# Last lecture (1)

- Course info
- Definition of plasma
- Solar interior and atmosphere

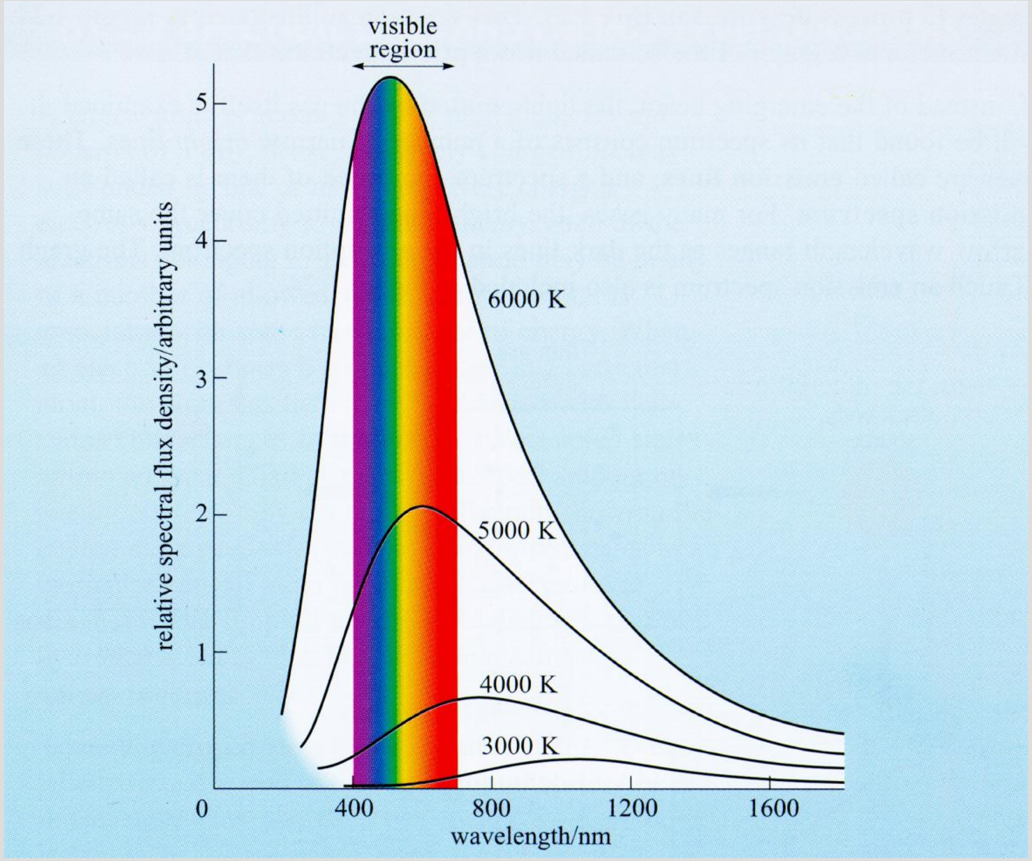
# Today's lecture (2)

- Plasma physics I
- Solar activity

# Today

| Activity | Date | Time | Room | Subject | Litterature |
|---|---|---|---|---|---|
| L1 | 28/8 | 15-17 | Q21 | Course description, Introduction, The Sun 1 | **CGF** Ch 1.1,1.2, 1.4, 5, (p 110-113), 6.3 |
| L2 | 29/8 | 13-15 | Q2 | The Sun 2, Plasma physics 1 | **CGF** Ch 1.3, 5 (p 114-121) |
| L3 | 4/9 | 10-12 | E2 | Solar wind, The ionosphere and atmosphere 1, Plasma physics 2 | **CGF** Ch 6.1, 2, 3.1-3.2, 3.5, **LL** Ch III, Extra material |
| T1 | 6/9 | 8-10 | Q21 | Mini-group work 1 | |
| L4 | 6/9 | 15-17 | Q2 | The ionosphere 2, Plasma physics 3 | **CGF** Ch 3.4, 3.7, 3.8 |
| T2 | 10/9 | 15-17 | Q21 | Mini-group work 2 | |
| L5 | 11/9 | 10-12 | E3 | The Earth's magnetosphere 1, Plasma physics 4 | **CGF** 4-1-4.3, **LL** Ch I, II, IV.A |
| T3 | 17/9 | 8-10 | Q21 | Mini-group work 3 | |
| L6 | 18/9 | 13-15 | Q33 | The Earth's magnetosphere 2, Other magnetospheres | **CGF** Ch 4.6-4.9, **LL** Ch V. |
| L7 | 19/9 | 13-15 | Q2 | Aurora, Measurement methods in space plasmas and data analysis 1 | **CGF** Ch 4.5, 10**,** **LL** Ch VI, Extra material |
| T4 | 24/9 | 8-10 | Q2 | Mini-group work 4 | |
| L8 | 24/9 | 15-17 | V3 | Space weather and geomagnetic storms | **CGF** Ch 4.4, **LL** Ch IV.B-C, VII.A-C |
| T5 | 2/10 | 8-10 | Q31 | Mini-group work 5 | |
| L9 | 2/10 | 13-15 | Q2 | Alfvén waves, Interstellar and intergalactic plasma, Cosmic radiation | **CGF** Ch 7-9, Extra material |
| T6 | 8/10 | 15-17 | Q21 | | |
| L10 | 9/10 | 10-12 | Q2 | Guest Lecture by Swedish astronaut Christer Fuglesang | |
| Written examination | 16/10 | 14-19 | L21, L22, L31 | | |

# Black-body radiation




*Black-body good approximation for opaque bodies where emitted light is much more likely to interact with the material of the source than to escape.*

Wien's displacement law

$$\lambda_{peak} = \frac{2.90 \times 10^{-3}}{T}$$

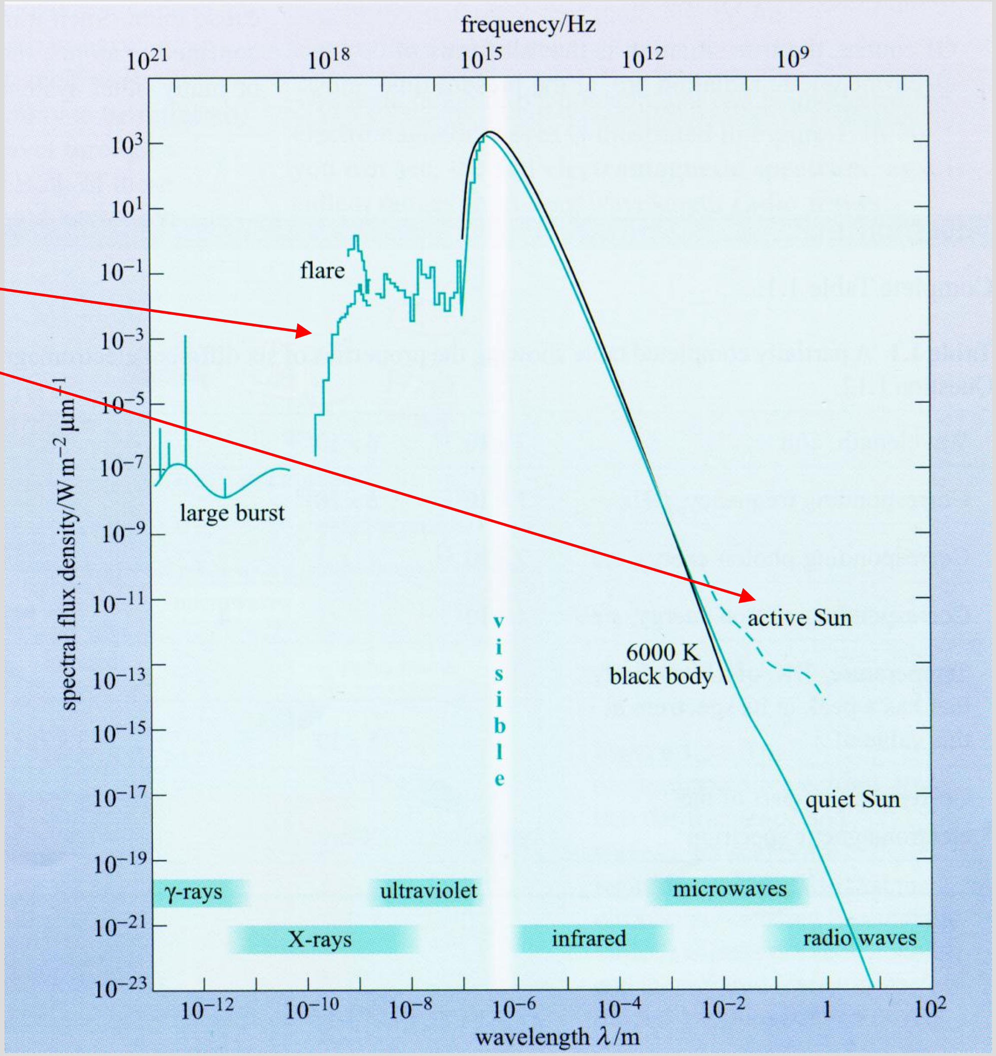
Stefan-Bolzmanns law

$$J = \sigma_{SB} T^4$$

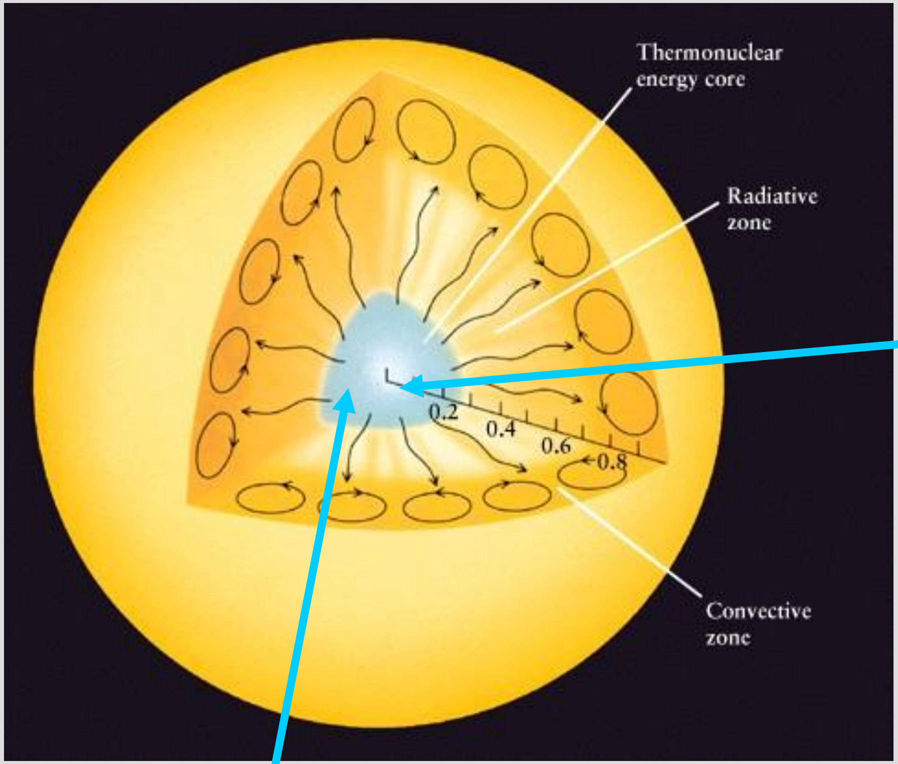
*(J = total energy radiated per unit area per unit time )*

# The solar spectrum

Non-blackbody contributions



# Sun's interior

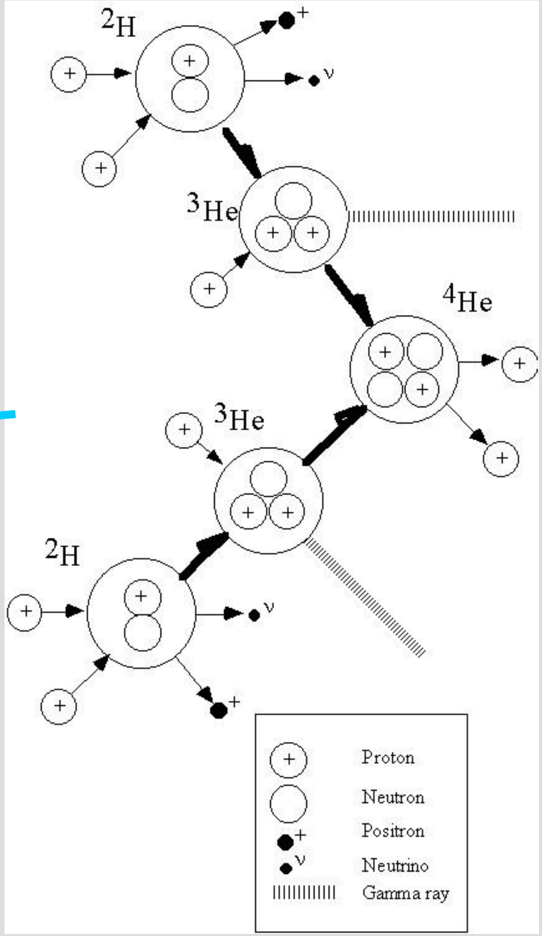



$T = 15 \cdot 10^6$ K

$P = 4 \cdot 10^{26}$ W

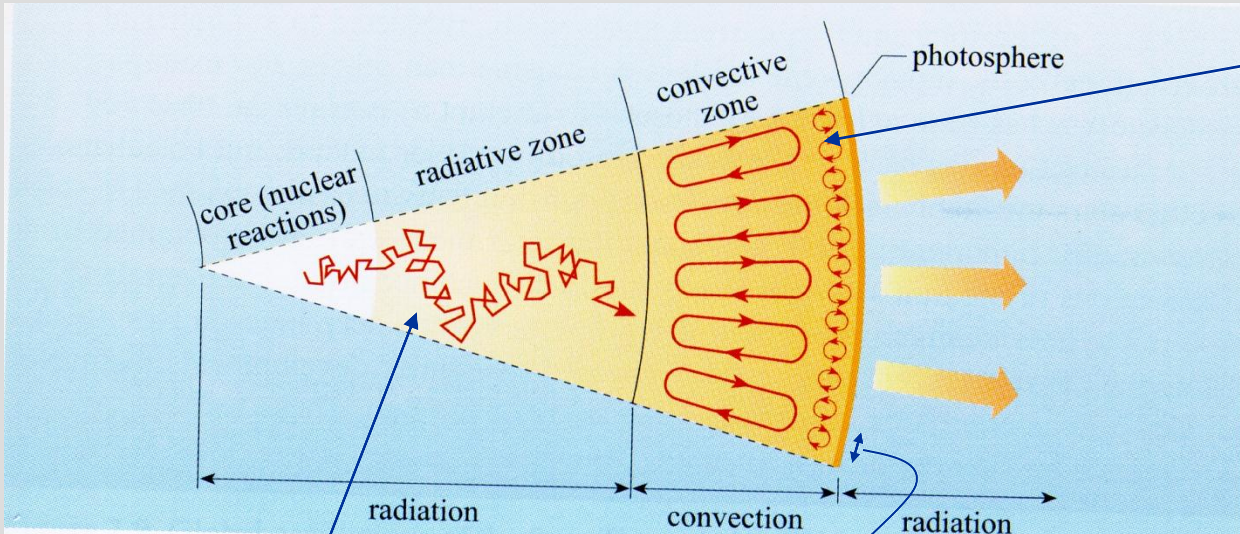
*(P/m ~ 1mW/kg)*

## The proton cycle



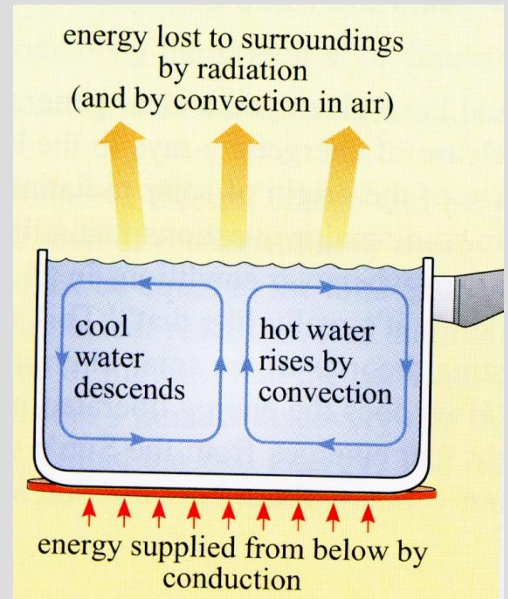

$$4\,{}^1_1\text{H} \rightarrow {}^4_2\text{He} + 2e^+ + 2\nu_e + 2\gamma$$

# Energy transport in the sun




Transport by convection

~1000 km



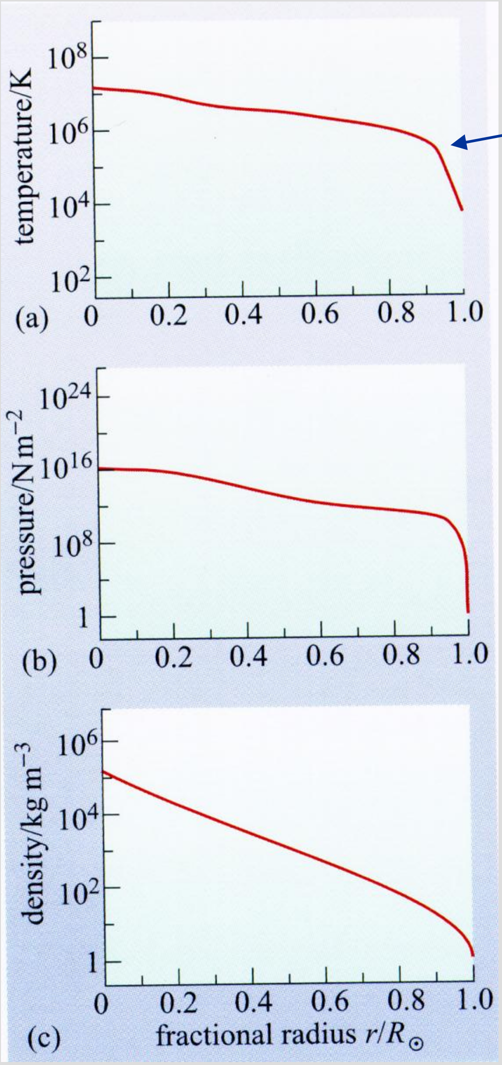

Transport by radiation, which interacts with the dense solar matter (scattering and absorption/re-emission).

I takes on average 200 000 years for a photon to reach the photosphere!

These convection cells are called *granulation*.

At the photosphere the mean free path of the photons becomes so large that they can reach directly out into space.

# Sun's interior




At the photosphere the mean free path of the photons becomes so large that they can reach directly out into space.

As a consequence also the temperature, and pressure drops.

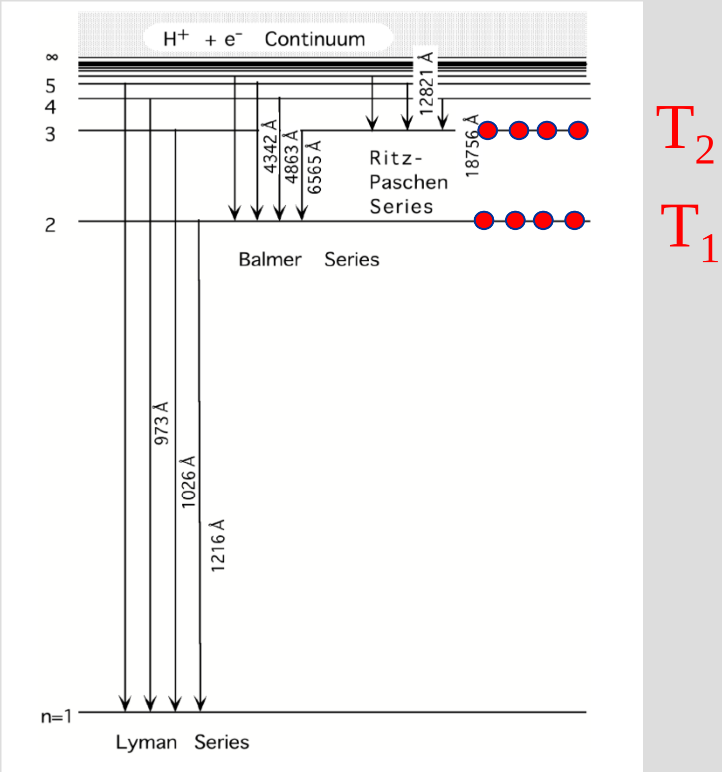
$$p_{pl} = nk_B T$$

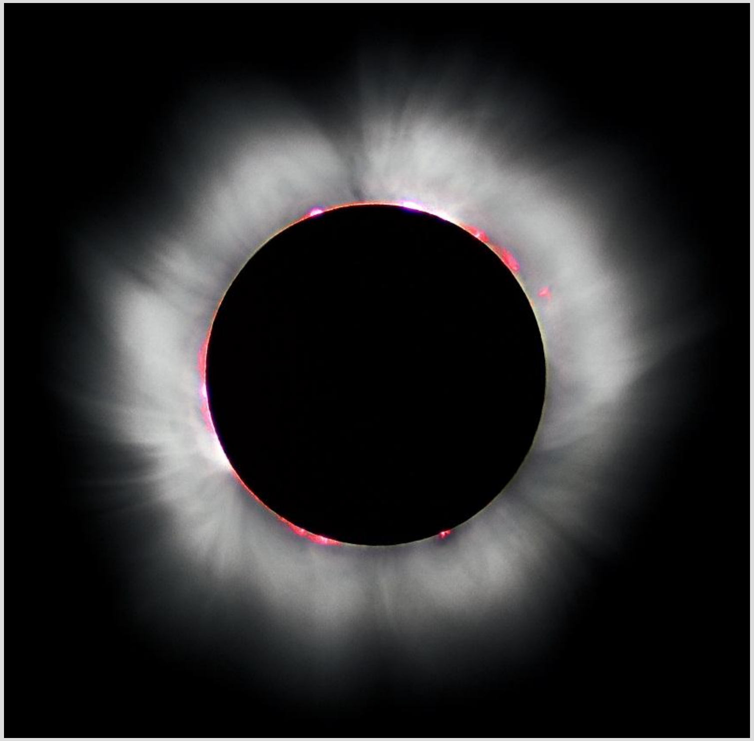
Example of exponential density variation in balance between pressure and gravity

$$\rho_m = const \cdot e^{-z/(k_B T / gm)} = const \cdot e^{-z/H}$$

# Why is the chromosphere red?

Hydrogen spectrum



Hγ
434 nm

Hβ
486 nm

Hα
656 nm

# The layers of the solar atmosphere

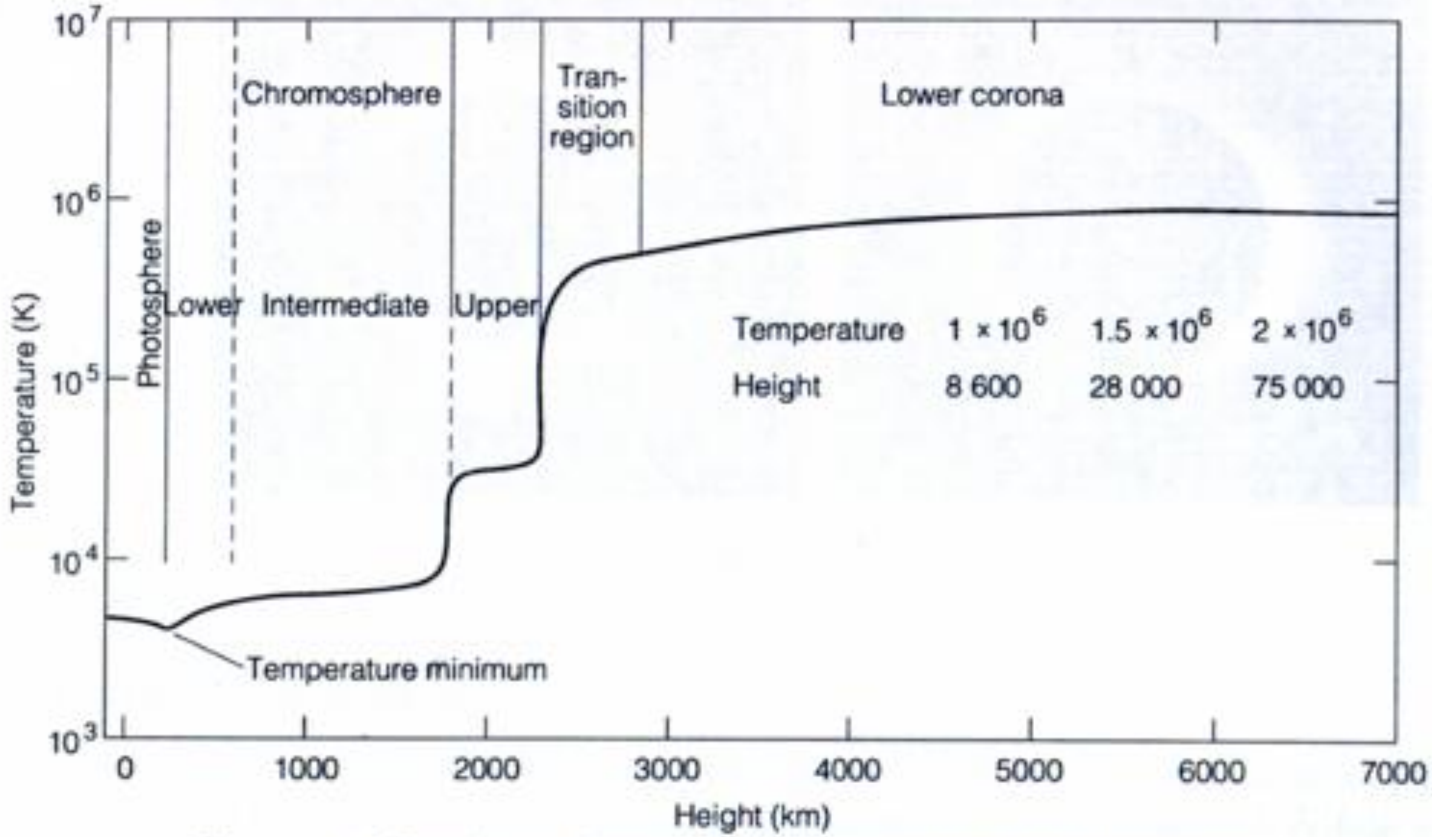



**Figure 5.3.** Distribution of average temperature in the solar atmosphere (Athay 1976).

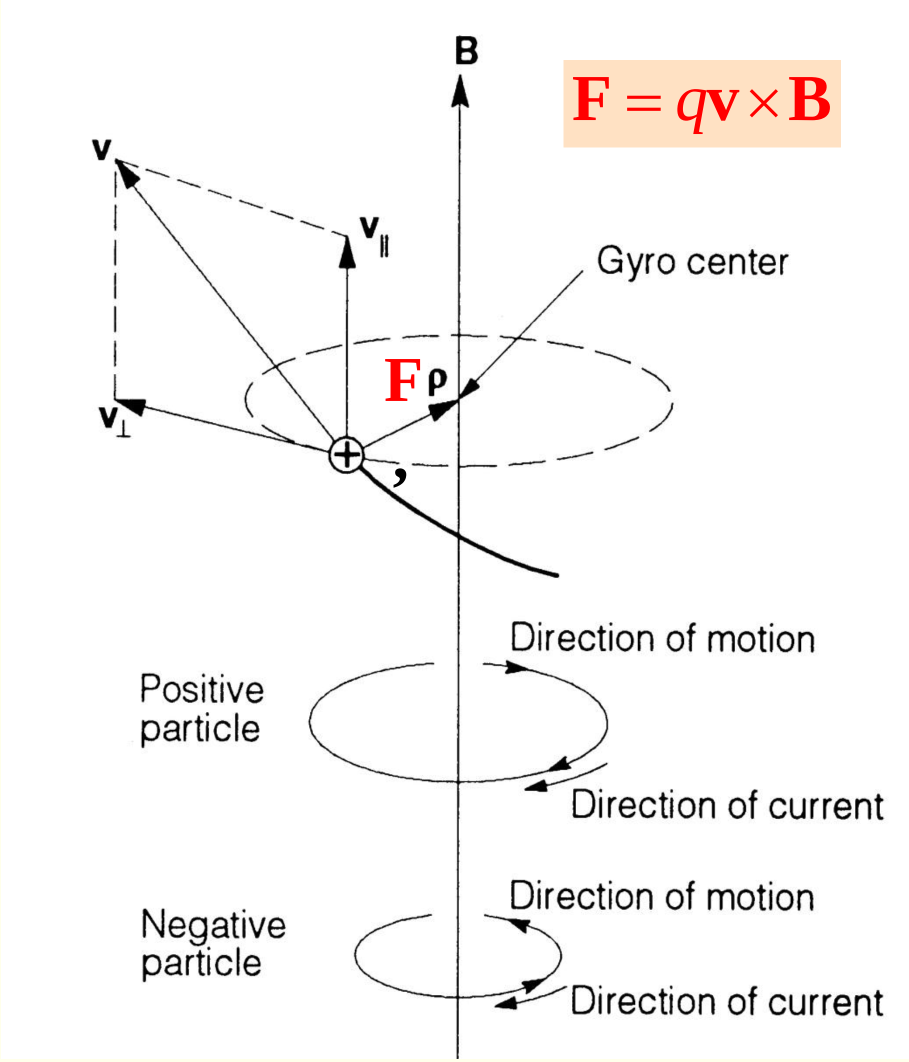
# Coronal loops

*Why does the plasma follow the magnetic field lines?*

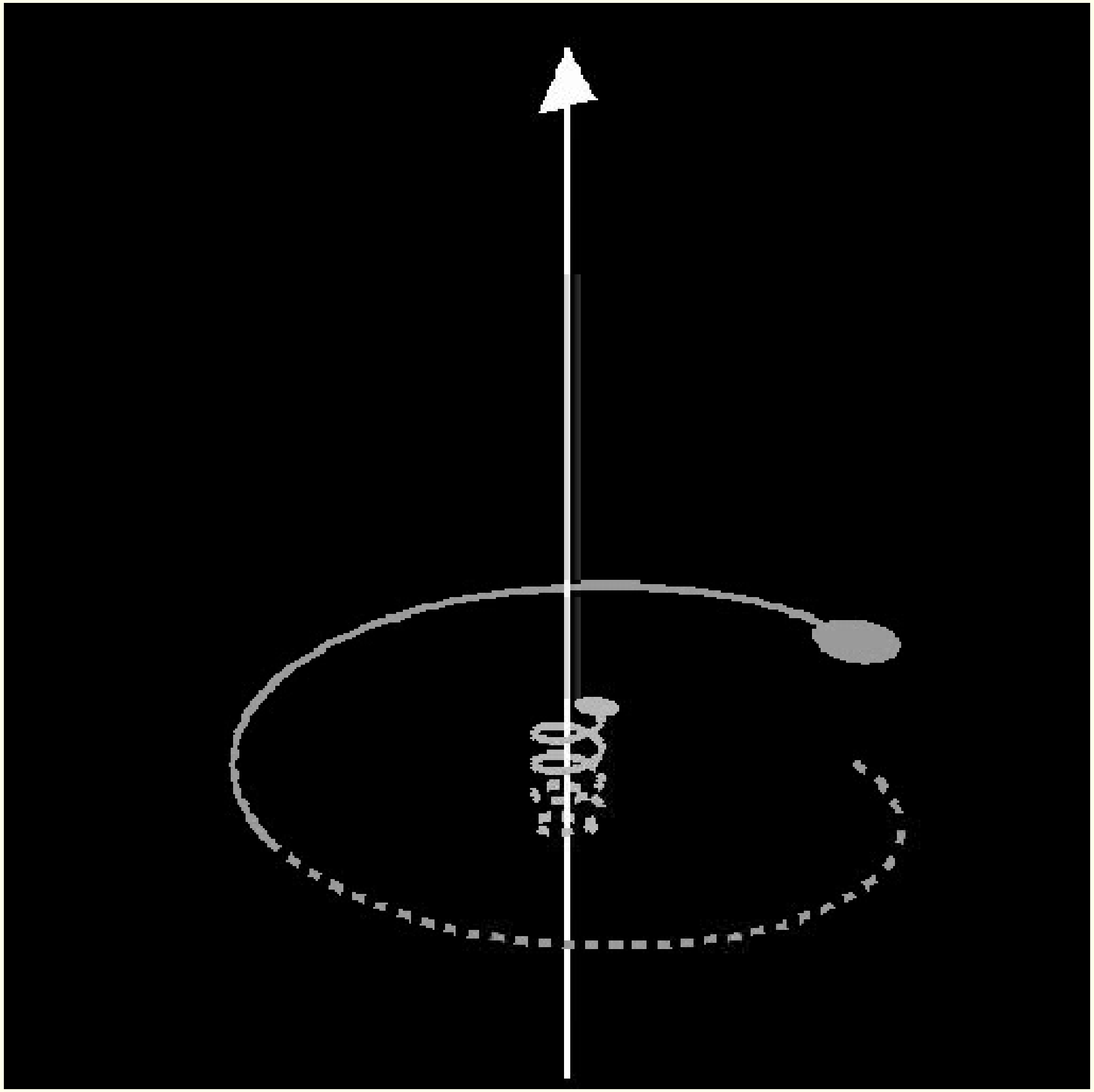
# Magnetized plasma

Extremely common in space.

In single particle description of plasma, the particles gyrate in the plane perpendicular to **B**.

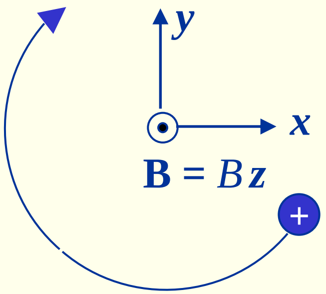


# Gyro motion



# Gyro motion

Consider a positively charged particle in a magnetic field.




Assume that the magnetic field is in the z-direction.

$$m\frac{d\mathbf{v}}{dt} = q\mathbf{v} \times \mathbf{B} \Rightarrow$$

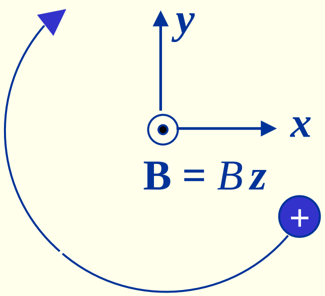
$$\begin{cases} m\dfrac{dv_x}{dt} = qv_yB \\ m\dfrac{dv_y}{dt} = -qv_xB \\ m\dfrac{dv_z}{dt} = 0 \end{cases} \Longrightarrow$$

*Constant velocity along* $\mathbf{z}$

$$\begin{cases} \dfrac{d^2v_x}{dt^2} = \dfrac{qB}{m}\dfrac{dv_y}{dt} = \omega_g\dfrac{dv_y}{dt} = -\omega_g^2 v_x \\ \dfrac{d^2v_y}{dt^2} = -\dfrac{qB}{m}\dfrac{dv_x}{dt} = -\omega_g\dfrac{dv_x}{dt} = -\omega_g^2 v_y \end{cases}$$

$$\Longrightarrow$$

# Gyro motion




$$\begin{cases} \dfrac{d^2 v_x}{dt^2} = -\omega_g^2 v_x \\[2ex] \dfrac{d^2 v_y}{dt^2} = -\omega_g^2 v_y \end{cases}$$

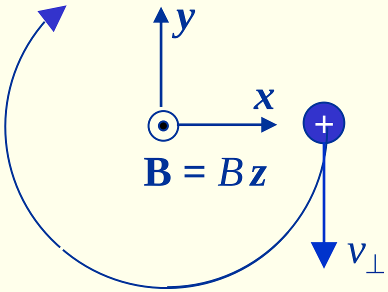
$\Rightarrow$

$$\begin{cases} v_x = Re\left(v_{0x} e^{i(\omega_g t + \delta_x)}\right) = v_{0x} cos(\omega_g t + \delta_x) \\[2ex] v_y = Re\left(v_{0y} e^{i(\omega_g t + \delta_y)}\right) = v_{0y} cos(\omega_g t + \delta_y) \end{cases}$$

and

$$\begin{cases} x = \dfrac{v_{0x}}{\omega_g} sin(\omega_g t + \delta_x) \\[2ex] y = \dfrac{v_{0y}}{\omega_g} sin(\omega_g t + \delta_y) \end{cases}$$

# Gyro motion




$$\begin{cases} v_x = v_{0x}cos(\omega_g t + \delta_x) \\ v_y = v_{0y}cos(\omega_g t + \delta_y) \end{cases}$$

$$\begin{cases} x = \dfrac{v_{0x}}{\omega_g} sin(\omega_g t + \delta_x) \\ y = \dfrac{v_{0y}}{\omega_g} sin(\omega_g t + \delta_y) \end{cases}$$

For a particle starting at time $t$=0 at ($x_0$,0) with velocity (0,-$v_\perp$)
we get (by definition $v_{0x}, v_{0x} > 0$)

$$\begin{cases} v_y(0) = v_{0y}cos\delta_y = -v_\perp & \Rightarrow\ v_{0y} = v_\perp, \delta_y = \pi \\ v_x(0) = v_{0x}cos\delta_x = v_{0x}cos\delta_x = 0 & \Rightarrow\ \delta_x = \dfrac{\pi}{2}, \dfrac{3\pi}{2} \end{cases}$$

and

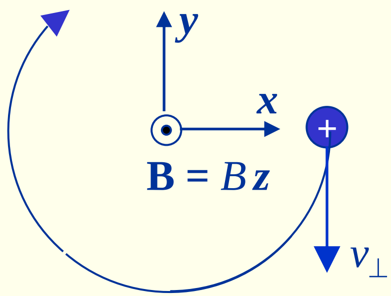
$$\begin{cases} x(0) = \dfrac{v_{0x}}{\omega_g}\sin\delta_x = x_0 & \Rightarrow\ \delta_x = \dfrac{\pi}{2},\ x_0 = \dfrac{v_{0x}}{\omega_g} \\ y(0) = \dfrac{v_\perp}{\omega_g}\sin\pi = 0 & \end{cases}$$

So

$$\begin{cases} v_x = v_{0x}cos\left(\omega_g t + \dfrac{\pi}{2}\right) = -v_{0x}sin(\omega_g t) \\ v_y = v_\perp cos(\omega_g t + \pi) = -v_\perp cos(\omega_g t) \end{cases}$$

$$\begin{cases} x = \dfrac{v_{0x}}{\omega_g} sin\left(\omega_g t + \dfrac{\pi}{2}\right) = \dfrac{v_{0x}}{\omega_g} cos(\omega_g t) = \dfrac{v_{0x}}{\omega_g} cos(-\omega_g t) \\ y = \dfrac{v_\perp}{\omega_g} sin(\omega_g t + \pi) = -\dfrac{v_\perp}{\omega_g} sin(\omega_g) = \dfrac{v_\perp}{\omega_g} sin(-\omega_g t) \end{cases}$$

# Gyro motion




$$\begin{cases} v_x = -v_{0x} \sin(\omega_g t) \\ v_y = -v_\perp \cos(\omega_g t) \end{cases}$$

$$\begin{cases} x = \dfrac{v_{0x}}{\omega_g} \cos(-\omega_g t) \\ y = \dfrac{v_\perp}{\omega_g} \sin(-\omega_g t) \end{cases}$$

Then (because the force is all the time perpendicular to the velocity)

$$v_x{}^2 + v_y{}^2 = v_{0x}{}^2 \sin^2(\omega_g t) + v_\perp{}^2 \cos^2(\omega_g t) = v_\perp{}^2$$

so

$$v_{0x} = v_\perp$$

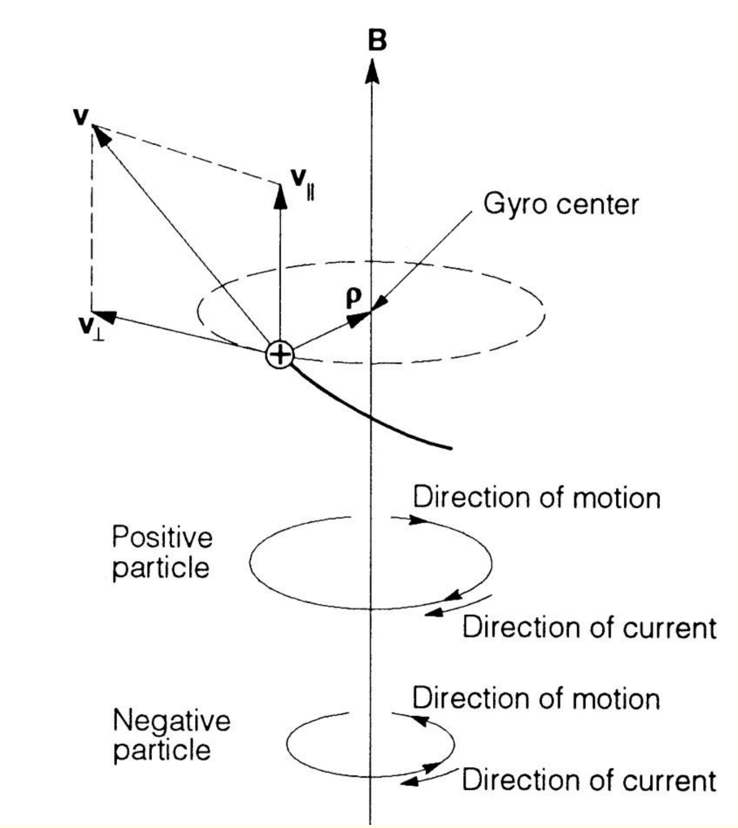
So

$$\begin{cases} x = \dfrac{v_\perp}{\omega_g} \cos(-\omega_g t) \\ y = \dfrac{v_\perp}{\omega_g} \sin(-\omega_g t) \end{cases}$$

and

$$x^2 + y^2 = \frac{v_\perp{}^2}{\omega_g{}^2} \equiv r_L{}^2 = \varrho^2$$

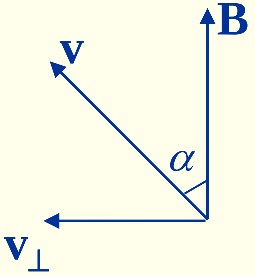
# Gyro (Larmor) radius




Magnetic force: $$\mathbf{F} = q\mathbf{v}_\perp \times \mathbf{B}$$

Centripetal force: $$\mathbf{F} = \frac{mv_\perp^2}{\rho}\hat{\boldsymbol{\rho}}$$

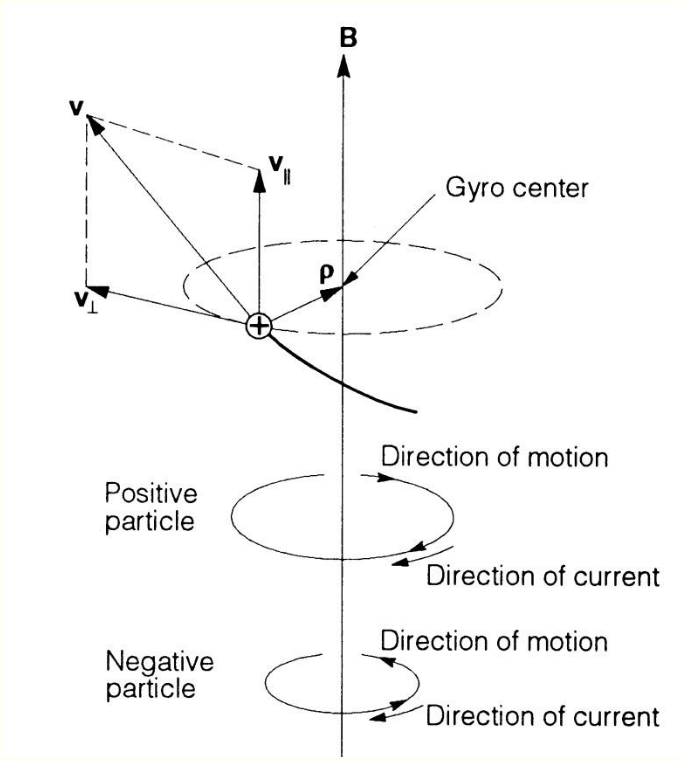
$$\Leftrightarrow$$




$$v_\perp = v \cdot \sin\alpha$$

$$\boxed{\rho = \frac{mv_\perp}{qB}}$$

# Gyro frequency




$$\rho = \frac{mv_\perp}{qB}$$

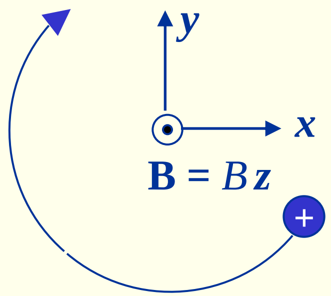
$$\omega\rho = v_\perp$$

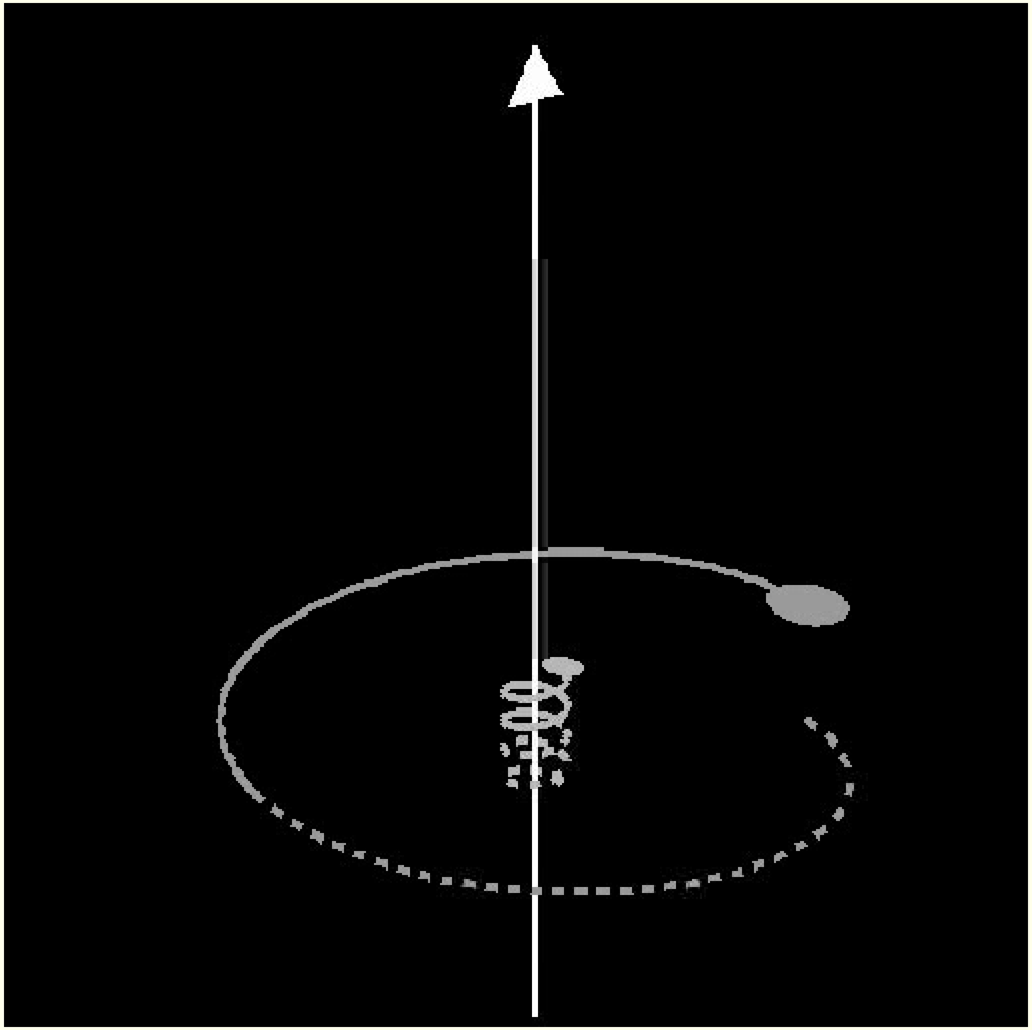
$$\Rightarrow$$

$$\boxed{\omega_g = \frac{qB}{m}}$$

$$\omega = 2\pi f$$

# Drift motion



Then

$$x = r_L cos(-\omega_g t)$$
$$y = r_L sin(-\omega_g t)$$

$$\omega_g = \frac{qB}{m}$$

$$r_L = \frac{mv_\perp}{qB}$$

# Magnetized plasma

A magnetic field drastically changes some of the plasma properties because the plasma particles are tightly bound to the magnetic field lines.

It is difficult for the particles to move perpendicular to **B**, but easy to move along **B**.

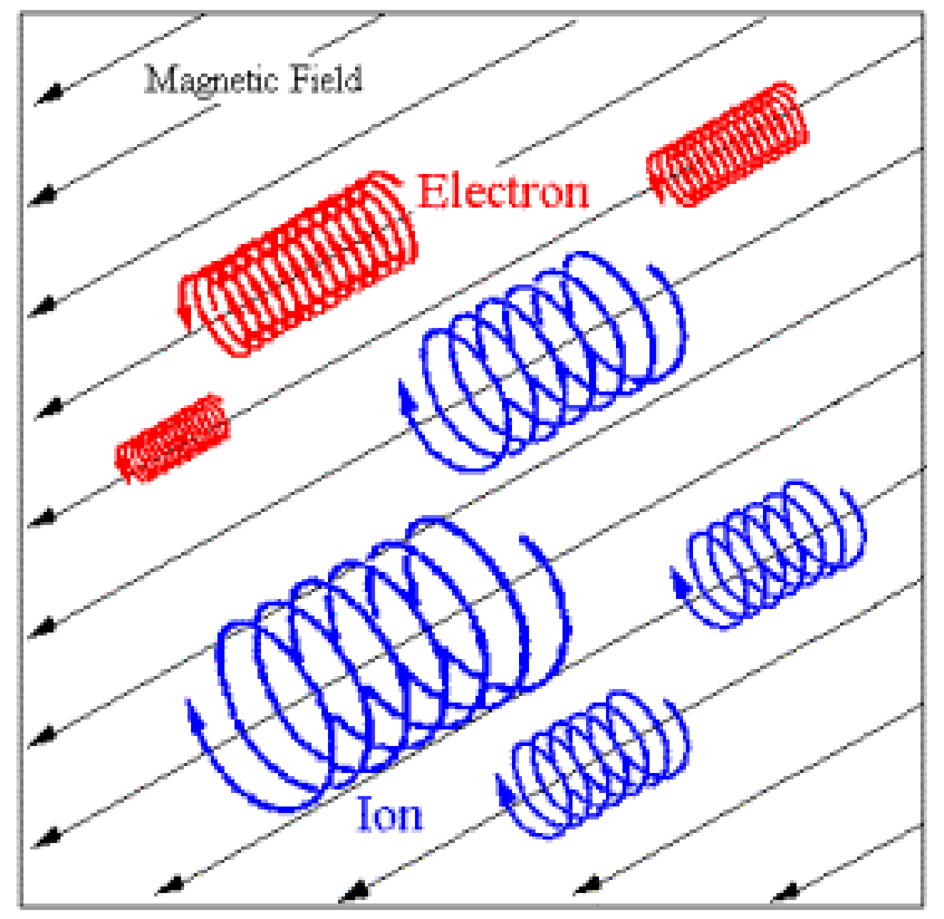
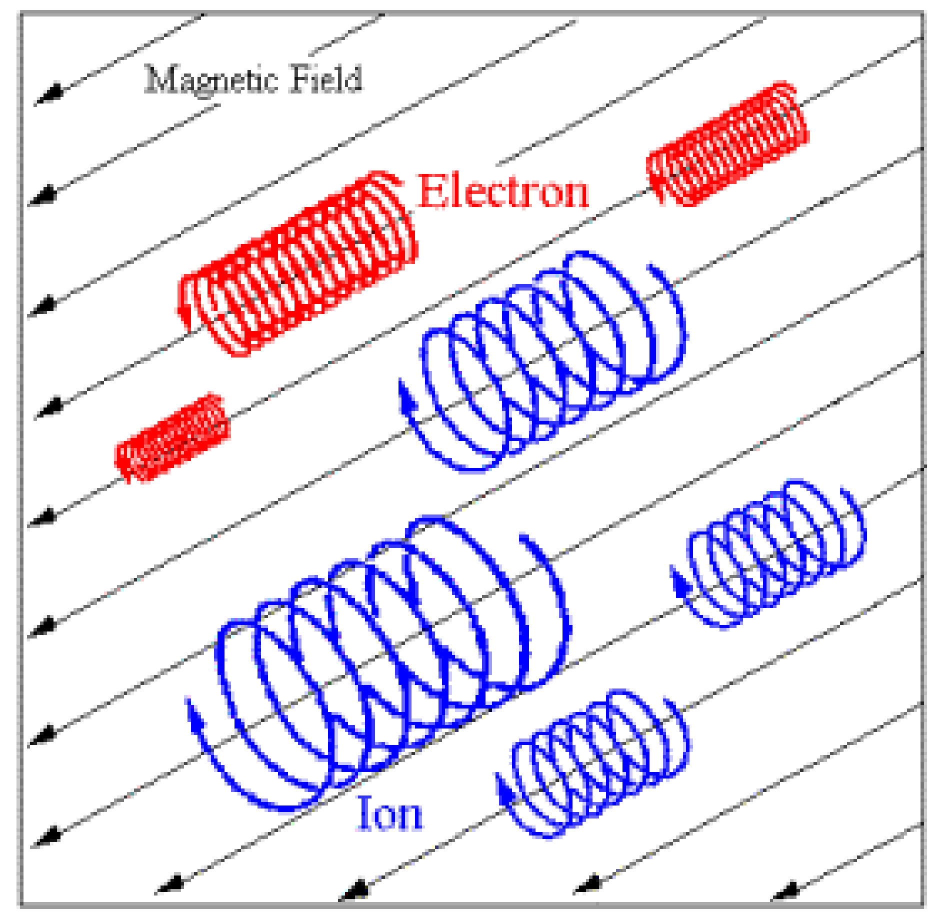



*Figure 10: Gyration of charged particle along magnetic field lines.*

# Think about this:



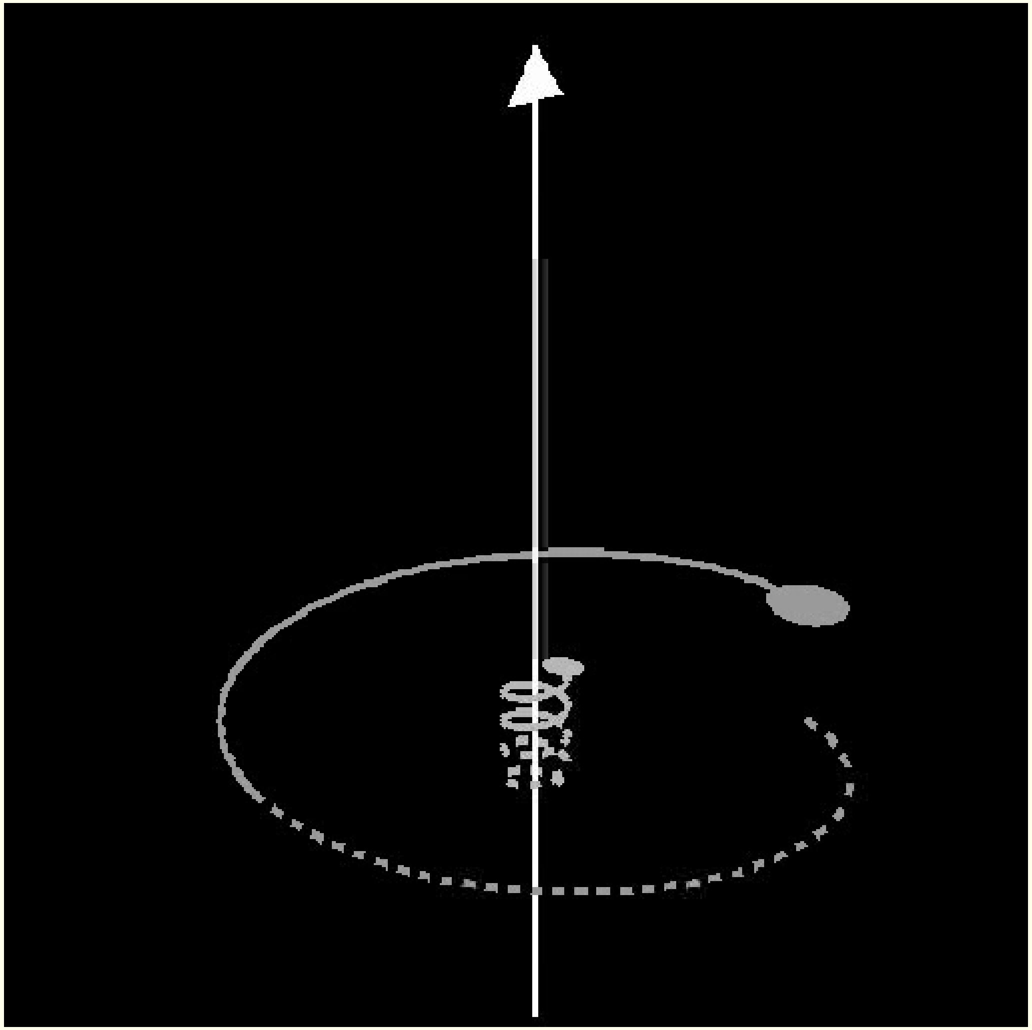

Can you think about a physical property of the plasma that varies with the direction?

(Such a property is called *anisotropic*.)

# Coronal loops


*Why does the plasma follow the magnetic field lines?*

# Gyro motion



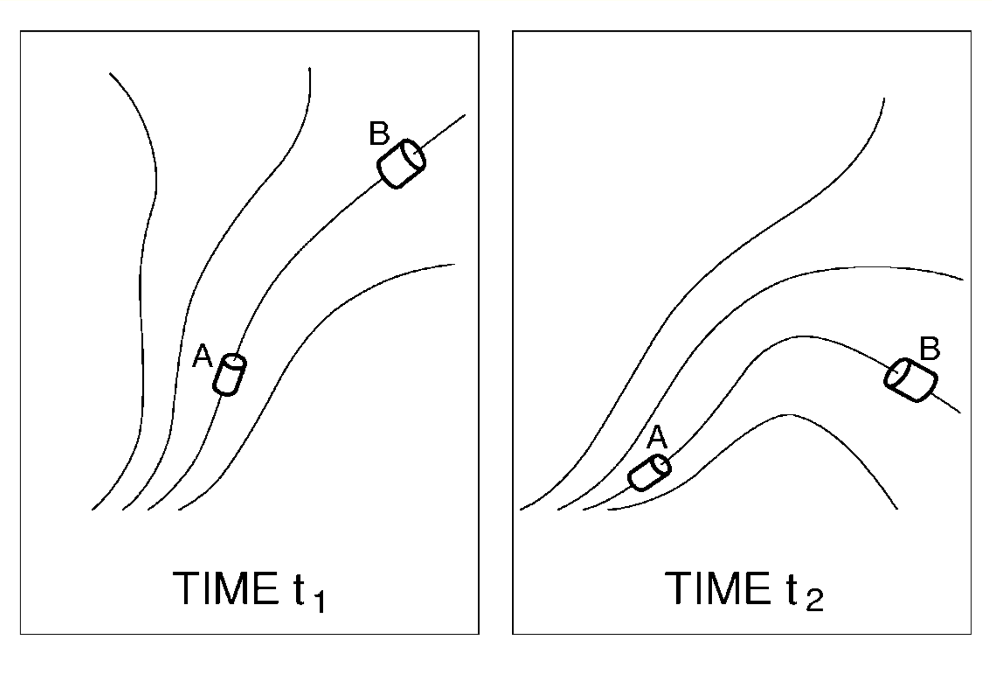
## Equipartion principle

Statistically the kinetic energy is equally distributed along the three dimensions:

$$E_{\parallel} = \frac{1}{2} k_B T$$

$$E_{\perp} = \frac{2}{2} k_B T$$

# Frozen in magnetic field lines




In fluid description of plasma two plasma elements that are connected by a common magnetic field line at time $t_1$ will be so at any other time $t_2$.

This applies if the magnetic Reynolds number is large:

$$R_m = \mu_0 \sigma l_c v_c >> 1$$

***An example of the collective behaviour of plasmas.***

# Maxwell's equations

*Gauss' law*

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

*No magnetic monopoles*

$$\nabla \cdot \mathbf{B} = 0$$

*Faraday's law*

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

*Ampére's law*

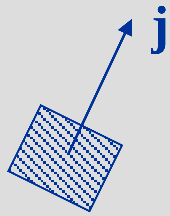
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

## Lorentz' force equation

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

## Ohm's law

$$\mathbf{j} = \sigma \mathbf{E}$$




## Energy density

$$W_B = \frac{B^2}{2\mu_0}, \quad W_E = \varepsilon_0 \frac{E^2}{2}$$

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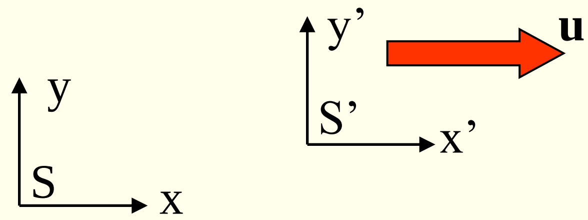
$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}, \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

# Field transformations (relativistic)




*Relativistic transformations*
*(perpendicular to the velocity* $\mathbf{u}$*):*

$$\mathbf{E}' = \frac{\mathbf{E} + \mathbf{u} \times \mathbf{B}}{\sqrt{1 - u^2/c^2}}$$

$$\mathbf{B}' = \frac{\mathbf{B} - \left(\mathbf{u}/c^2\right) \times \mathbf{E}}{\sqrt{1 - u^2/c^2}}$$

*For* $u \ll c$*:*

$$\mathbf{E}' = \mathbf{E} + \mathbf{u} \times \mathbf{B}$$

induced electric field

$$\mathbf{E} = \mathbf{E}' - \mathbf{u} \times \mathbf{B}$$

$$\mathbf{B}' = \mathbf{B}$$

# Frozen in magnetic flux *PROOF*

(1) $\mathbf{j} = \sigma \mathbf{E}' = \sigma\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$ Ohm's law

(2) $\mu_0 \mathbf{j} = \nabla \times \mathbf{B} - \cancel{\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}}$ Ampère's law

(3) $\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$ Faraday's law

$$(1) \Rightarrow \quad \mathbf{E} = \frac{\mathbf{j}}{\sigma} - \mathbf{v} \times \mathbf{B}$$

$$(3{+}1) \Rightarrow \quad \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left(\frac{\mathbf{j}}{\sigma} - \mathbf{v} \times \mathbf{B}\right)$$

$$(2) \Rightarrow \quad \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left(\frac{\nabla \times \mathbf{B}}{\mu_0 \sigma} - \mathbf{v} \times \mathbf{B}\right)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{v} \times \mathbf{B}\right) - \frac{1}{\mu_0 \sigma} \nabla \times \left(\nabla \times \mathbf{B}\right) =$$

$$\nabla \times \left(\mathbf{v} \times \mathbf{B}\right) - \frac{1}{\mu_0 \sigma}\left(\cancel{\nabla\left(\nabla \cdot \mathbf{B}\right)} - \nabla^2 \mathbf{B}\right)$$

$$\therefore \quad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{v} \times \mathbf{B}\right) + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

# Frozen in magnetic flux *PROOF II*

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{v} \times \mathbf{B})}_{A} + \underbrace{\frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}}_{B}$$

Order of magnitude estimate:

$$\frac{A}{B} = \frac{\nabla \times (\mathbf{v} \times \mathbf{B})}{\frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}} \approx \frac{\frac{v \Delta B}{L}}{\frac{\Delta B}{\mu_0 \sigma L^2}} = v L \mu_0 \sigma \equiv R_m$$

Magnetic Reynolds number $R_m$:

$$R_m >> 1 \Rightarrow \quad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

Frozen-in fields!

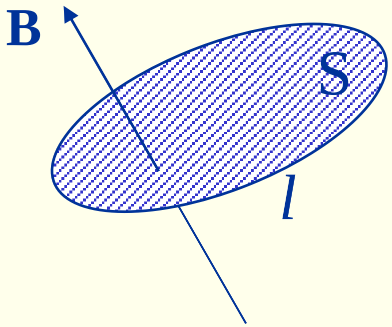
$$R_m << 1 \Rightarrow \quad \frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

Diffusion equation!

# Frozen in magnetic flux *PROOF III*

$$R_m >> 1 \Rightarrow \boxed{\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})}$$

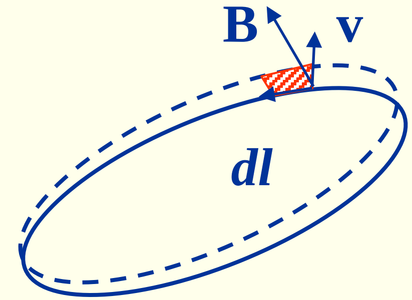
Consider the change of magnetic flux $\Phi$ through a surface S with contour $l$ which follows plasma motion




$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$\frac{d\Phi}{dt} = \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} + \frac{d\Phi_c}{dt}$$

$\dfrac{d\Phi_c}{dt}$ This term is due to change in the surface S due to plasma motion

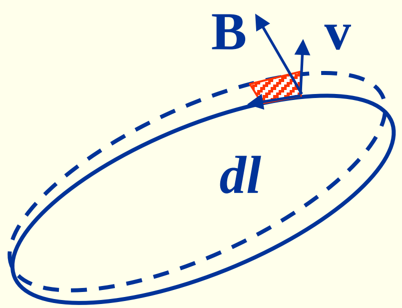



has an area of $(\mathbf{v} \cdot dt) \times d\mathbf{l}$

The flux through is $(\mathbf{v} \cdot dt) \times d\mathbf{l} \cdot \mathbf{B}$

$$\because \frac{d\Phi_c}{dt} = \int_l \mathbf{v} \times d\mathbf{l} \cdot \mathbf{B} =$$

# Frozen in magnetic flux *PROOF IV*




$$\frac{d\Phi_c}{dt} = \int_l \mathbf{v}\times d\mathbf{l}\cdot\mathbf{B} =$$

$$-\int_l \mathbf{v}\times\mathbf{B}\cdot d\mathbf{l} = -\int_S \nabla\times(\mathbf{v}\times\mathbf{B})\cdot d\mathbf{S}$$

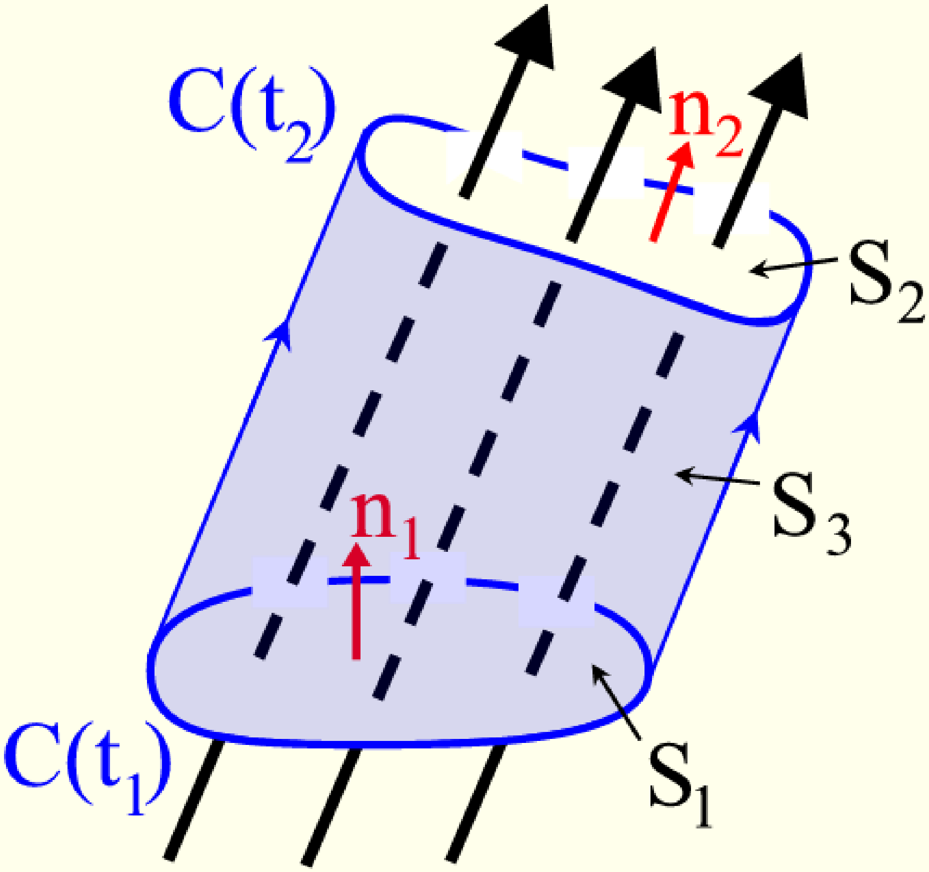
$$\because \frac{d\Phi}{dt} = \int_S \frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S} - \int_S \nabla\times(\mathbf{v}\times\mathbf{B})\cdot d\mathbf{S} =$$

$$\int_S \left[\frac{\partial\mathbf{B}}{\partial t} - \nabla\times(\mathbf{v}\times\mathbf{B})\right]\cdot d\mathbf{S} = 0$$

★

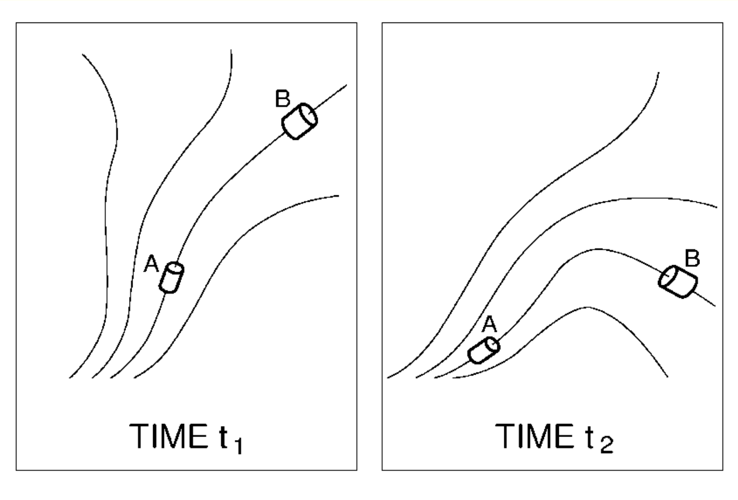
$$\boxed{\because \frac{d\Phi}{dt} = 0}$$

# Frozen in magnetic field lines




A *flux tube* is defined by following **B** from the surface S. Due to the frozen-in theorem the flux tube keeps its identity and the plasma in a flux tube stays in it for ever.

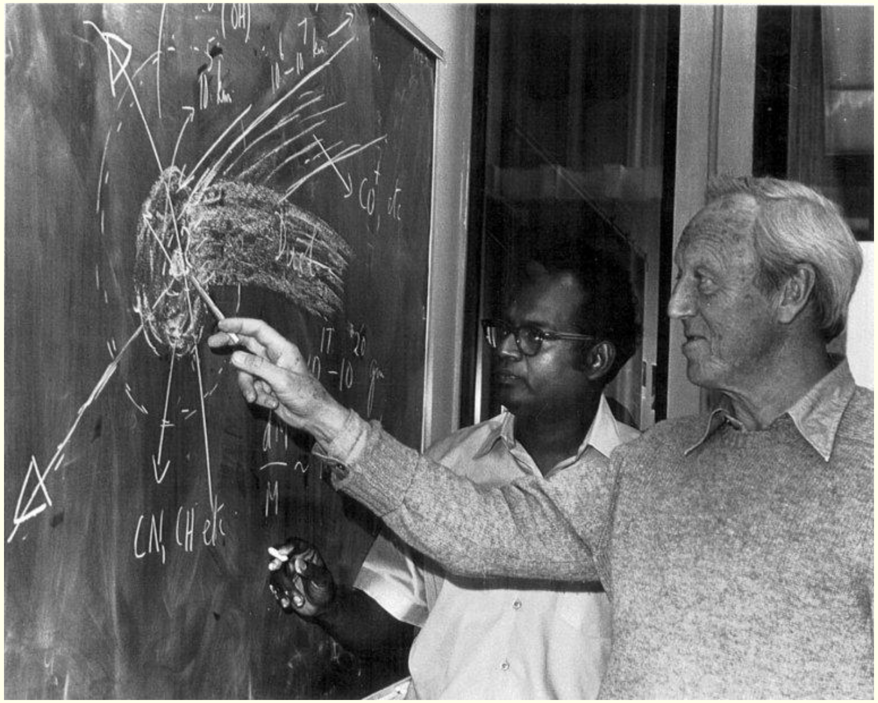
In particular if we let the tube become infinitely thin we have the theorem of frozen-in field lines.



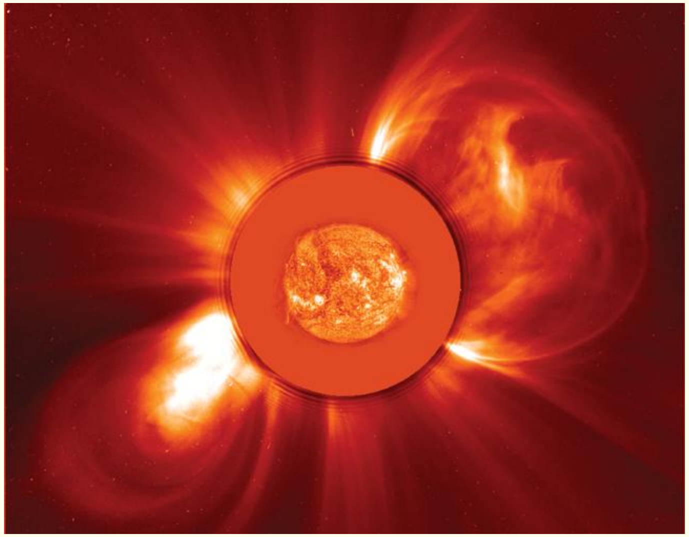
# Frozen in magnetic field lines: some history

- Also known as *Alfvén's theorem*
- Hannes Alfvén (1908-1995), professor at KTH
- Alfvén received the Nobel prize in 1970

  *'for fundamental work and discoveries in magneto-hydrodynamics with fruitful applications in different parts of plasma physics'*



# Magnetized plasma



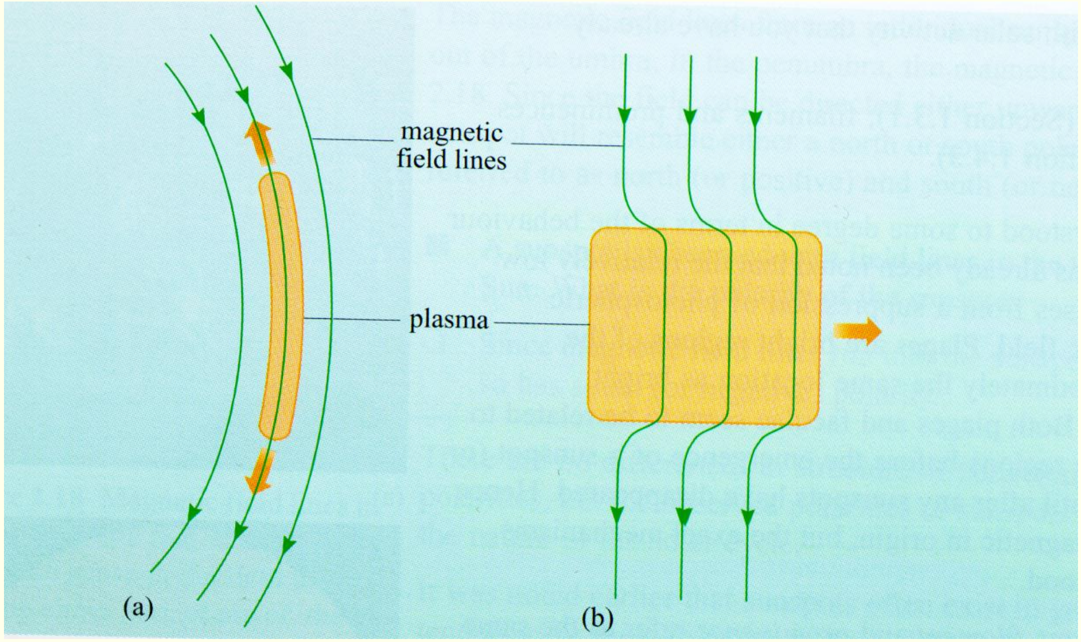
*Solar magnetic field*

*Northern lights (aurora)*

Different *plasma populations* (plasmas with different temperature and density) keep to their own field line, and thus "paint out" the magnetic field lines.

*Coronal loop*

# Does the plasma follow the magnetic field (a) or the other way around (b)?




$\beta << 1$ $\beta >> 1$

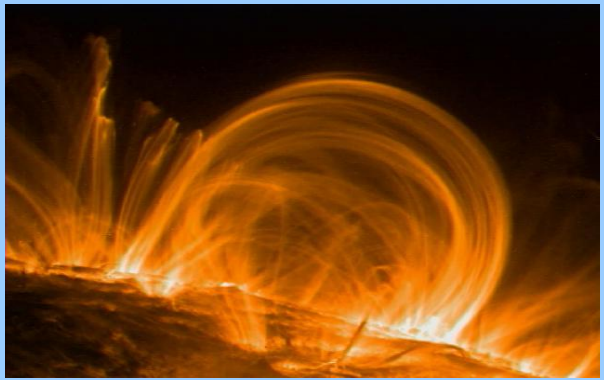
Depends on relative energy density (pressure)

$$p_{pl} = nk_B T$$

$$p_B = \frac{B^2}{2\mu_0}$$

$$\beta = \frac{p_{pl}}{p_B}$$

# Plasma beta



***Coronal loop***

B = 0.2 T
$n = 10^{23}\ m^{-3}$ (~1% of density at Earth surface)
T = 6000 K

Plasma (thermal) pressure/energy density: $p_{pl} = nk_bT$

Magnetic pressure/energy density: $p_b = B^2/2\mu_0$

$$\beta = \frac{p_{pl}}{p_B}$$

Green $\beta >> 1$ The plasma dominates the magnetic field

Red $\beta \sim 1$ Some complicated in-between behaviour

Blue $\beta << 1$ The magnetic field dominates the plasma

# Plasma beta

$B = 0.2$ T
$n = 10^{23}$ m$^{-3}$ (~1% of density at Earth surface)
$T = 6000$ K

Plasma (thermal) pressure/energy density: $p_{pl} = nk_bT = \quad 10^{23} \cdot 1.38 \cdot 10^{-23} \cdot 6000 \approx 8.3\,\text{kPa}$

Magnetic pressure/energy density: $p_b = B^2/2\mu_0 = \dfrac{0.2^2}{2 \cdot 4\pi \cdot 10^{-7}} \approx 16\,\text{kPa}$

Red

$\beta \sim 1$ Some complicated in-between behaviour