

Analysis of Metabolic Dynamics in *Saccharomyces cerevisiae*

SA104X Spring 12 - Degree Project in Engineering Physics, First Level
Royal Institute of Technology (KTH), Department of Mathematics

Alexander Axelsson, alax@kth.se
Maria Båkestad, bmaria@kth.se
Viktor Kull, vkull@kth.se
Jesper Welander, jeswel@kth.se

Supervisors: Hildur Æsa Oddsdóttir, Xiaoming Hu

This thesis conducts a study on the stability of steady states in the glycolysis of in silico models of *Saccharomyces cerevisiae*. Computational models can in theory reach steady states which are unstable and thus would be unlikely to stay steady in nature. Little work has previously been done to check whether the calculated steady states are actually stable or not. This thesis investigates the stability of some of the possible steady states.

The glycolysis is modelled as a system of nonlinear differential equations. This is done by using the rate equations describing the rate of change in concentration of each metabolite involved in glycolysis. By linearizing this system around different steady states and calculating the eigenvalues of the associated jacobian matrices the stability of the steady states can be determined.

Given the large range of possible initial conditions which result in different steady states, a few physiologically feasible ones are chosen to be the subject of this study. A steady state is stable if all the eigenvalues of the Jacobian matrix are negative. By making use of SBML (System Biology Markup Language) databases of *S.cerevisiae*, existing models are converted into matlab code and simulated.

For comparison a separate model was also studied. This model is based on in vivo and in vitro analysis made on the cell. The SBML system does not have strictly negative eigenvalues: some of them are zero. This means that linear analysis cannot determine whether the system is stable. Nonlinear terms must be considered too. The non-SBML model reaches a steady state with only negative eigenvalues. However this system needs additional external constraints to reach a steady state. We suspect these constraints are contributing to the stability.

KUNGLIGA TEKNISKA HÖGSKOLAN

INSTITUTIONEN FÖR MATEMATISK STATISTIK

Multipel regression av energiindex

Deltagare:

Magnus Bergroth,
bergrot@kth.se

Alexander Keder,
keder@kth.se

Handledare:

Harald Lang

I denna rapport undersöks ett amerikanskt energiindex genom regressionsanalys. Modellerna som framställs genom regression är baserade på data hämtad från börsen och anges i procentuell ändring respektive absolut ändring. Validitet för varje enskild kovariat testas och en risknivå avgör om en kovariat skall inkluderas i modellen eller inte. Detta leder till två modeller med bättre kurvanpassning till den givna mängd data över det valda energiindexet. Målet med regressionsanalysen är att med givet data framställa en kurvanpassning som bäst beskriver den valda regressorn. De tester som utfördes för relativa värden påvisade en korrelation mellan energiindex och kovariater för råvaror och större marknadsindex, medan för absoluta värden påvisades en korrelation mellan energiindex och nästan samtliga kovariater.

Mathematical modeling of flocking behavior

Optimization and Systems Theory

Authors:

Richard Zauer,
Sebastian Bremberg,
Oscar Blomkvist

Supervisors:

Xiaoming Hu
Yuecheng Yang

Abstract

In this thesis the flocking behaviour of prey, when threatened by a group of predators, is investigated using dynamical systems. By implementing the unicycle model, a simulation is created using Simulink and Matlab. A set of forces are set up to describe the state of the prey, that in turn determines their behaviour in different scenarios. An effective strategy is found so that all members of the flock can survive the predator attack, taking into account the advantages of the predator's greater translational velocity and the prey's higher angular velocity. Multiple obstacles and an energy constraint are added to make the model more realistic. The objective of this thesis is to develop a strategy that maximizes the chance of survival of each flock member by not only staying together in a group but also making use of environmental advantages.

Trajectory Tracking for Mobile Manipulators:

A Comparison of Control Laws and Optimization
of the Reachable Workspace

Authors:

Mikael Brudfors, Clara Lynn & Robert Rogersten

Supervisors:

Xiaoming Hu & Johan Markdahl

Department of Mathematics,
Optimization and Systems Theory

Abstract

This report investigates how mobile manipulators can be controlled when an end-effector is required to follow some certain trajectory. The robot platform is of type (2,0) and is restricted to planar motion and under non-holonomic constraints. The manipulator of the robot are modeled as a series of chains, connected by revolute joints. Mathematical theory are presented for both the maximum working range and the reachable workspace of the manipulator. Three different control laws; nonlinear dynamic system with linear and nonlinear feedback control and input/output linearization are derived. Finally, the performance of these control laws are compared in simulations by analysis of their trajectory tracking errors and their stability.

Kandidatexamsarbete

By Anton Lund & Marcus Cordi

On fitting the multifractal model to the SEK/USD exchange rate

Abstract

Understanding the processes that determine price variations is important in determining risks in the financial system. Many of the conventional models used to model price variations are based on the Brownian motion, although it does not catch large price deviations, dependence and clustering that are present in financial market data. Several attempts corrections of this model have been made (e.g. GARCH). This thesis attempts to explain an alternative method, Multifractal Model of Asset Returns (MMAR), based mainly on the papers by B. Mandelbrot A. Fisher and L. Calvet. MMAR allows for large price deviations, clustering and dependence of price variation. The theoretical framework is reviewed and then tested empirically on the SEK/USD exchange rate.

KANDIDATEXAMENSARBETE I TEKNISK FYSIK
INSTITUTIONEN FÖR MATEMATIK, KTH

Inre biljarder och tvistavbildningar

Författare:

Viktor Ek

Handledare:

Maria Saprykina

Sammanfattning

Denna rapport diskuterar inre biljarder, som är studiet av rörelsen hos en punktpartikel som rör sig inom ett begränsat område och reflekteras mot områdets rand utan att tappa i fart. Först introduceras en lämplig parametrisering av rörelsen. Sedan införs biljardavbildningen, som beskriver hur man går från en studspunkt till nästa i rörelsen. Det visar sig att denna avbildning för vissa former på området tillhör en väl studerad typ av avbildning, en så kallad tvistavbildning.

Dessutom diskuteras vilka periodiska banor som finns när området har olika former och egenskaper och hur man kan hitta dessa med hjälp av den så kallade genererande funktionen. Vidare bevisas Birkhoffs sats för periodiska banor, som beskriver existens av periodiska banor för konvexa områden med glatt rand och positiv krökning.

Manipulability Index Optimization for a Planar Robotic Arm

Max Engardt, Axel Heimbürger & Philip Sydhoff

Supervisors:
Johan Markdahl and Xiaoming Hu

Degree Project in Engineering Physics, First Level, SA104X
Department of Mathematics, Optimization and Systems Theory
Royal Institute of Technology
Stockholm, Sweden

April 29, 2012

Abstract

The subject of robotic manipulability studies the relationship between robotic configurations and their ability to move and interact with objects. It is of importance to understand and optimize the manipulability of robotic structures in order to achieve improved performance. Therefore, manipulability indices are introduced which allows for a quantization of the manipulability. The manipulability generally depends on the planar arm structure and the relative position of the different links in the planar arm.

The report starts off with general theory regarding the robotic arm model and optimization. Thereafter, planar robotic arm manipulators are considered from the viewpoint of optimizing the manipulability. Specifically, some important properties of the manipulability indices are analytically derived. Numerical optimization using MATLAB with different constraints are conducted and a method for calculating an optimal path is defined and performed with satisfactory results. General optimization methods are developed and the specific problem formulations are to be considered as arbitrary examples.

Är det möjligt att förutspå överavkastning?

En statistisk undersökning av svenska global-aktiefonder

Oskar Ericsson
oericss@kth.se

Andreas Lagerqvist
lagerqv@kth.se

SA104X Examensarbete i Teknisk Fysik, Grundläggande nivå
Matematikinstitutionen
Kungliga Tekniska Högskolan
Handledare: Harald Lang

Sammanfattning

Syftet med detta arbete är att försöka beskriva vilka kvantitativa faktorer som kan förklara fonders överavkastning gentemot jämförelseindex. Detta genom att analysera globala aktiefonder noterade i svenska kronor genom statistiska metoder, bland annat minsta kvadrat-skattningar och logistiska regressioner. Den undersökta modellen innehåller variabler för fondförmögenhet, delägarskap i förvaltningsbolaget, snittvärde på P/E-tal hos fondens underliggande aktieinnehav, allokeringsstorlek, omsättningshastighet, aktiv risk, avgift och ålder på fond. Resultaten har en förklaringsstyrka på omkring 66 % och signifikant positiv samband med överavkastning fanns för allokeringsstorlek, aktiv risk och ålder på fond. Medan motsvarande negativa relation fanns för omsättningshastighet och fondavgift. En sammantagen observation av resultaten är att överavkastning verkar finnas hos de fonder som har en stabil investeringsprofil med allokering mot stora bolag och som tidigare har stora avvikeler mot jämförelseindex.

Support Vector Machines for Optimizing Speaker Recognition Problems

Jennie Falk
jennli@kth.se

Gabriella Hultström
ghu@kth.se

Supervisor: Per Enqvist

SA104X, Degree Project in Engineering Physics, First Level
Department of Mathematics, Optimization and Systems Theory

Abstract

Classification of data has many applications, amongst others within the field of speaker recognition. Speaker recognition is the part of speech processing concerned with the task of automatically identifying or verifying speakers using different characteristics of their voices. The main focus in speaker recognition is to find methods that separate data, in order to differentiate between different speakers. In this thesis, such a method is obtained by building a support vector machine. The first version of the support vector machine is used to separate linearly separable data using linear hyperplanes, and it is then modified to separate linearly non-separable data, by allowing some data points to be misclassified. Finally, the support vector machine is improved further, through a generalization to higher dimensions by the use of different kernels and thus higher order hyperplanes. The developed support vector machines is in the end used on a set of speaker recognition data.



Trajectory Planning for a Rigid Body Based on Voronoi Tessellation and Linear-Quadratic Feedback Control

Authors:

Gustav FRIBERG

gfriberg@kth.se

Fredrik WAHLBERG

fwahl@kth.se

Hans FLODIN

hansfl@kth.se

Supervisors:

Johan THUNBERG

johan.thunberg@math.kth.se

Xiaoming HU

hu@kth.se

Degree Project in Engineering Physics, First Level, SA104X
Department of Mathematics, Optimization and Systems Theory
Royal Institute of Technology Stockholm, Sweden

April 29, 2012

Abstract

When a rigid body moves in 3-dimensional space, it is of interest to find a trajectory such that it avoids obstacles. With this report, we create an algorithm that finds such a trajectory for a 6-DOF rigid body. In the trajectory, both the rotation and translation of the rigid body are included.

This is a trajectory planning problem in the space of Euclidean transforms $SE(3)$. Because of the complexity of this problem, we divide it into two consecutive parts; path planning in Euclidean space and control design.

In the first part we create *virtual spheres* surrounding each infeasible region to obtain approximate central points for each obstacle, which are then used as input for our increased and partly redesigned *Voronoi tessellation*. This results with a graph of all feasible paths along the surfaces of convex polytopes containing the central points of the virtual spheres. A simple global graph-search algorithm is then used to find the shortest path between the nodes of the polytopes, which is then further optimized.

During the work process, we also discovered that our path finding method easily could be generalized to be used in n -dimensional space, which we have also done.

In the second part, the control law for the rigid body to follow the rotation and translation trajectory is designed to minimize the cost, this is done by designing a *Linear-Quadratic* feedback loop.

Potential theoretic skeletons

Author: Robert Gelotte
Degree Project SA106X
Department of Mathematics
Supervisor: Björn Gustafsson

Abstract

Potential theoretic skeletons (also called mother bodies) are minimal bodies that generate the same gravitational potential as some given heavy body. They are minimal in the sense that they have Lebesgue measure zero, and must satisfy some additional requirements. A mother body does not necessarily exist for a heavy body, and if it does, it is not necessarily unique. Despite these issues, the study of these objects is motivated by inverse problems of geophysics and Hele-Shaw flows. In this text, mother bodies are studied from a potential theoretic point of view as well as with PDE methods, and the equivalence of these approaches is explained.

Prediction of Stock Return Using Linear Regression

Author: Fredrik Giertz

Under the Guidance of: Harald Lang

Department: KTH Mathematical Statistics

Abstract

In this bachelor thesis multiple linear regression was used to examine the relationship between variables with respect to an oil company index and making predictions of the returns in the latter. The included variables were crude oil spot prices, natural gas spot prices, petrol spot prices and currency exchange rates. The oil company index consisted of oil company equities belonging (8th march 2012) to the S&P 500 Index. Heteroskedasticity was present in the regression results and were proven by using *White's test*. Autocorrelated errors were shown to be present using the *Durbin-Watson test*. The effect of the latter was corrected using *Newlyn-West's autocorrelation consistent standard errors* and the former using *Whites robust errors*. The results show that it is, with a very narrow confidence interval, possible to determine relationships between all the variables and the oil company index. It was concluded that the oil spot price had the largest positive effect on the oil equity index and that the Chinese Renminbi currency had the largest negative effect on the same.

Sammanfattning

In denna kandidatsexamen används multipel linjär regression för att analysera relationen mellan ett antal variabler och ett oljeaktieindex samt göra prediktion av avkastningen i den senare nämnda. Variablerna bestod av petroleumspotpriser, naturgasspotpriser, bensinspotpriser samt valutor. Oljeaktiesindexet bestod av oljeaktier som (den 8:e Mars 2012) återfanns i S&P 500 indexet. Heteroskedasticitet återfanns i regressionsresultatet och bevisades med *White's test* samt korrigeras med hjälp av *White's robust errors*. Autokorrelerade feltermen bevisades med *Durbin Watson's test* och korrigares med hjälp av *Newlyn West's autocorrelation consistent standard errors*. Resultaten visade att det är möjligt att, med mycket hög precision, fastställa relationer mellan variablerna och oljeaktieindexet. Slutsatsen är att petroleumspotpriset hade störst positiv inverkan på avkastningen i oljeaktieindexet och att den kinesiska Renminbi valutan hade störst negativ inverkan på den samma.

Abstract

This report aims to study the condominium prices in central Stockholm and the factors that affect these prices. A linear regression model was set up and data about relevant covariates such as floor area, mortgage rates and where the condominium is situated were gathered to assess if and how they influence the price. The raw data was processed and enhanced to gain as much relevant information as possible to the model. To determine the covariates' influence on the final price, regressions were ran and various tests were performed on the output data. This led to that the covariates were either kept or excluded from the original model depending on the outcome of these tests.

The above resulted in a model where all remaining covariates are significant for the final price of the condominium.

Sammanfattning

Rapportens syfte är att studera bostadsrättspriser i Stockholms innerstad och de faktorer som påverkar dessa priser. En linjär regressionsmodell införs och data om relevanta kovariater såsom golvyta, bolåneräntor och vart bostadsrätten ligger samlades in för att bedöma om och hur de påverkar priset. Rådata behandlades och förbättrades för att få ut så mycket relevant information som möjligt till modellen. För att bestämma kovariaternas påverkan på slutpriset så kördes regressioner och olika test användes på resulterande data. Detta ledde till att kovariaterna antingen behölls eller utesluts från originalmodellen beroende på testresultaten. Ovan resulterade i en modell där alla kvarvarande kovariater är signifikanta för bostadsrättens slutpris.

Titel: Estimation av bostadsrättspriser m.h.a. multipel regressionsanalys

Deltagare: Patrik Gunnvald, Rickard Gunnvald

Handledare: Harald Lang

Institution: Institutionen för Matematik, inriktning Matematisk Statistik



EXAMENSARBETE INOM MASKINTEKNIK, GRUNDNIVÅ,
SA108X, 15 HP

Finansiella Signaler och Marknadsjusterad Aktieavkastning

FÖRFATTARE:

Hannah Lindberg & Patrik Hansson
hannahli@kth.se, pahan@kth.se

HANLEDARE:

Harald Lang
lang@kth.se

KUNGLIGA TEKNISKA HÖGSKOLAN
INSTITUTIONEN FÖR MATEMATISK STATISTIK
2012

Sammanfattning

I det här arbetet har linjär regression använts för att undersöka hur olika fundamentala finansiella nyckeltal påverkar ett företags överavkastning. Undersökningen baseras på observerad data från årsredovisningar och börsvärdet från år 2007-2009. Faktorerna som undersöks är F_SCORE, Book-to-Market-ratio, affärsverksamhet samt företagsstorlek. För att göra detta har även Bootstrap och Logit metoden tillämpats.

Det primära begreppet i vår analys är Piotroskis F_SCORE, relevansen av denna variabel undersöks initialt i en linjär OLS regression på överavkastningen där den visat sig vara starkt positivt korrelerad med överavkastning. Vidare undersöktes även signifikansen av de ingående komponenterna till F_SCORE med liknande analys. Vår modell är inte användbar för prediktioner av framtida överavkastning, utan är tillämpbar för att avgöra hur F_SCORE påverkar aktieutvecklingen under samma år. Mer direkt undersöks data från år t och avkastning år t.

Statistiskt signifikanta resultat har erhölls för affärsår och F_SCORE, de signaler inom F_SCORE som visat statistik signifikans är *ACCRUAL*, $\Delta LEVEL$ samt $\Delta MARGIN$. Resultaten för resterande av undersökta faktorer var inte statistiskt signifikanta och det går därför inte att säga något säkert om dessa.

Vår studie visar att F_SCORE är positivt korrelerad med överavkastning och att F_SCORE påverkar aktieutvecklingen under samma år. Det framgår även att de finansiella signalernas påverkan på framtida aktieavkastning delvis redan absorberats genom dess korrelation till historiska signaler.

THE ROYAL INSTITUTE OF TECHNOLOGY (KTH)

DEPARTMENT OF MATHEMATICS

BACHELOR THESIS, SA104X

On the asymptotics of models for a homogeneous and isotropic spacetime

Ludvig HULT

Eric LARSSON

Supervisor:

Hans Ringström

Abstract

We present an analysis of the asymptotics of two cosmological models: One with the standard fluid matter model, describing dust and radiation, and one with Vlasov matter. We assume that the cosmological principle holds, and work with spacetimes that model an isotropic, homogeneous and flat universe. We determine the conditions under which the two models have similar asymptotics, both past and future, and to what extent the models are equivalent. We also determine the difference in asymptotics of the models.

We conclude by discussing some physical implications of the results.

A MARKOV CHAIN APPROACH TO MONETARY POLICY DECISION MAKING

Marcus Erik
Josefsson Rasmusson
mjose@kth.se erikras@kth.se

April 29, 2012

SA104X Degree Project in Engineering Physics, First Level
Department of Mathematics
Royal Institute of Technology (KTH)
Supervisor: Tobias Rydén

Abstract

Through monetary policy, central banks aim to prevent societal costs associated with high or unstable inflation. Forecasts and several other tools are used to provide guidance to this end, as outcomes of interest rate decisions are not fully predictable.

This report presents a statistical approach, viewing the development of the economy as a Markov chain. The economy is thus represented by a finite number of states, composed of inflation and short-term variations in GDP. The Markov property is assumed to hold, that is, the economy moves between states over an appropriately chosen time period and the transition probabilities depend only on the initial state. Using the Markov Decision Process (MDP) framework, the transition probabilities between such states are evaluated using historical data, distinguished by the interest rate decision preceding the transition. Completing the model, a cost of inflation is defined for each state as the deviation from a set target. An optimal policy is then determined as a fixed decision for each state, minimizing the expected average cost incurred while using the model.

The model is evaluated on data from Sweden and the U.S., for periods 1994-2008 and 1972-2008 respectively. The results are assessed by the estimated transition probabilities as well as by the optimal policy suggested. While the Swedish observations are concluded to be too few in number to render valuable results, outcomes using the U.S. data agree in most aspects with what would have been expected from macroeconomic theory. In conclusion, the results suggest that the model can be applied to the problem, granted sufficient data is available for reliable transition probabilities to be estimated.

Virvlars rörelse i två dimensioner

Sammanfattning

Aku Kammonen och Marc Tobal
kammo@kth.se tobal@kth.se
Kandidatexamensarbete SA106X
KTH Matematik

Handledare och examinator: professor Björn Gustafsson

Innan begreppet punktvirvel introduceras gäs allmän flödesmekanik igenom. Av speciellt intresse är Eulers ekvationer som används flitigt inom tillämpad flödesmekanik. För att härleda Eulers ekvationer gäs tre satser igenom, som behöver uppfyllas. I flödesmekaniken dyker behovet av att kunna derivera med avseende på både position och tid, därav blir materialderivatan ett naturligt redskap.

När vi talar om flöden stöter vi ofta på benämningen vorticitet. Vorticitetens förhållande till flödets hastighetsfält studeras; även beräkningen av vorticiteten studeras genom att titta på komponenterna av hastighetsfältet. Olika typer av virvelkoncept såsom virveltub, virvelfilament och virvelträd gäs även igenom. Slutligen behandlas begreppet punktvirvel, som är en diskret virvelapproximation med styrka i en punkt.

Punktvirvlarnas rörelseekvationer—system av ickelinjära första ordningens ODE:er—beskrivs i både komplex och reell form. De uttrycker vardera virvels hastighet i form av de andra virvlarnas positioner och styrkor. Ett exempel ges där två virvlar, i ett plan utan vare sig ränder eller hål, rör sig längs en linje vilket leder fram till en sats som säger att två virvlar i planet utan ränder eller hål följer en linje om och endast om de har samma styrka till beloppet men motsatt tecken. Satsen bevisas sedan.

Hamiltonfunktionen och Hamiltons ekvationer introduceras först ur ett allmänt perspektiv för att sedan snäva in på de punktvirvelspecifika Hamiltonekvationerna. Energins bevarande i ett punktvirvelsystem visas genom tidsderivering av Hamiltonfunktionen. Med hjälp av Noethers sats visas att virvelrörelsemängdens och virvelrörelsemängdsmomentets bevarande impliceras av translations- och rotationsinvarians.

Genom en rad exempel illustreras var vorticitetscentret befinner sig i ett system av punktvirvlar.

Ett system av punktvirvlar kan utvecklas självliknande. Om virvelpositionerna uttrycks på komplex form är det då intressant att känna till deras fas och amplitud. Dessa lösas ut men en skalfaktor lämnas obestämd. En känd självliknande utveckling är Kimuras så kallade trippelkollision: tre virvlar kolliderar och det blir stiltje. Trippelkollisionen simuleras i simulatorn som utvecklats parallellt med rapportskrivandet, men det visar sig att de tre punktvirvlarna kommer ut igen från kollisionspunkten. Detta antas hända då Matlabs ODE45, som används, når sin feltoleransgräns. Det bör även nämnas att stiltjelösningen är svag och det väldigt lätt uppstår turbulens.

Relativa jämviktspositioner för virvlar studeras med moderna metoder framtagna av Aref. Virvelpositionerna kan associeras med ett genererande polynom vars rötter är just virveljämviktspositionerna de själva. Slutligen visas ett antal intressanta specialfall. Av speciellt intresse är de positioner som ger jämvikt då punktvirvlarna antingen befinner sig på en linje eller i hörnen av en polygon.

Modelling the Dynamics of Cooperative Predators

Fredrik Lannsjö, Lionel Nurweze, Kristina Sörensen

Supervisors: Xiaoming Hu, Yuecheng Yang

Spring, 2012

Abstract

Modelling the behaviour of flocks is an established field in the mathematical world. A related, but so far relatively unexplored area, the modelling of predators hunting flocks of animals is the main focus of this thesis. The modelling is based on biological studies of the behaviour of specific predators and their preys, in this case lions and zebras. Taking the biological data in consideration, general rules are established in order to model the overall hunting and escape strategies, with a focus on the former. The dynamics of the predator during the hunt are modelled using the unicycle model applied locally, i.e., applied to each agent. Subsequently a computer simulation based on the mathematical models is constructed and numerous simulations performed. The obtained data of hunting outcomes are then compared to the biological data obtained from published field studies to assess the validity of the model.

Department of Mathematics, Optimization and Systems Theory
Royal Institute of Technology, Stockholm, Sweden

Feature extraction of nasal speech with the pole-zero model

Máté Szekér Rickard Norlander
szeker@kth.se norla@kth.se

May 2012

SA104X Bachelor Thesis in Engineering Physics
Department of Mathematics
Royal Institute of Technology (KTH)
Supervisors: Per Enqvist, Anders Möller

Abstract

It is believed that nasality has low intraspeaker and high interspeaker variation, and that pole-zero filters are good at capturing the nasal characteristics of speech. In this bachelor thesis, we performed a comparison between different methods of fitting pole-zero filters to data, and their usefulness for nasality-based speaker recognition in particular. We describe the speech production system to the extent that it is possible to understand the models commonly used. Basic concepts of optimization and signal processing will be explained. We describe to the extent possible, the theory underpinning the various methods, and then compare them in various ways. We used simulated speech data, to see to what extent the methods were able to come up with good estimates of the known filters, when noise had been introduced. Another way we compared the methods was to extract various features with the help of the computed filters, and determine how well we could classify real speech data from the Waxholm speech corpus using the extracted features.

When we compared the models on nasalized speech by determining their frequency responses, we found evidence that the number of zeros influence the shape and number of antiformants, but had little effect on the frequencies of the formant peaks.