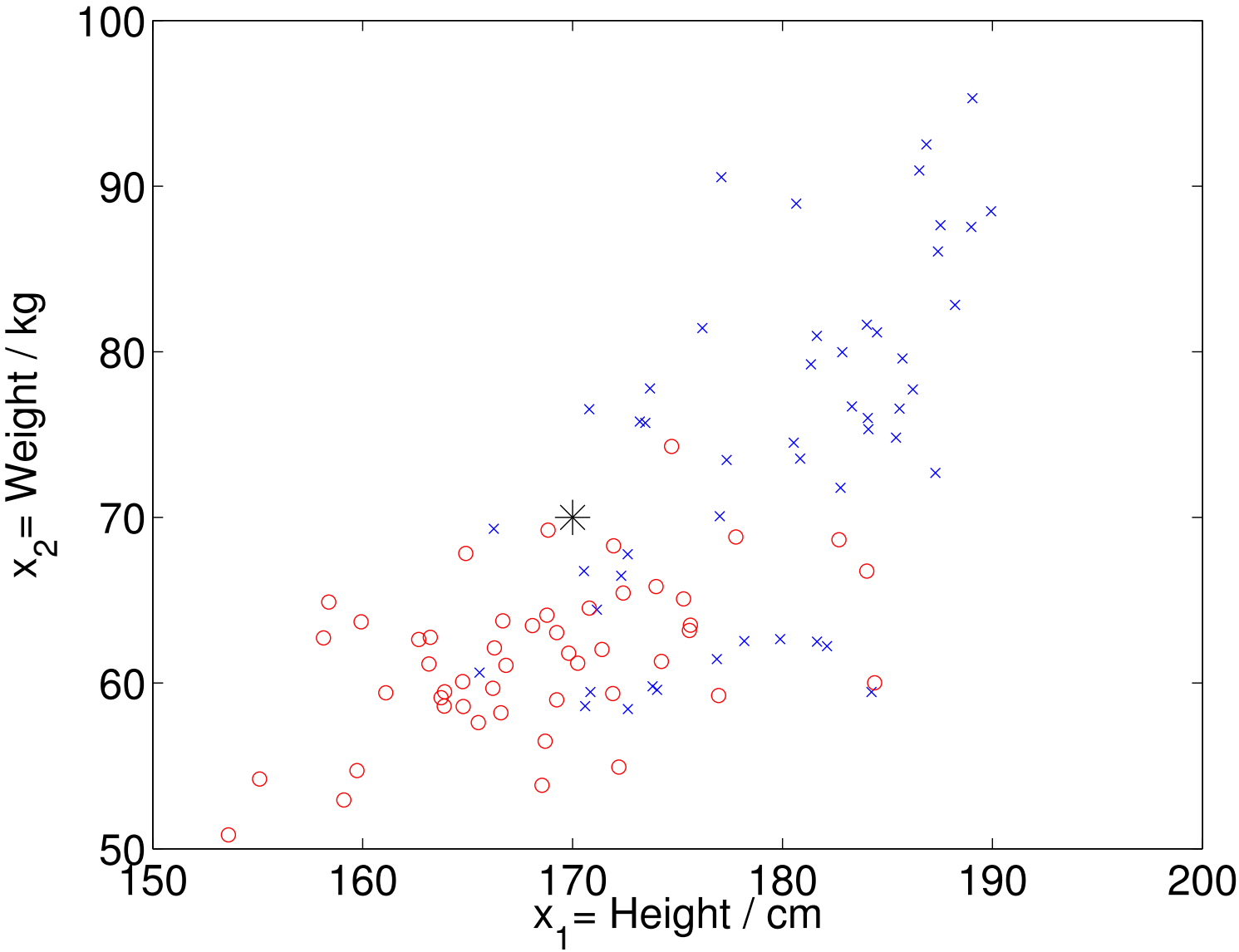
Source Categories



Optimal Classification Mechanism

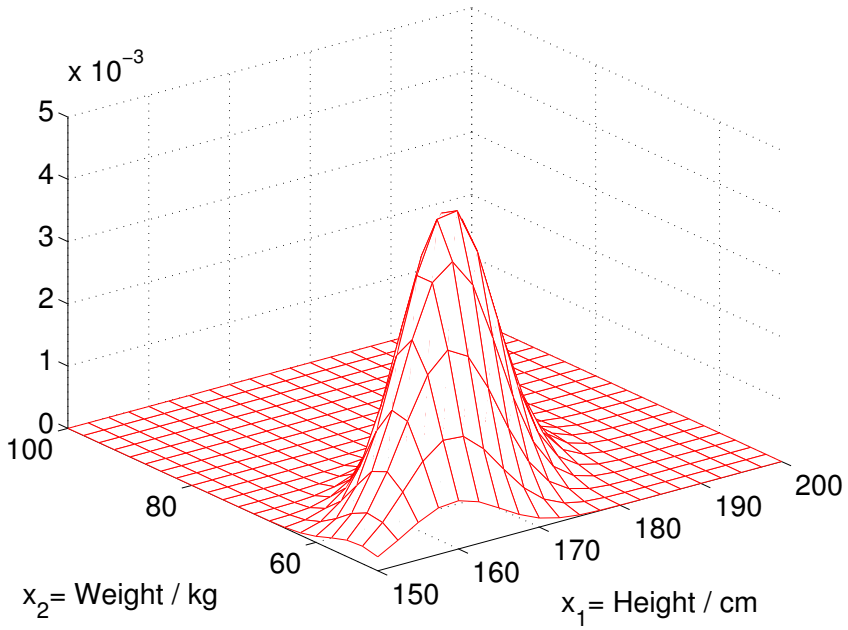


First Classifier Example: Man or Woman?

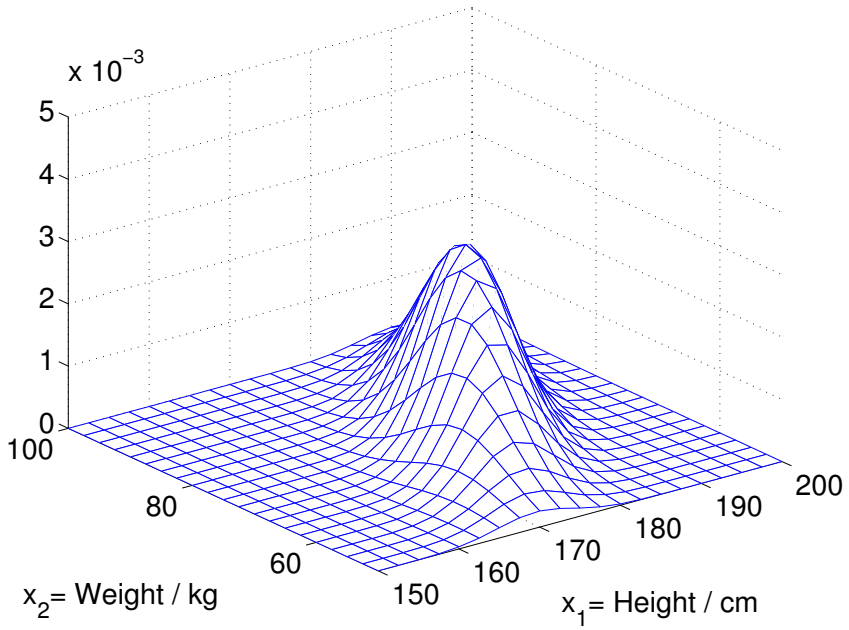


Two Gaussian Feature Probability Density Functions

Women

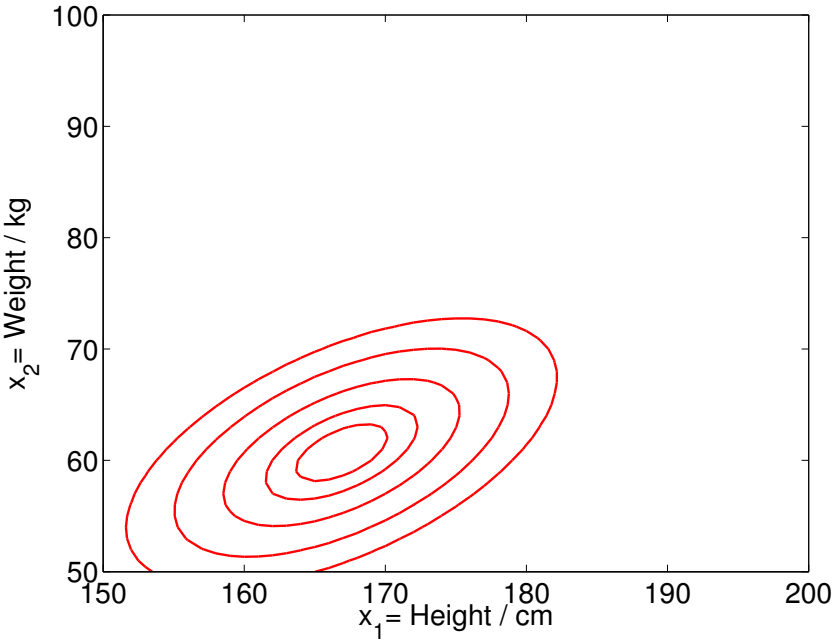


Men

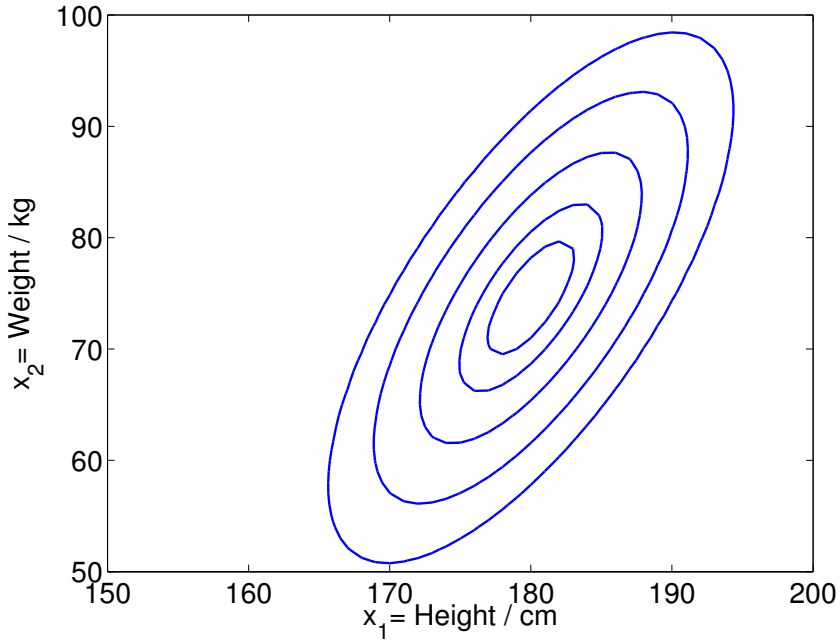


Contours of Equal Probability Density

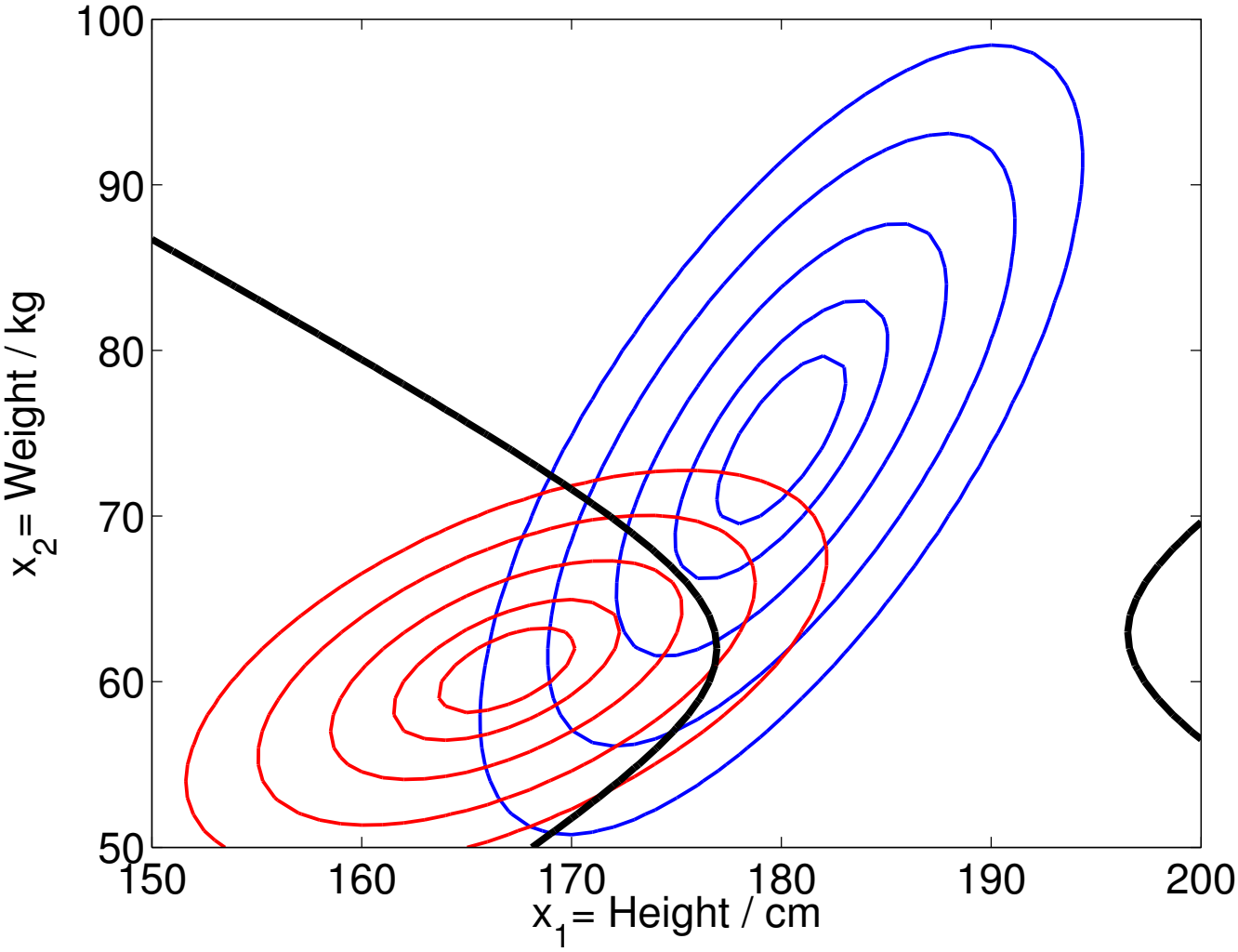
Women



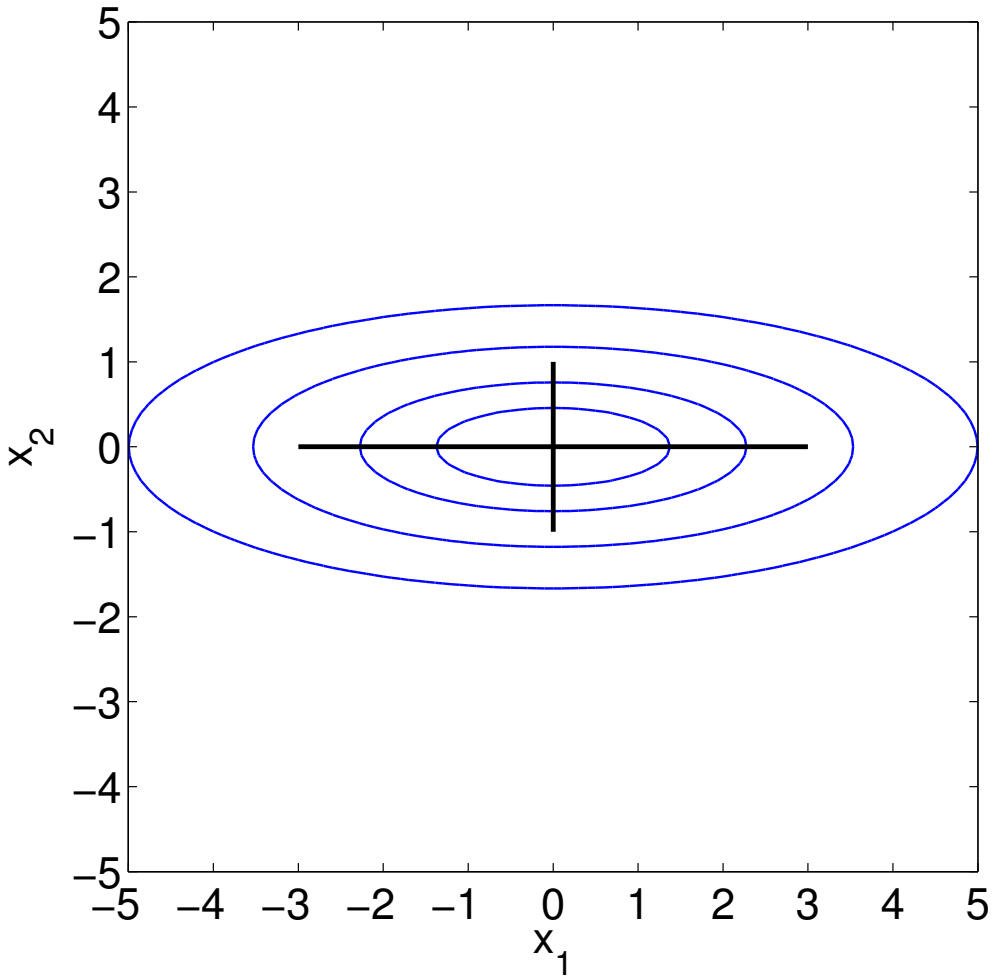
Men



Decision Boundary and Decision Regions

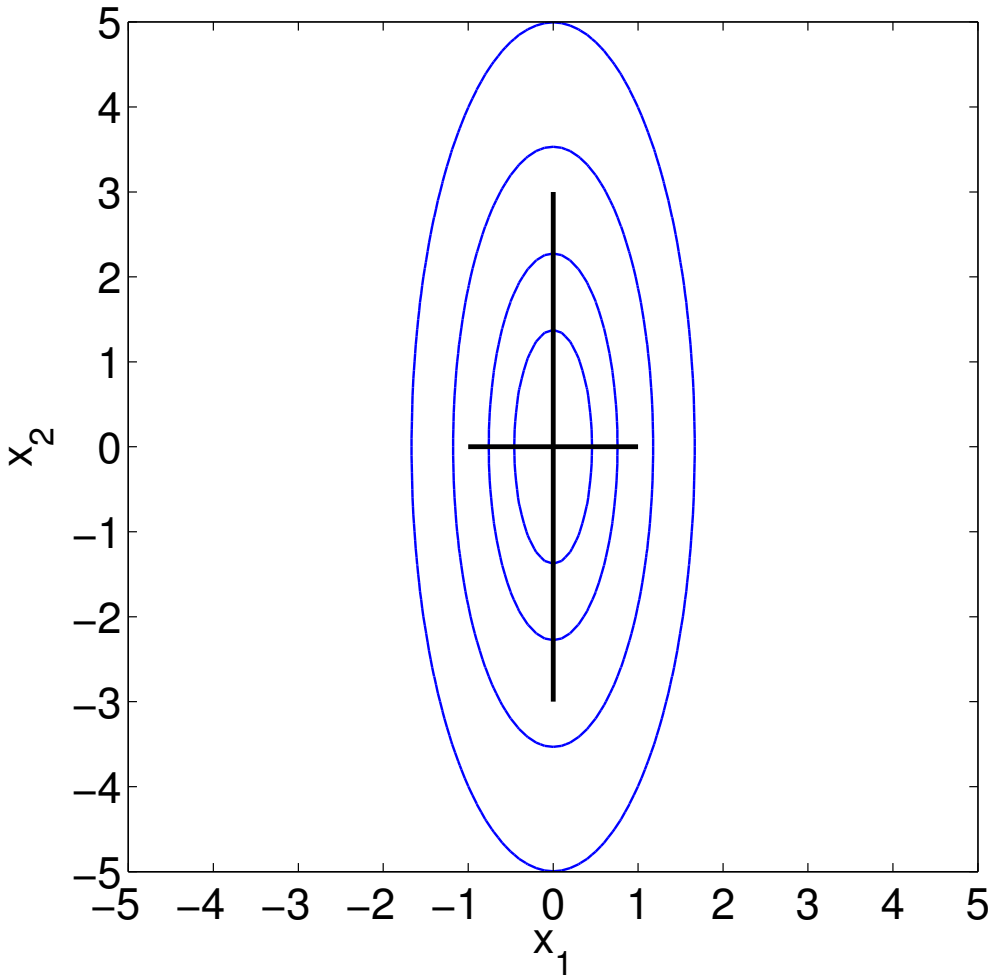


Gaussian Probability Density (2-dim)



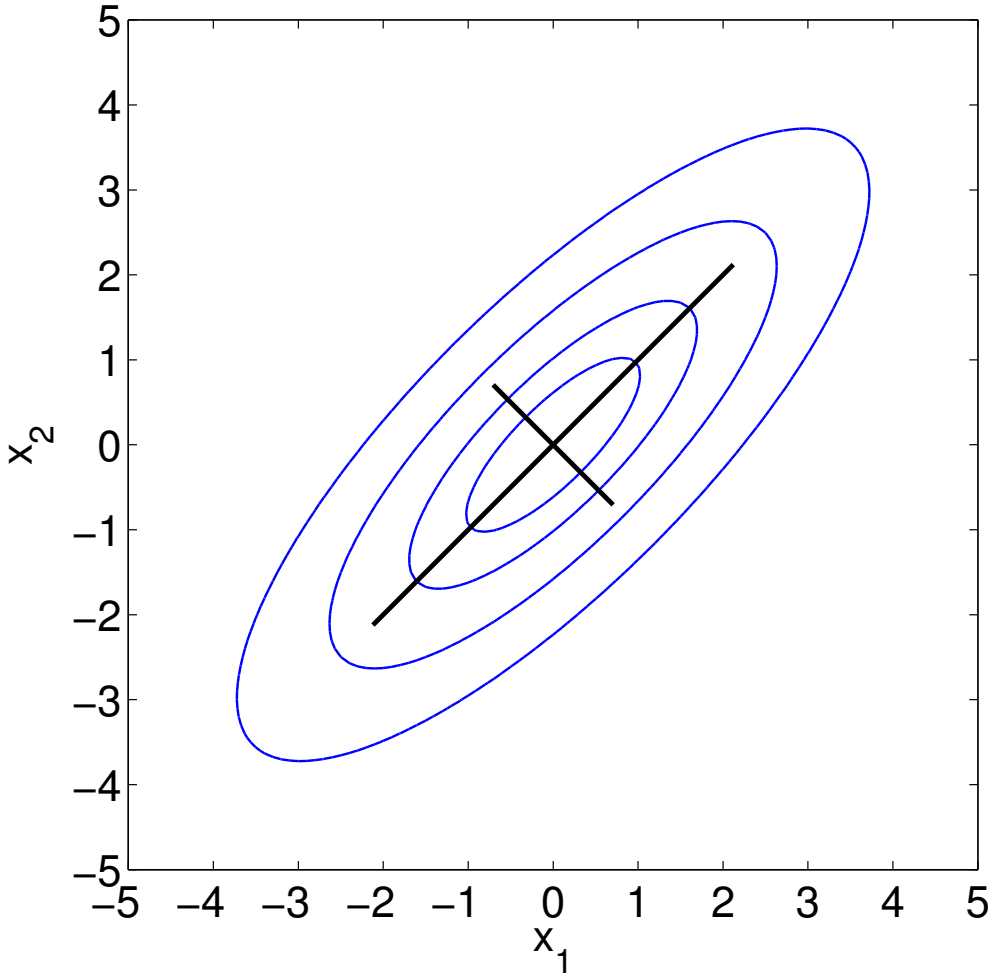
$$\text{cov}\,[X] = \begin{pmatrix} 3^2 & 0 \\ 0 & 1^2 \end{pmatrix} = \begin{pmatrix} 9 & 0 \\ 0 & 1 \end{pmatrix}$$

Gaussian Probability Density (2-dim)



$$\text{cov}\,[X] = \begin{pmatrix} 1^2 & 0 \\ 0 & 3^2 \end{pmatrix} = \\ = \begin{pmatrix} 1 & 0 \\ 0 & 9 \end{pmatrix}$$

Gaussian Probability Density (2-dim)



$$\text{cov}\,[X] = P \begin{pmatrix} 3^2 & 0 \\ 0 & 1^2 \end{pmatrix} P^T = \\ = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$$

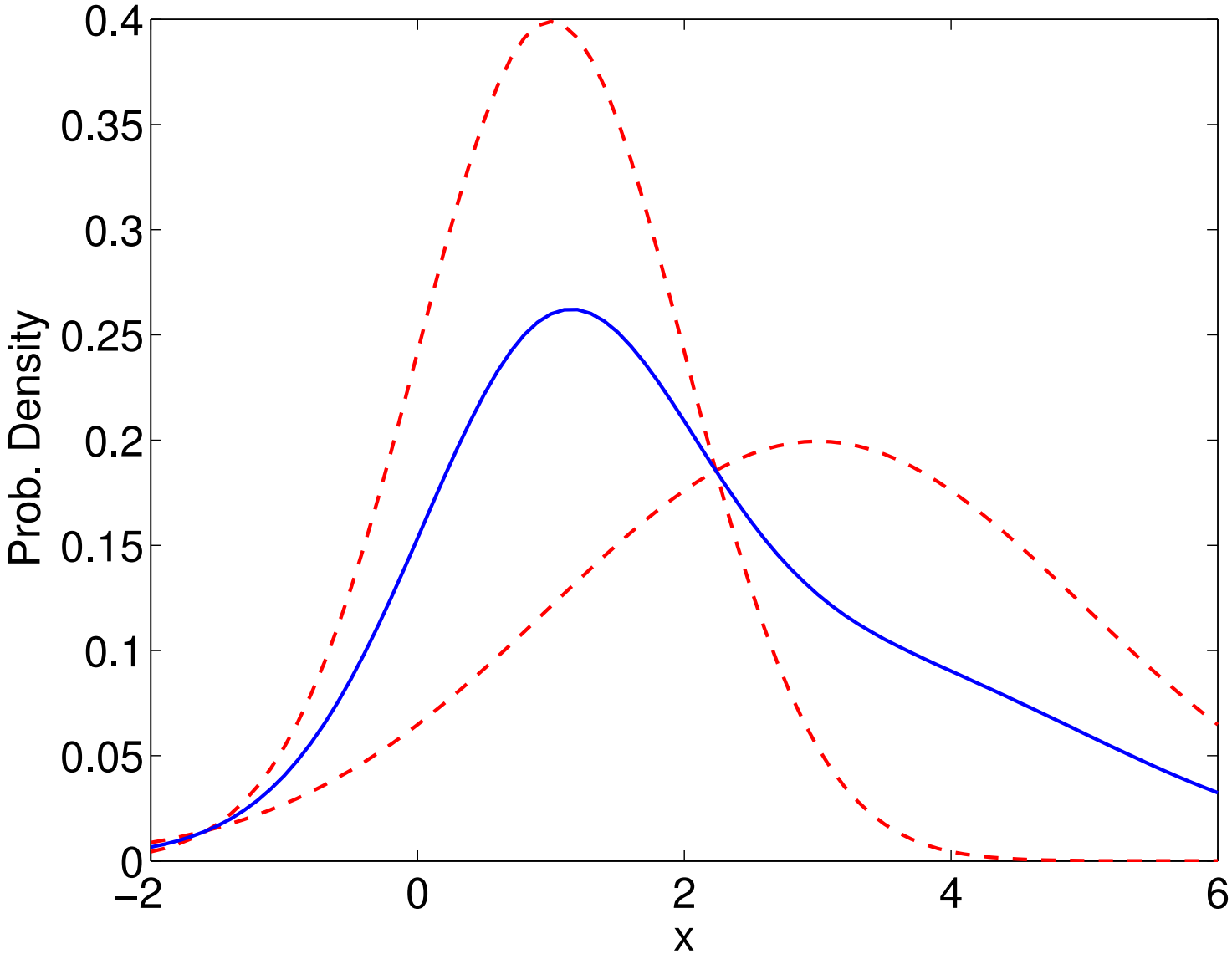
where

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

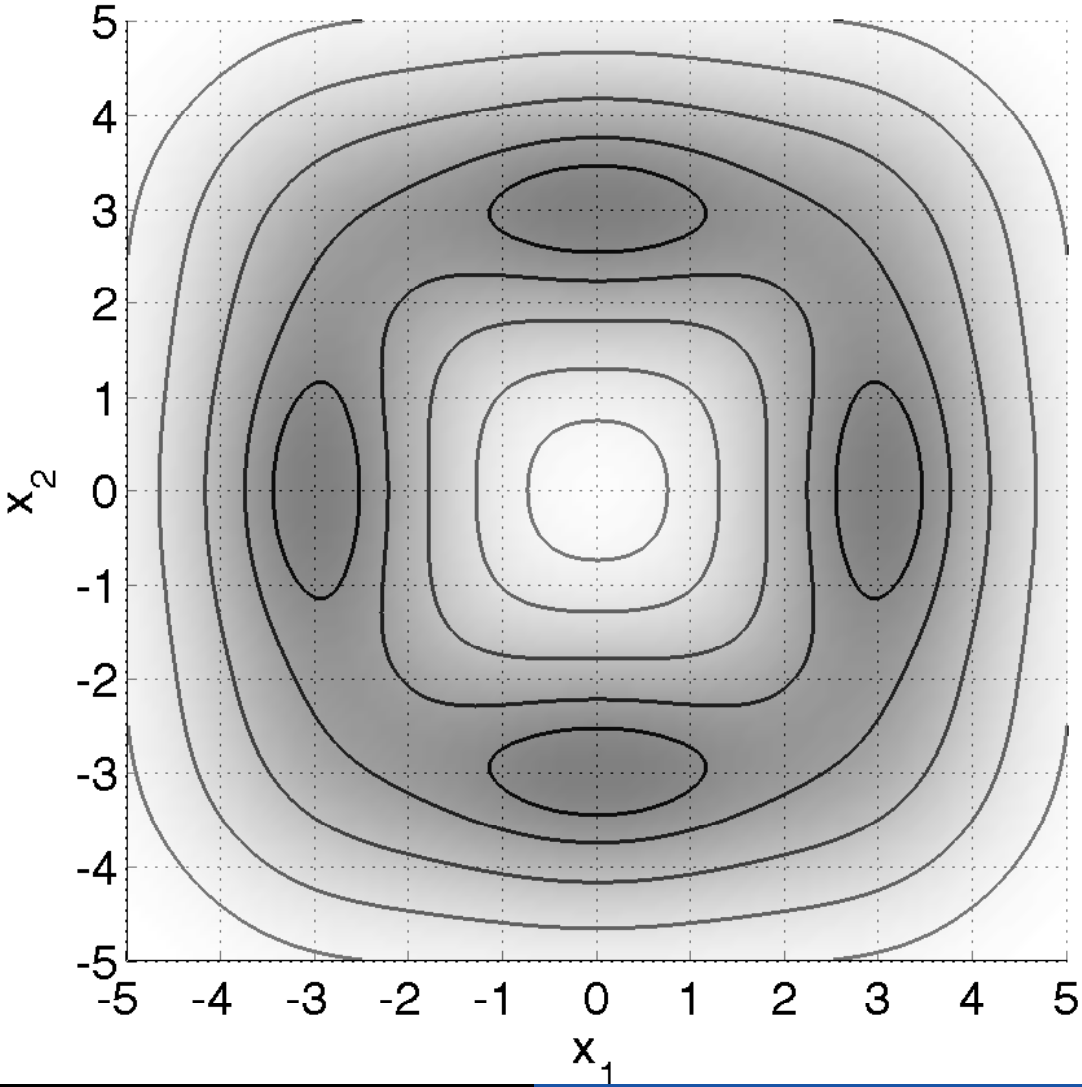
Gaussian Probability Density (K-dim)

$$f_X(x) = \frac{1}{(2\pi)^{K/2}\sqrt{\det C}} e^{-\frac{1}{2}(x-\boldsymbol{\mu})^T C^{-1}(x-\boldsymbol{\mu})}$$

General Probability Density: Gaussian Mixture (1-dim)



General Probability Density: Gaussian Mixture (2-dim)



General Probability Density: Gaussian Mixture (K-dim)

$$f_X(x) = \sum_{m=1}^{M} w_m \frac{1}{(2\pi)^{K/2}\sqrt{\det C_m}} e^{-\frac{1}{2}(x-\boldsymbol{\mu}_m)^T C_m^{-1}(x-\boldsymbol{\mu}_m)}$$

Summary: Important Concepts

Feature Vector: vector x containing all input data to the classifier

Observation Space: set of all possible feature-vector values

Decision Function: $d(x)$ maps feature vector x into discrete decision output, coded as integer

Decision Region: set of feature-vector values giving same decision: $\Omega_i = \{x : d(x) = i\}$

Discriminant Function: $g_i(x)$ maps any feature vector x into real number, used for classification

General Optimal Classifier: $d(x) = \text{argmax}_k \, g_k(x)$

Two-category Classifier: can be implemented using a single Discriminant Function: $d(x) = \text{sgn} \, g(x)$

Gaussian Mixture Model: can approximate any distribution