



Last lecture (3)

- Solar activity
- Solar wind

Today's lecture (4)

- Ionosphere
 - layers
 - radio wave reflection
 - electrical conductivity in magnetized plasma



Today

Activity	Date	Time	Room	Subject	Litterature
L1	28/8	15-17	Q21	Course description, Introduction, The Sun 1	CGF Ch 1.1,1.2, 1.4, 5, (p 110-113), 6.3
L2	29/8	13-15	Q2	The Sun 2, Plasma physics 1	CGF Ch 1.3, 5 (p 114-121)
L3	4/9	10-12	E2	Solar wind, The ionosphere and atmosphere 1, Plasma physics 2	CGF Ch 6.1, 2, 3.1-3.2, 3.5, LL Ch III, Extra material
T1	6/9	8-10	Q21	Mini-group work 1	
L4	6/9	15-17	Q2	The ionosphere 2, Plasma physics 3	CGF Ch 3.4, 3.7, 3.8
T2	10/9	15-17	Q21	Mini-group work 2	
L5	11/9	10-12	E3	The Earth's magnetosphere 1, Plasma physics 4	CGF 4-1-4.3, LL Ch I, II, IV.A
T3	17/9	8-10	Q21	Mini-group work 3	
L6	18/9	13-15	Q33	The Earth's magnetosphere 2, Other magnetospheres	CGF Ch 4.6-4.9, LL Ch V.
L7	19/9	13-15	Q2	Aurora, Measurement methods in space plasmas and data analysis 1	CGF Ch 4.5, 10, LL Ch VI, Extra material
T4	24/9	8-10	Q2	Mini-group work 4	
L8	24/9	15-17	V3	Space weather and geomagnetic storms	CGF Ch 4.4, LL Ch IV.B-C, VII.A-C
T5	2/10	8-10	Q31	Mini-group work 5	
L9	2/10	13-15	Q2	Alfvén waves, Interstellar and intergalactic plasma, Cosmic radiation	CGF Ch 7-9, Extra material
T6	8/10	15-17	Q21		
L10	9/10	10-12	Q2	Guest Lecture by Swedish astronaut Christer Fuglesang	
Written examination	16/10	14-19	L21, L22, L31		

Mini groupwork 1

a)

$$h = \frac{42 \text{ mm}}{7 \text{ mm}} \cdot 6378 \text{ km} \cdot 2 = 77000 \text{ km}$$

The thermal energy is divided into motion in the three dimensions, two of which only give rise to a gyro motion around the magnetic field lines, with the motion along the magnetic field corresponding to an energy

$$E = \frac{k_B T}{2} = \frac{1.38 \cdot 10^{-23} \cdot 1.5 \cdot 10^6}{2} = 1 \cdot 10^{-17} \text{ J}$$

$$v = \sqrt{\frac{2E}{m_e}} = \sqrt{\frac{2 \cdot 10^{-17}}{0.91 \cdot 10^{-30}}} = 4.7 \cdot 10^6 \text{ ms}^{-1}$$

Approximating the loop with a quarter-circle, the electron has to travel a length

$$s = \pi h / 2 = 120\,000 \text{ km}$$

Then we get $t = 25 \text{ s}$.

Energy - temperature

Average energy of molecule/atom:

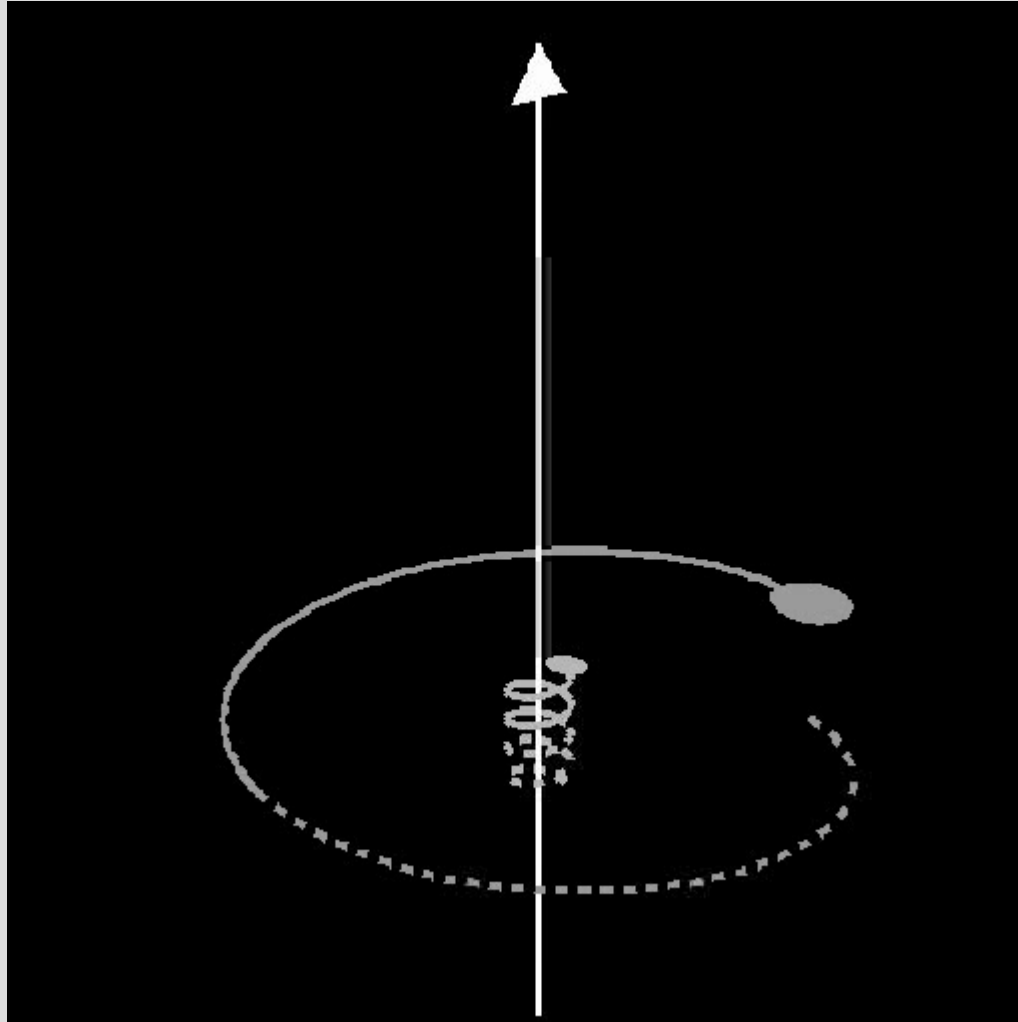
$$E = \frac{3}{2} k_B T \Rightarrow$$

$$T = \frac{2E}{3k_B}$$

$$1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ J} \Rightarrow$$

$$T = \frac{2E}{3k_B} = \frac{2 \cdot 1.6 \cdot 10^{-19} \text{ J}}{3 \cdot 1.38 \cdot 10^{-23} \frac{\text{J}}{\text{K}}} = 7729 \text{ K}$$

Gyro motion



Equipartition principle

Statistically the kinetic energy is equally distributed along the three dimensions:

$$E_{\parallel} = \frac{1}{2} k_B T$$

$$E_{\perp} = \frac{2}{2} k_B T$$

Mini groupwork 1

b)

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi} \frac{qB}{m} \Rightarrow$$

$$B = \frac{2\pi f_c m}{q} = \frac{2\pi \cdot 1 \cdot 10^{10} \cdot 0.91 \cdot 10^{-30}}{1.6 \cdot 10^{-19}} = 0.36 \text{ T}$$

The perpendicular energy is given by

$$E = 2 \cdot \frac{k_B T}{2} = 2 \cdot \frac{1.38 \cdot 10^{-23} \cdot 1.5 \cdot 10^6}{2} = 2 \cdot 10^{-17} \text{ J}$$

$\Rightarrow$

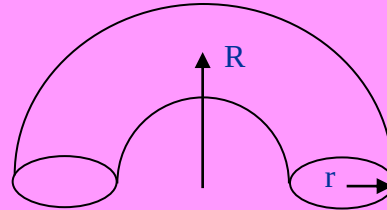
$$v = \sqrt{\frac{2E}{m_e}} = \sqrt{\frac{4 \cdot 10^{-17}}{0.91 \cdot 10^{-30}}} = 6.6 \cdot 10^6 \text{ ms}^{-1}$$

$$\rho = \frac{m_e v_{\perp}}{qB} = \frac{0.91 \cdot 10^{-30} \cdot 6.6 \cdot 10^6}{1.6 \cdot 10^{-19} \cdot 0.36} = 1.0 \cdot 10^{-4} \text{ m}$$

Mini groupwork 1

c)

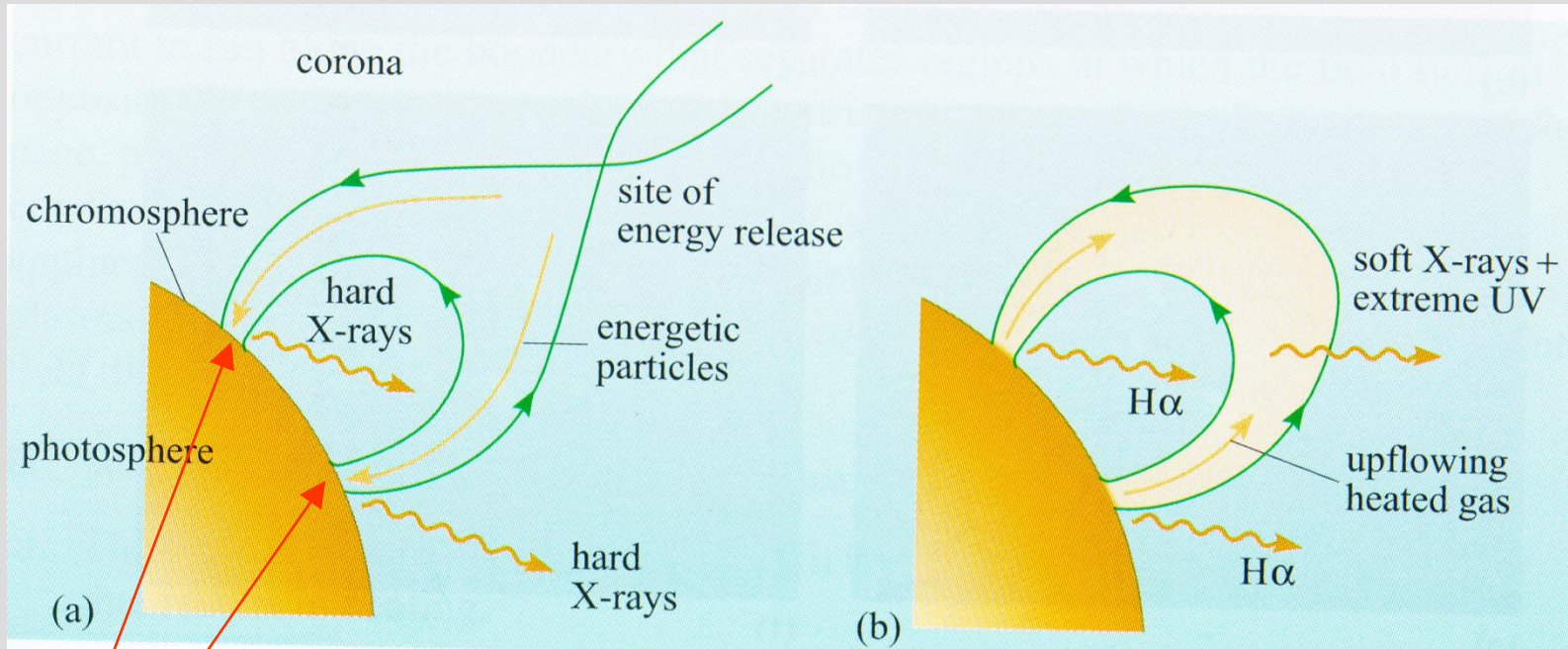
Model the flare by a half torus with minor axis r , and major axis. From the figure, estimate $R = 2.6 R_E$, and $r = 2 R_E$.



Let this half-torus be filled with a magnetic field of strength $B \sim 0.36$ T (using the value in b)). If the volume of the half-torus is V and the magnetic energy density is p_B , the total energy is

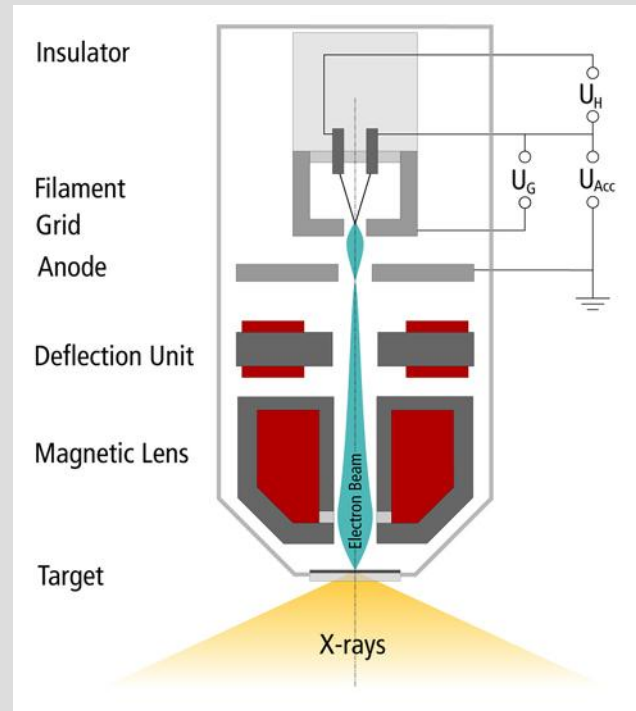
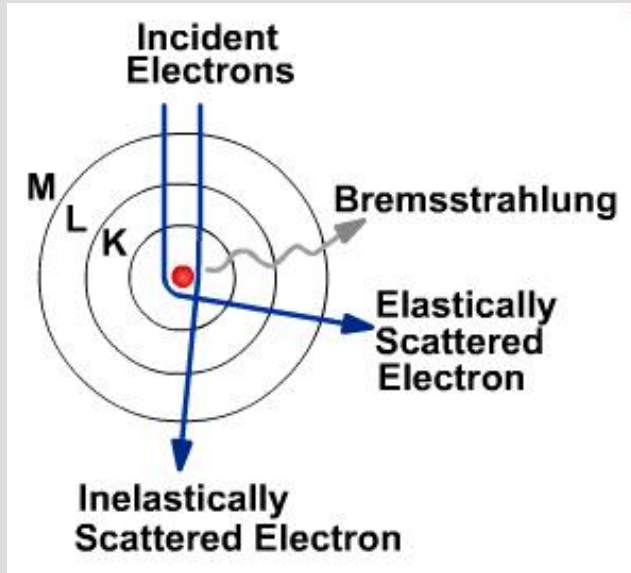
$$\begin{aligned}
 W &= V p_B = \pi R \pi r^2 \frac{B^2}{2\mu_0} = \pi^2 \cdot 2 \cdot 12^2 R_E^3 \frac{B^2}{2\mu_0} \\
 &= \pi^2 \cdot 2 \cdot 12^2 (6378 \cdot 10^3)^3 \frac{(0.36)^2}{2\mu_0} = 3.8 \cdot 10^{28} \text{ J}
 \end{aligned}$$

Solar flare mechanism



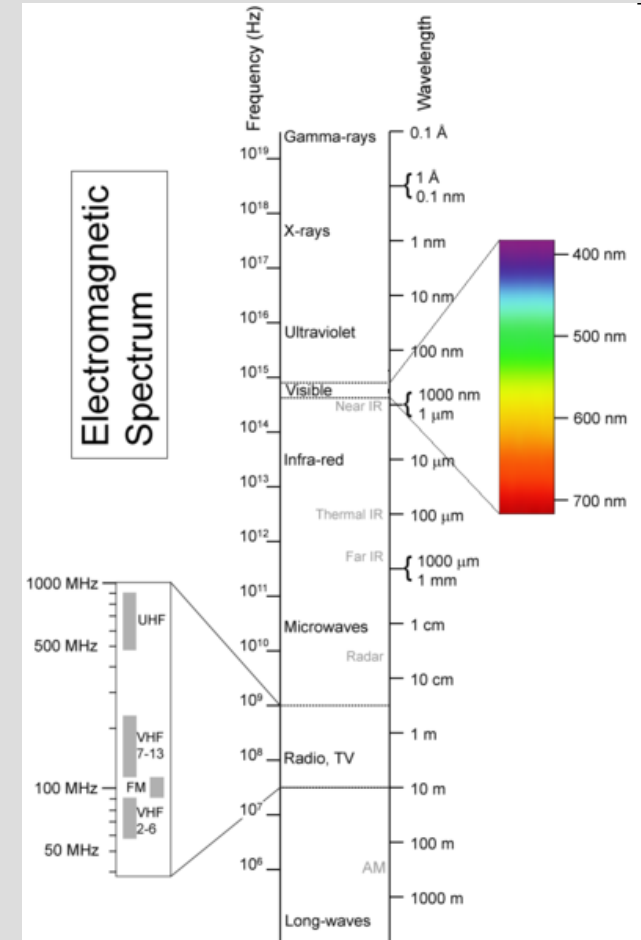
Electrons are accelerated, collide with solar surface (photosphere) and emit bremsstrahlung (X-rays).

X-rays, Bremsstrahlung

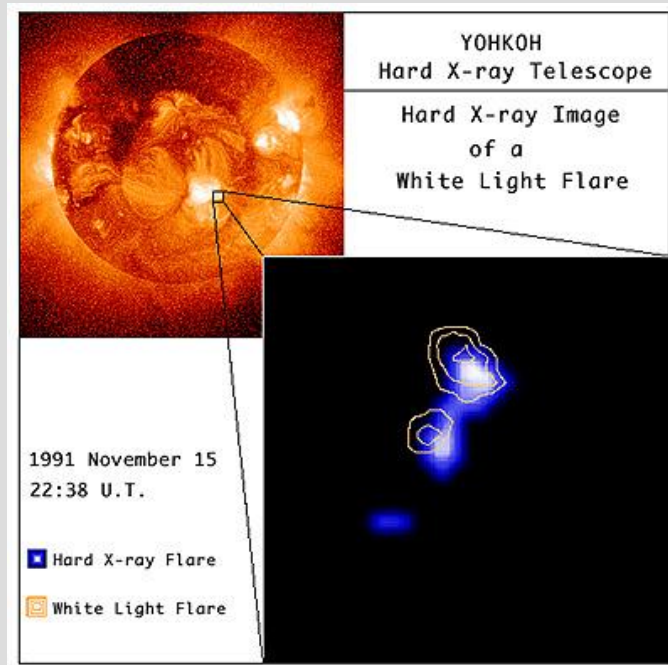


$$U_{acc} = hf = \frac{hc}{\lambda} \Rightarrow$$

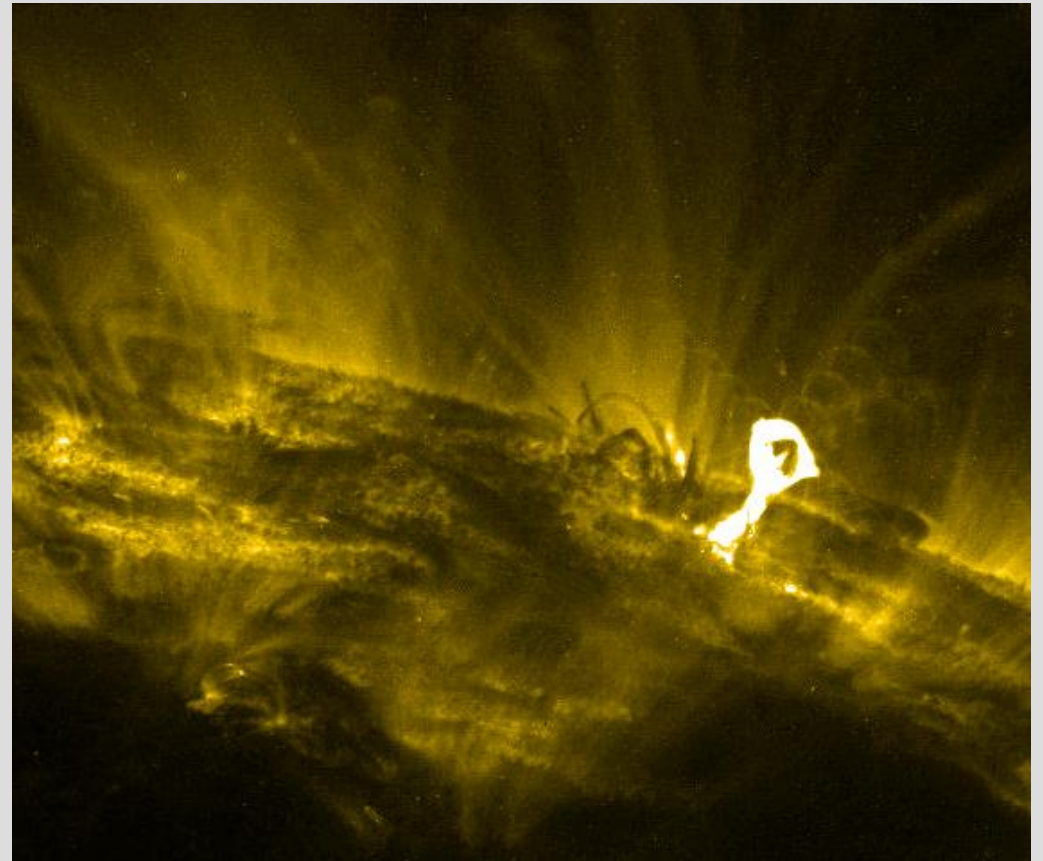
$$\lambda = \frac{hc}{U_{acc}} = \frac{6.62 \cdot 10^{-34} \cdot 3 \cdot 10^8}{100 \cdot 10^3 \cdot 1.6 \cdot 10^{-19}} = 1.2 \cdot 10^{-11} \text{ m} = 0.012 \text{ nm}$$



Solar flare observations



(a) double signature of x-ray emissions at foot of flare



(b) coronal loop filled with hot gas

Frozen in magnetic flux *PROOF II*

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{v} \times \mathbf{B})}_A + \underbrace{\frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}}_B$$

Order of magnitude estimate:

$$\frac{A}{B} = \frac{\nabla \times (\mathbf{v} \times \mathbf{B})}{\frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}} \approx \frac{\frac{v \Delta B}{L}}{\frac{\Delta B}{\mu_0 \sigma L^2}} = v L \mu_0 \sigma \equiv R_m$$

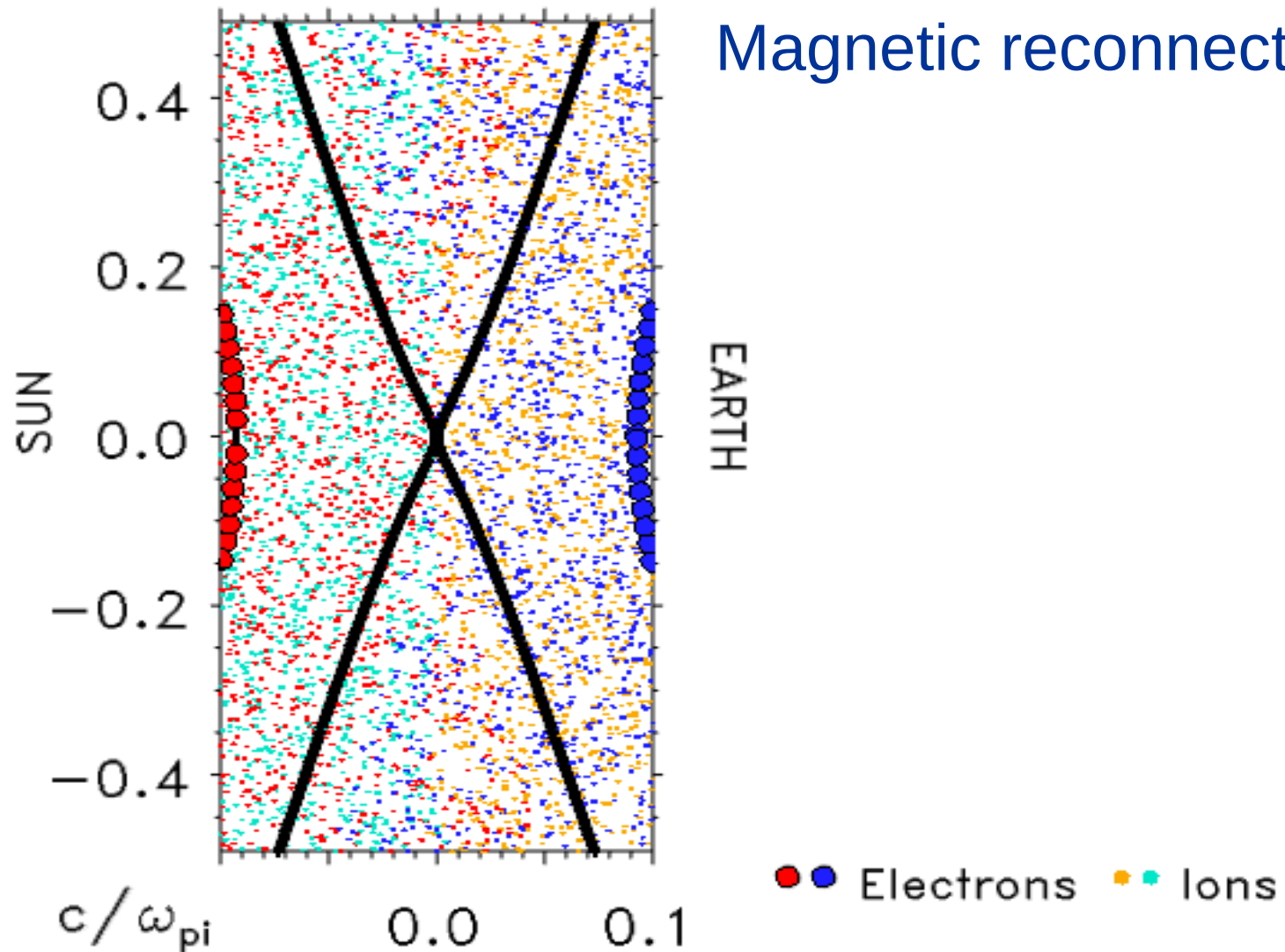
Magnetic Reynolds number R_m :

$$R_m \gg 1 \Rightarrow \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

Frozen-in fields!

$$R_m \ll 1 \Rightarrow \frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

Diffusion equation!



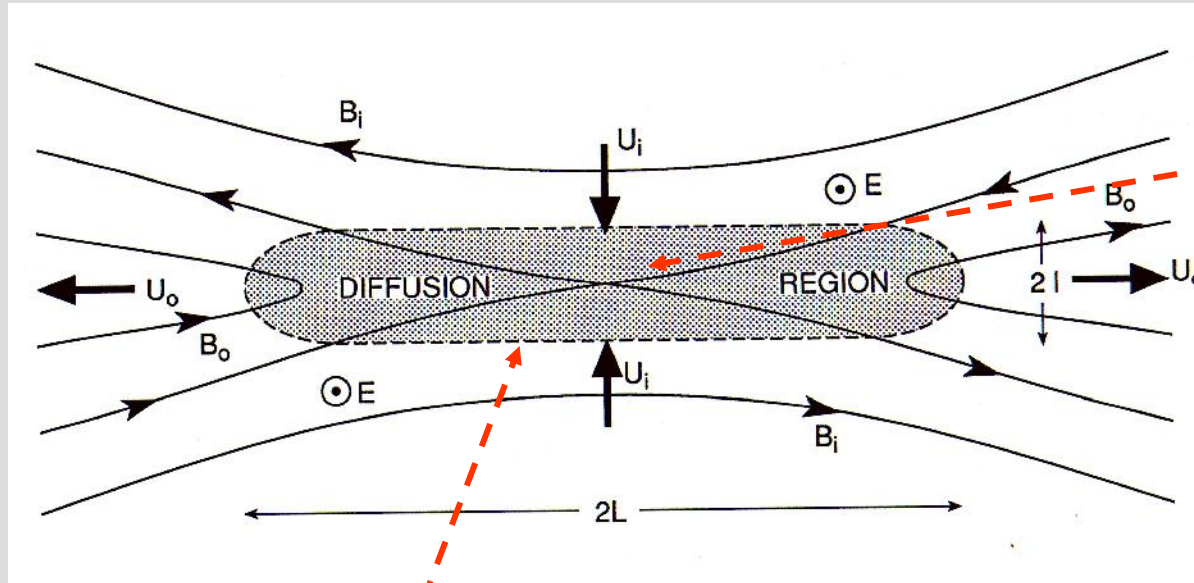
Reconnection

In 'diffusion region':

$$R_m = \mu_0 \sigma l v \sim 1$$

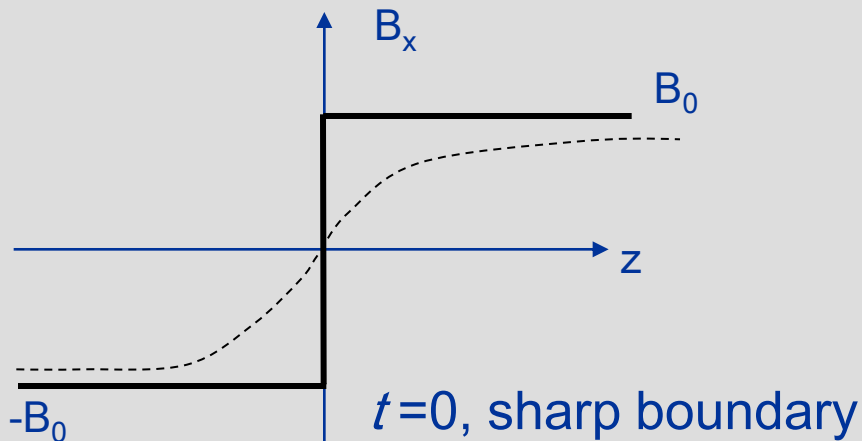
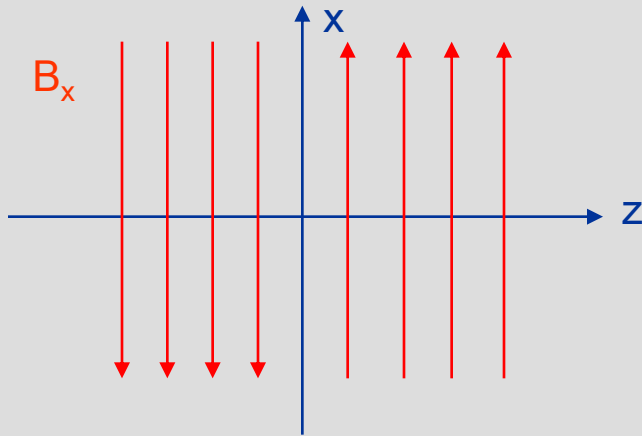
Thus: **condition** for frozen-in magnetic field breaks down.

A second **condition** is that there are two regions of magnetic field pointing in *opposite* direction:



- Field lines are “cut” and can be re-connected to other field lines
- Magnetic energy is transformed into kinetic energy ($U_o \gg U_i$)**
- Plasma from different field lines can mix**

Reconnection in 1D



$$\frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B} \quad \rightarrow \quad \frac{\partial B_x}{\partial t} = \frac{1}{\mu_0 \sigma} \frac{\partial^2 B_x}{\partial z^2}$$

Diffusion equation! Has solution

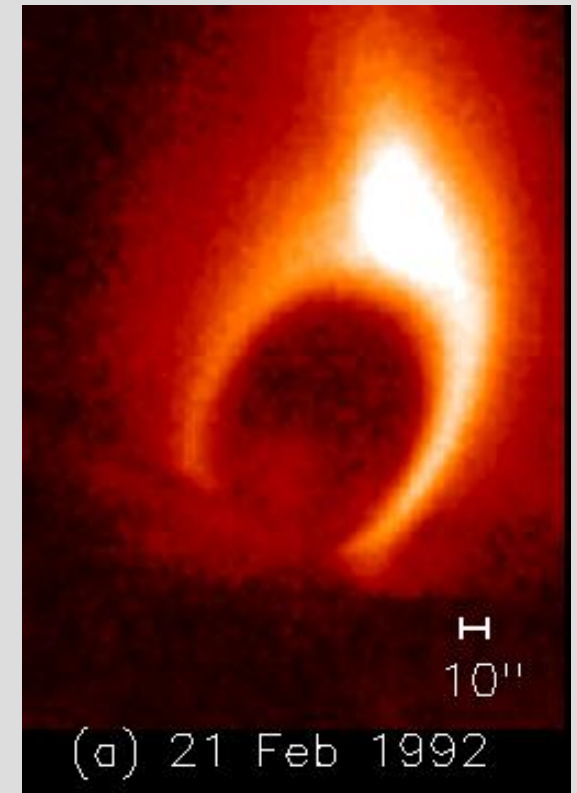
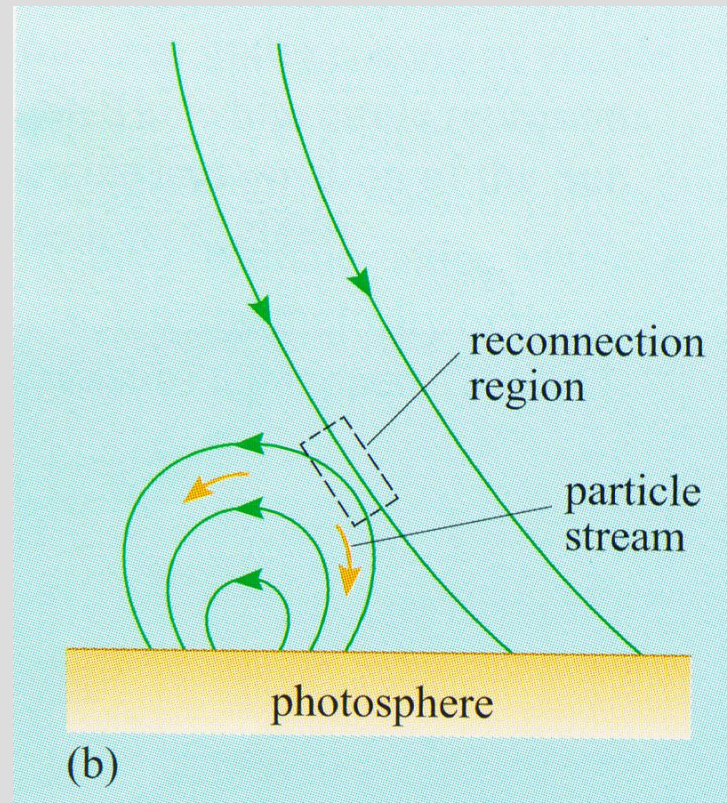
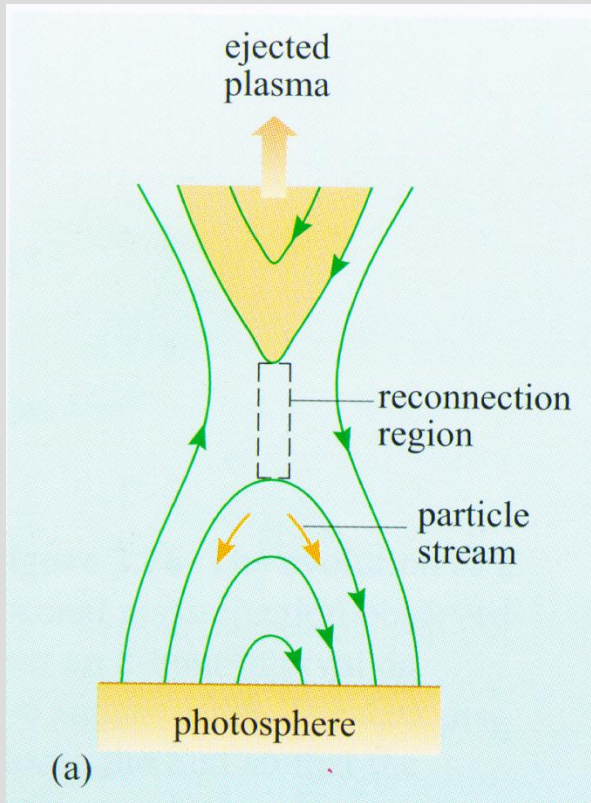
$$B_x(z, t) = B_0 \operatorname{erf} \left(\left[\frac{\mu_0 \sigma}{4t} \right]^{1/2} z \right)$$

The total magnetic energy then decreases with time:

$$W_B = \int_{-\infty}^{\infty} \frac{B^2}{2\mu_0} dz$$

The magnetic energy is converted into heat and kinetic energy in 2D

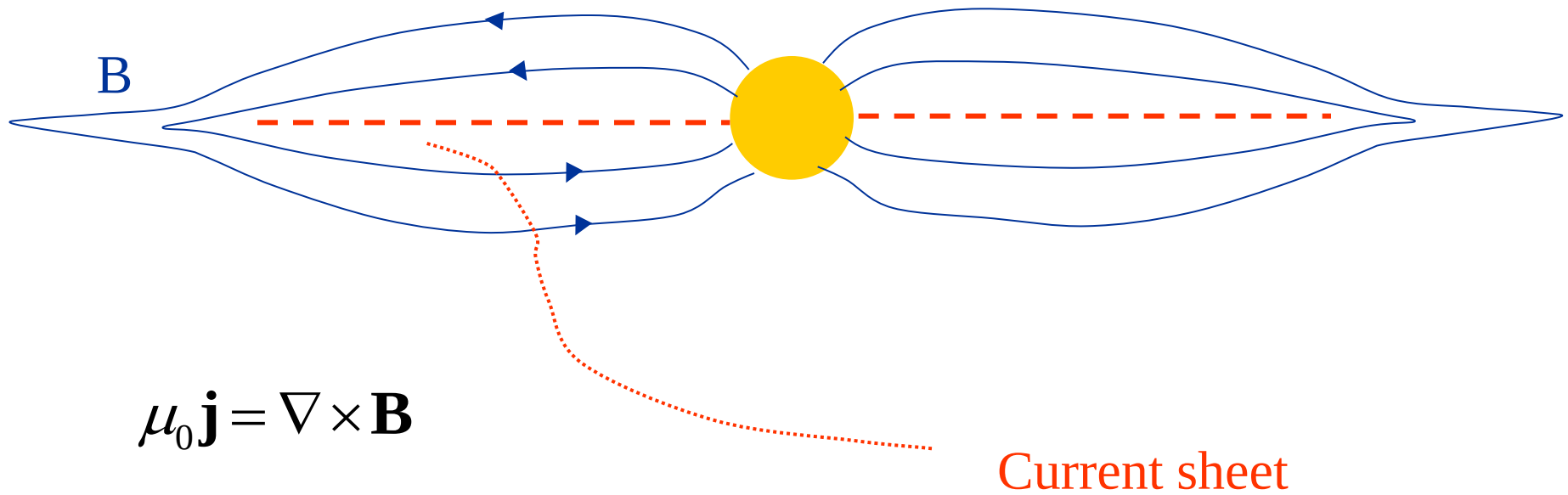
Solar flare *energization mechanism*



Two possible reconnection geometries

Solar wind

Interplanetary current sheet



Solar wind

Some basic facts

Average values

$$n_p = 8 \text{ cm}^{-3}$$

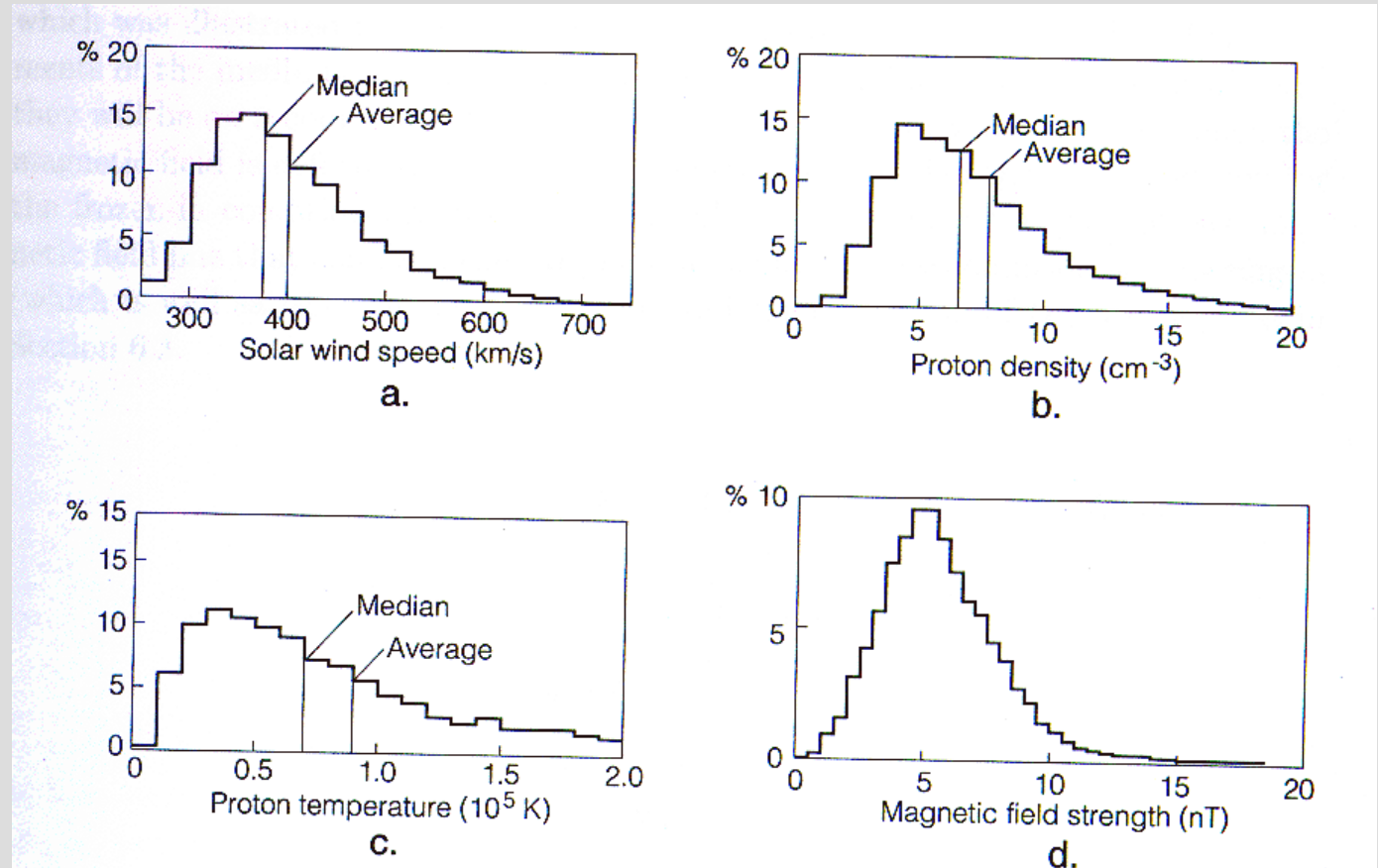
$$v = 320 \text{ km/s}$$

$$T_p = 4 \cdot 10^4 \text{ K}$$

$$T_e = 10^5 \text{ K}$$

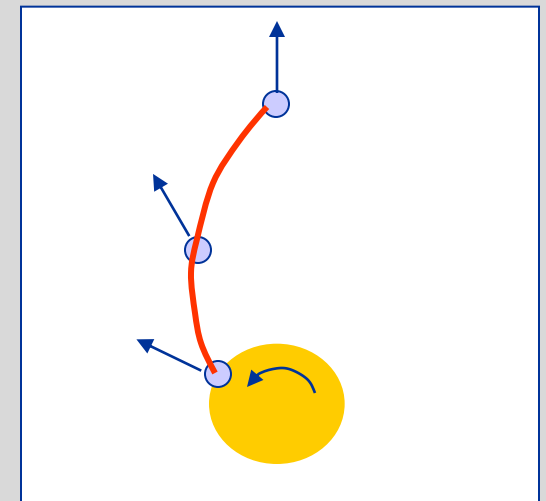
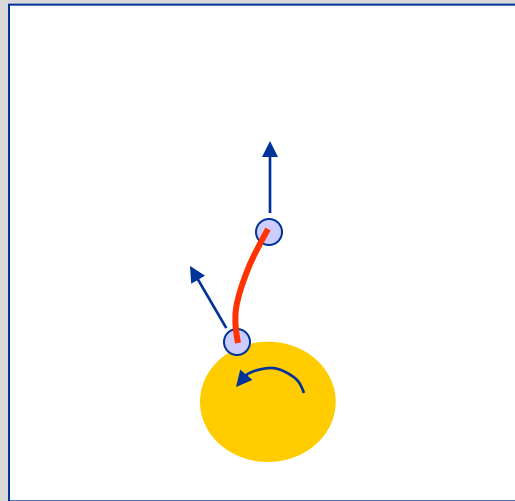
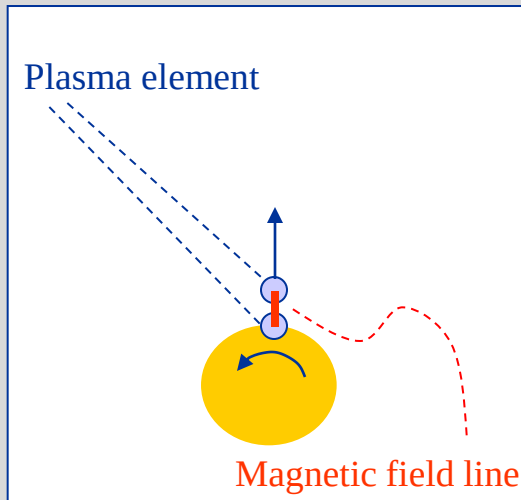
$$B = 5 \text{ nT}$$

$$\Phi_K = \rho v^3 / 2 = 0.22 \text{ mW/m}_2$$



Solar wind

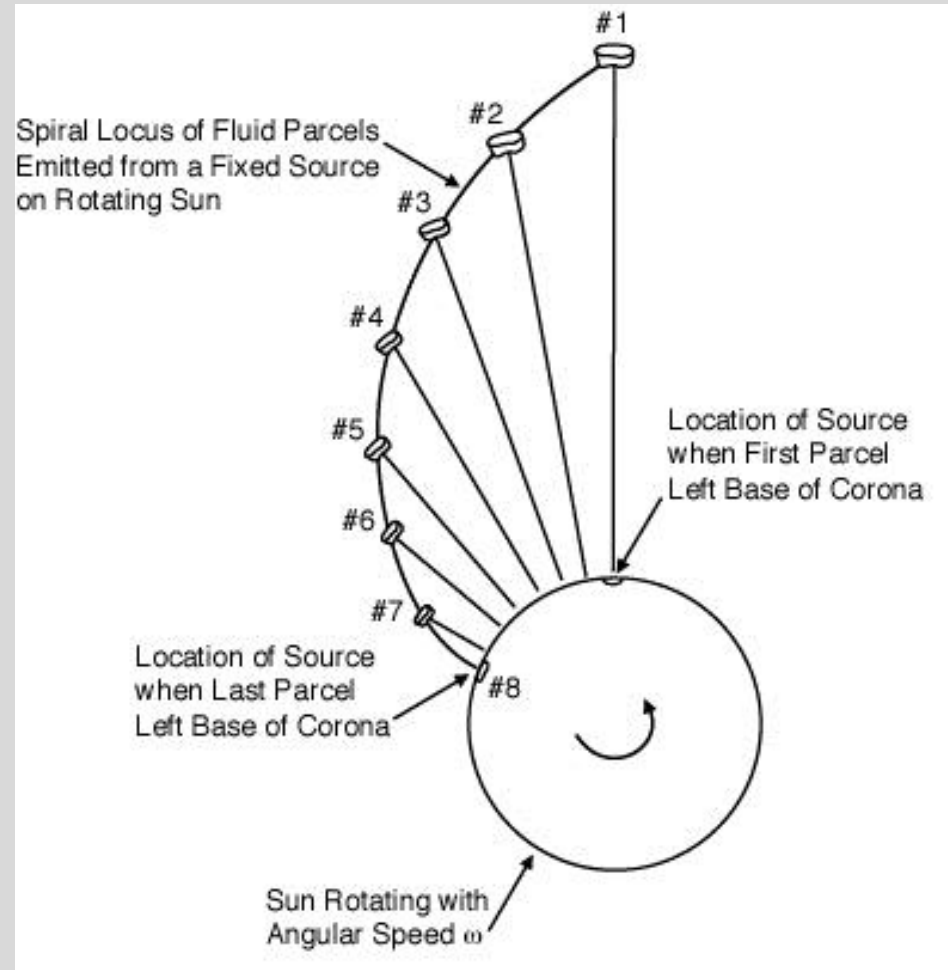
Magnetic field frozen into solar wind



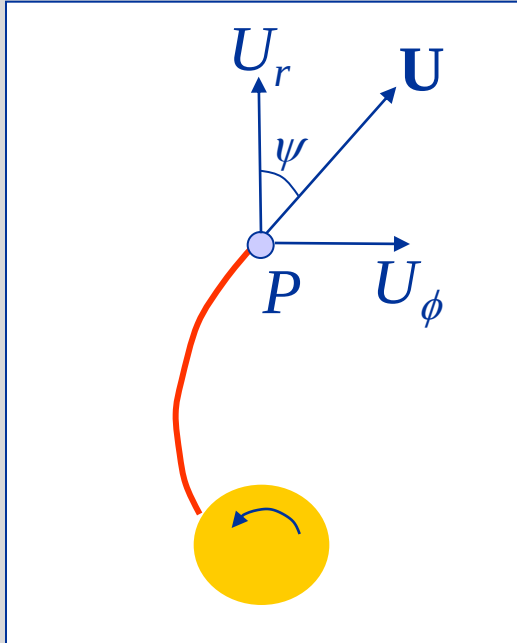
This is now seen from "above"! (Looking down on the ecliptic plane from the pole.)

Solar wind

Parker spiral



Parker spiral

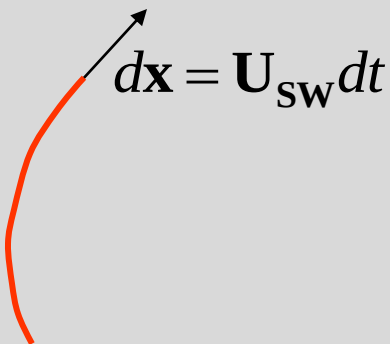


Derivation of Ψ (Parker angle)

Consider a coordinate system rotating with the sun. The plasma element P in this coordinate system has two velocity components: U_r and U_ϕ .

Since the magnetic field is frozen into the solar wind, and follows the orbit of the plasma element P , at any time B has to be parallel to U . Then we have:

$$\tan \psi = \frac{B_\phi}{B_r} = \frac{U_\phi}{U_r} = \left(\frac{\omega r}{u_{SW}} \right)$$



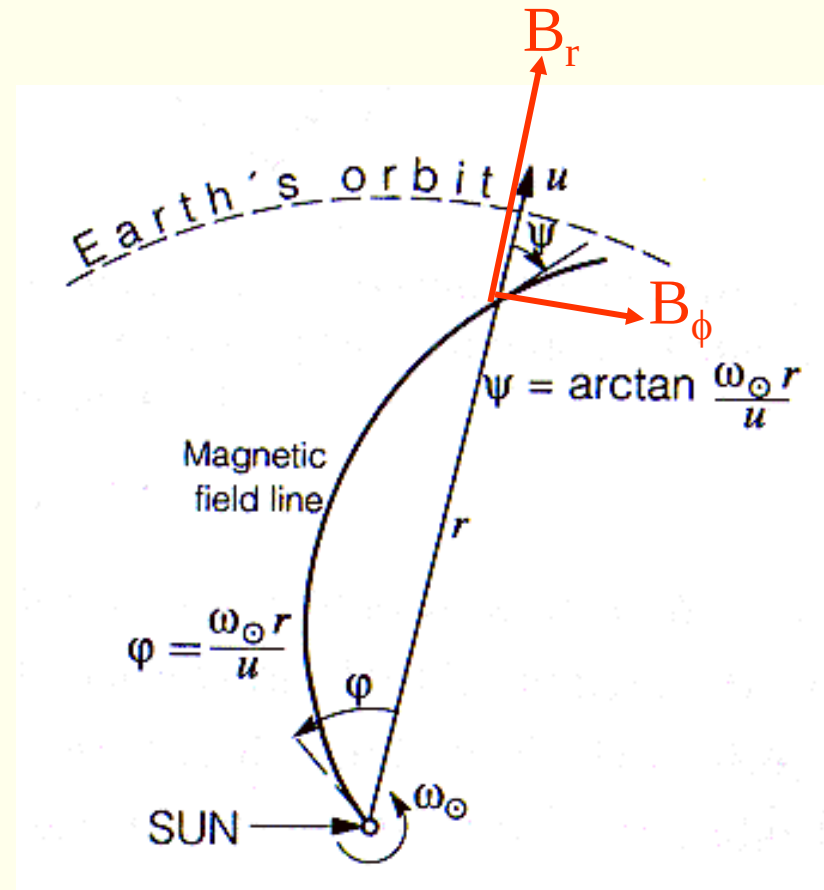
$$d\mathbf{x} = \mathbf{U}_{sw} dt$$

Solar wind

Parker spiral

Archimedean spiral:

$$\frac{B_{\phi}}{B_r} = \tan \psi = \left(\frac{\omega r}{u_{SW}} \right)$$



Archimedean spiral

An Archimedean spiral (also arithmetic spiral), is a spiral named after the 3rd-century-BC Greek mathematician Archimedes; it is the locus of points corresponding to the locations over time of a point moving away from a fixed point with a constant speed along a line which rotates with constant angular velocity.

Equivalently, in polar coordinates (r, ϕ) it can be described by the equation

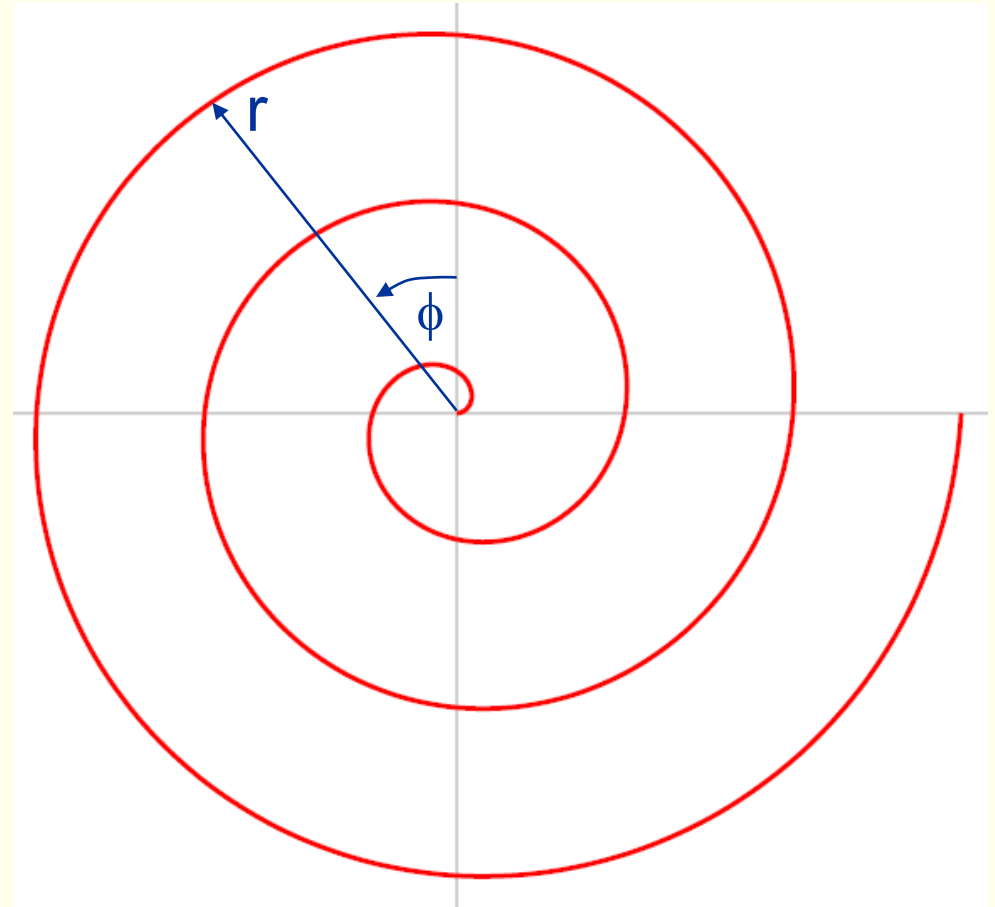
$$r = a + b\phi$$

$$r = a + b\omega t$$

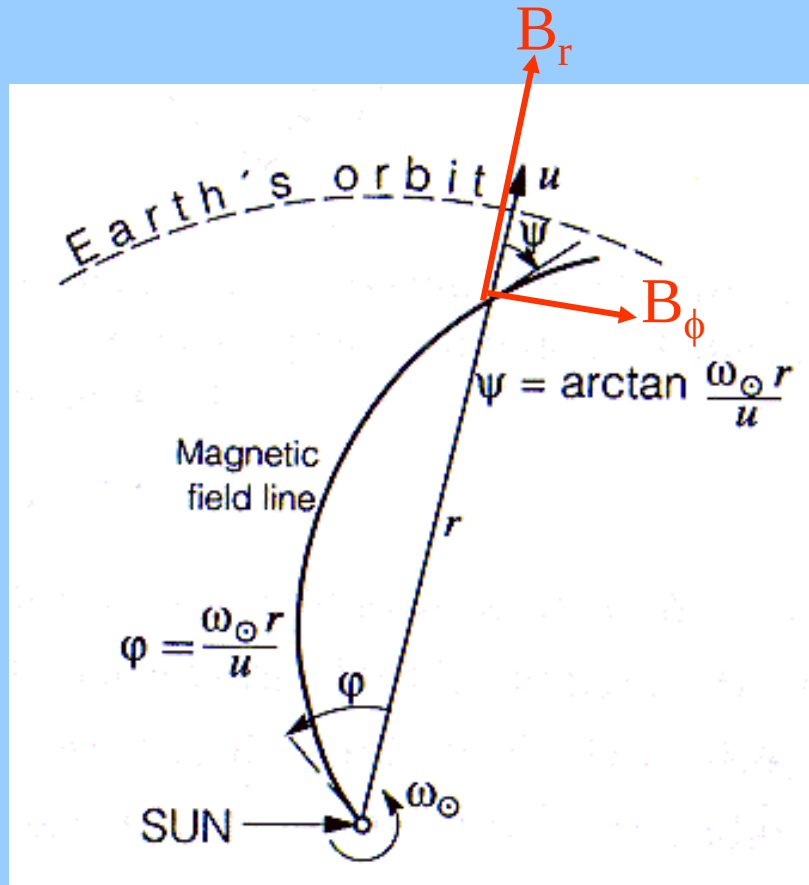
$$\frac{dr}{dt} = b\omega = u_{sw}$$

$$b = \frac{u_{sw}}{\omega}$$

$$r = R_{sun} + \frac{u_{sw}}{\omega} \phi$$



Use rotation period
 T of sun: $T = 27$ days



$$r = 1 \text{ A.U.}$$

What is the angle ψ
at Earth's orbit for a
typical solar wind
speed?

Yellow $\approx 50^\circ$

Red $\approx 80^\circ$

Blue $\approx 1^\circ$

Green $\approx 10^\circ$

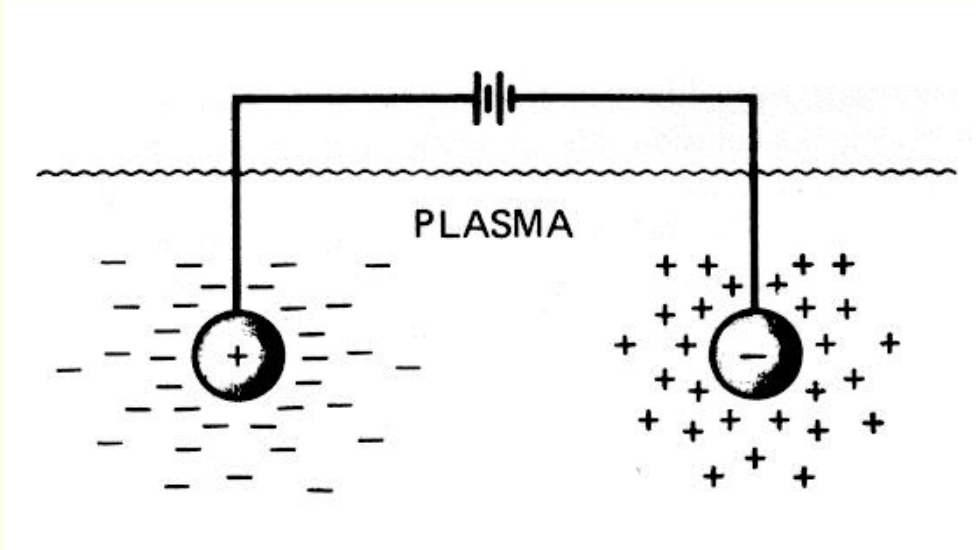
$$\Psi = \arctan\left(\frac{\omega r}{u}\right)$$

What is ω ? $\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{27 \cdot 24 \cdot 60 \cdot 60} = 2.7 \cdot 10^{-6} \text{ s}^{-1}$

$$\Psi = \arctan\left(\frac{\omega r}{u}\right) = \arctan\left(\frac{2.7 \cdot 10^{-6} \cdot 1.5 \cdot 10^{11}}{320 \cdot 10^3}\right) = \arctan(1.27) = 52^\circ$$

Yellow

Quasineutrality



$$\Phi = \Phi_0 e^{-x/\lambda_D}$$

Debye length

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{n_e e^2}}$$

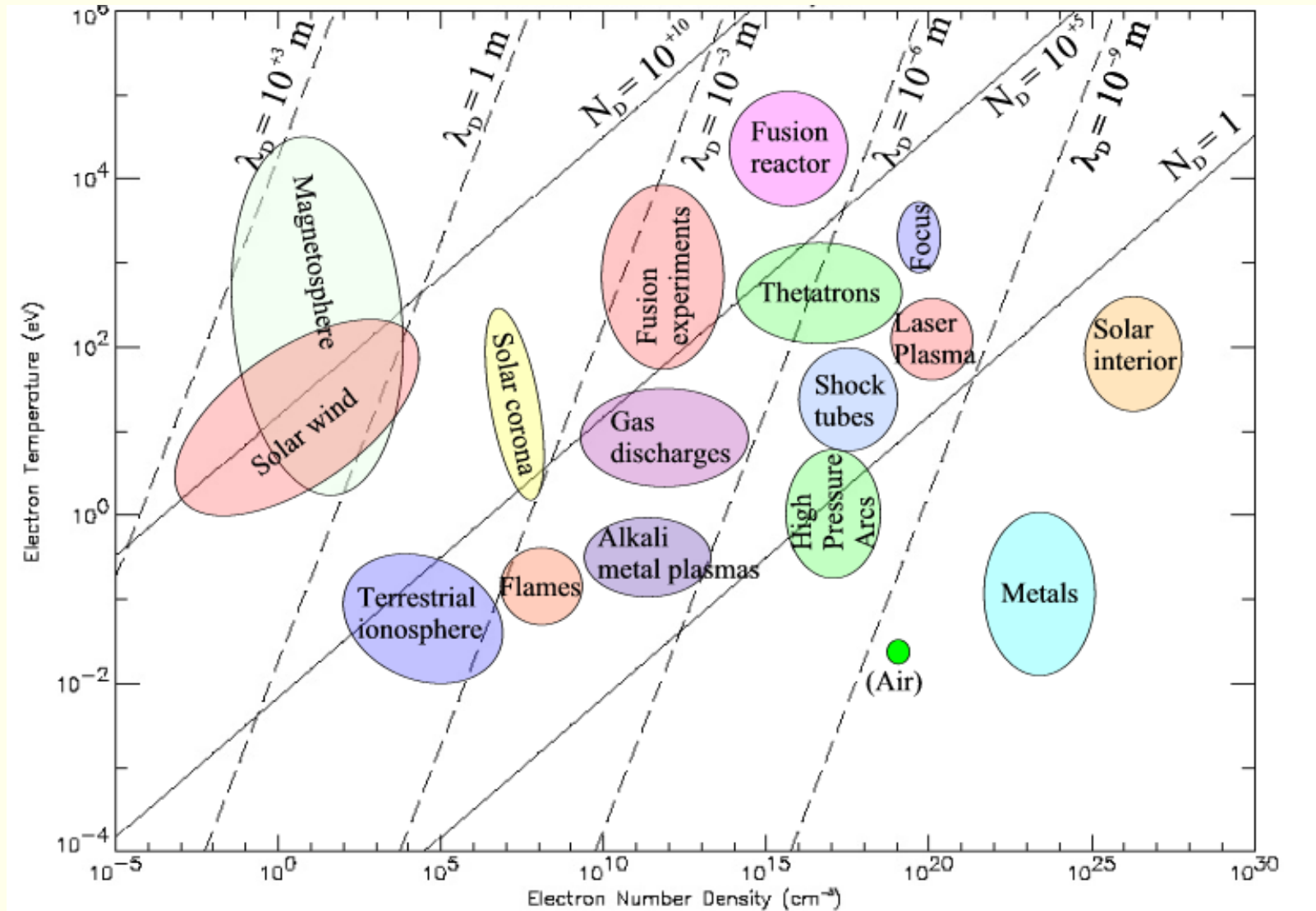
$$\frac{\Delta n}{n} = \frac{(n_e - n_i)}{n_e} < \left(\frac{\lambda_D}{l_c} \right)^2$$

$$l_C \gg \lambda_D \Rightarrow$$

Plasma close to neutral:

$$n_e \approx n_i$$

Debye lengths





Plasma models/descriptions

- Single particle motion
- Computer simulations of many-particle dynamics
- Generalization of statistical mechanics (kinetic theory)
- Generalization of fluid mechanics:
Magneto-hydrodynamics (MHD)

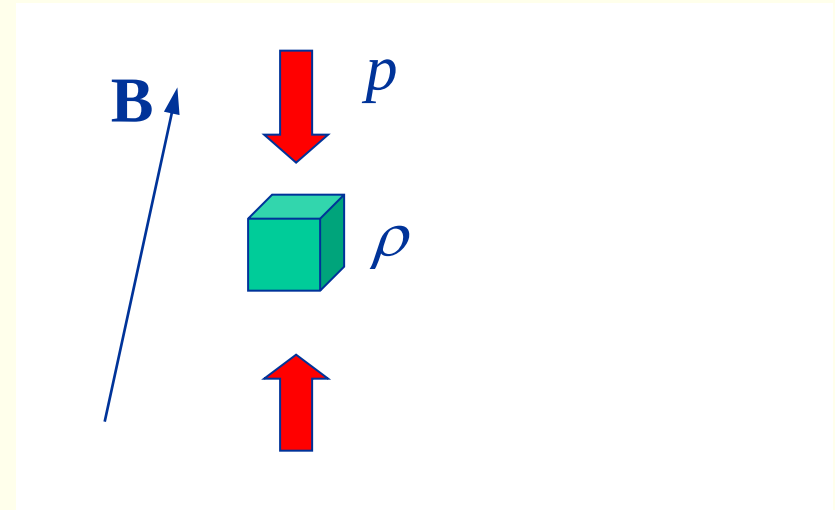
Plasma physics

Magnetohydrodynamics (MHD)

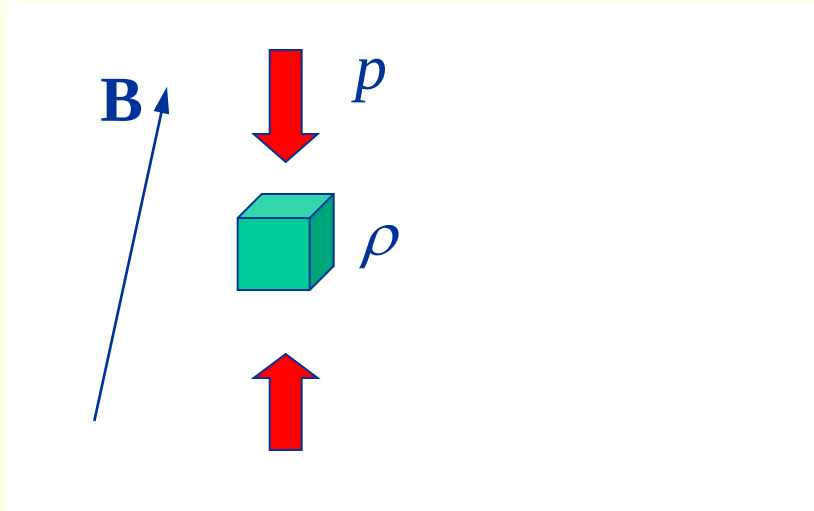
MHD is a combination of

- *fluid-/hydrodynamics* (which is based on Newton's laws of motion)
- *Maxwell's equations* (electrodynamics)

applied on a plasma volume element.



Magnetohydrodynamics (MHD)



For a volume element of plasma:

$$\mathbf{F} = m\mathbf{a} \quad \Rightarrow$$

$$-\nabla p + n_e q \mathbf{v}_e \times \mathbf{B} + \cancel{\rho q \mathbf{E}}^{\text{quasineutrality}} = \rho \frac{d\mathbf{v}}{dt} \quad \Rightarrow$$

$$(1) \quad \rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j} \times \mathbf{B}$$

Magnetohydrodynamics (MHD)

$$(1) \quad \rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j} \times \mathbf{B}$$

This together with two of Maxwell's equations and Ohm's law make up the most common MHD equations:

$$(2) \quad \mathbf{j} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$(3) \quad \nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

Only consider slow variations

$$(4) \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Magnetohydrodynamics (MHD)

$$(1) \quad \rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j} \times \mathbf{B}$$

In equilibrium:

$$0 = -\nabla p + \mathbf{j} \times \mathbf{B} \quad \Longleftrightarrow$$

$$-\nabla p + \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} = 0$$

$$-\nabla p - \nabla \left(\frac{B^2}{2\mu_0} \right) + \frac{1}{\mu_0} (\mathbf{B} \cdot \nabla) \mathbf{B} = 0$$

Represents tension in
magnetic field

If magnetic tension = 0

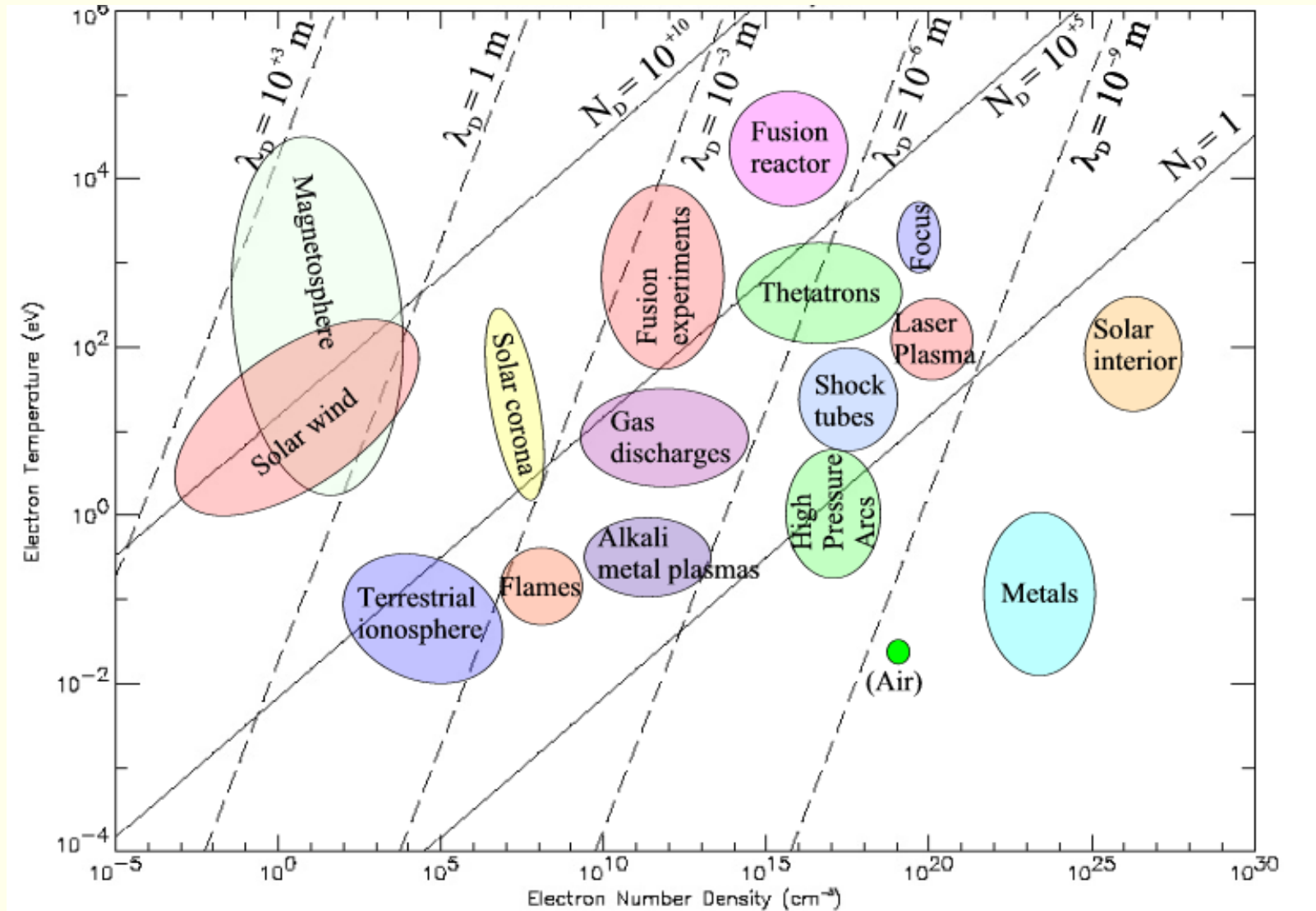
$$p + \frac{B^2}{2\mu_0} = \text{konst}$$

Magnetic pressure

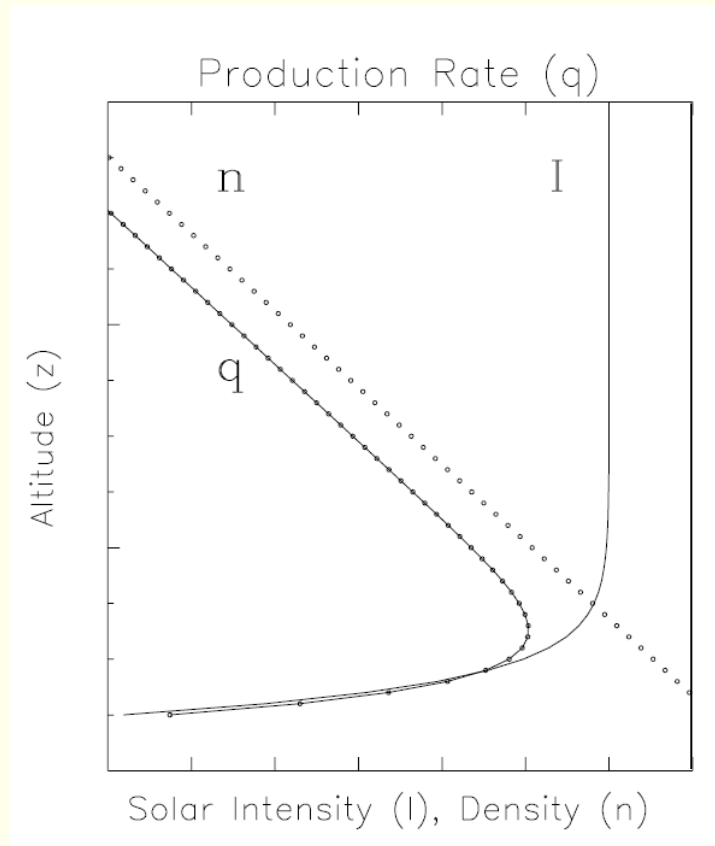


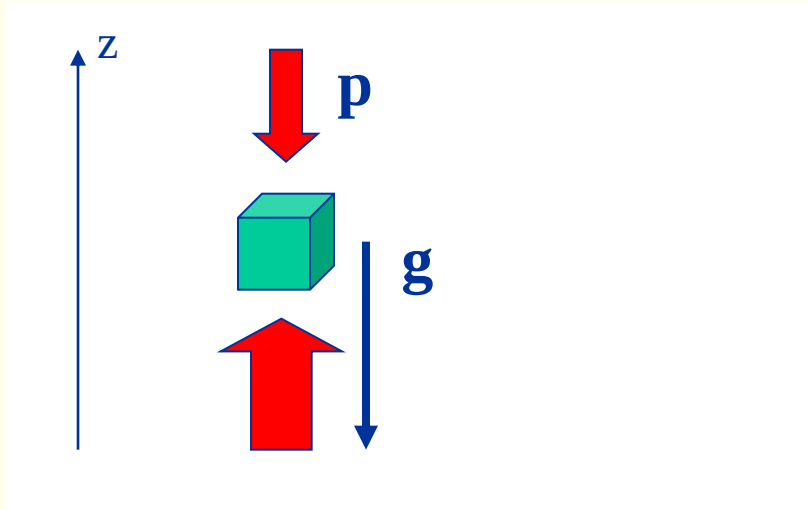
The ionosphere

Debye lengths



Basic principle for creation of ionospheric layer





Atmospheric scale height

$$-\frac{dp}{dz} = g\rho_m \quad \text{hydrostatic equilibrium for a volume element}$$

$$p = nk_B T = \frac{\rho k_B T}{m} \quad \text{ideal gas law}$$

$$-\frac{k_B T}{m} \frac{d\rho_m}{dz} = g\rho_m \quad \text{if } T \text{ is constant}$$

$$\rho_m = \text{const} \cdot e^{-z/(k_B T / gm)} = \text{const} \cdot e^{-z/H}$$

Scale height

$$H = k_B T / gm$$



Scale height

$$H = k_B T / g m$$

What is the approximate scale height in the atmosphere right here, right now?

(0° C = 273 K)

Blue

1 km

Yellow

30 km

Green

9 km

Red

100 km

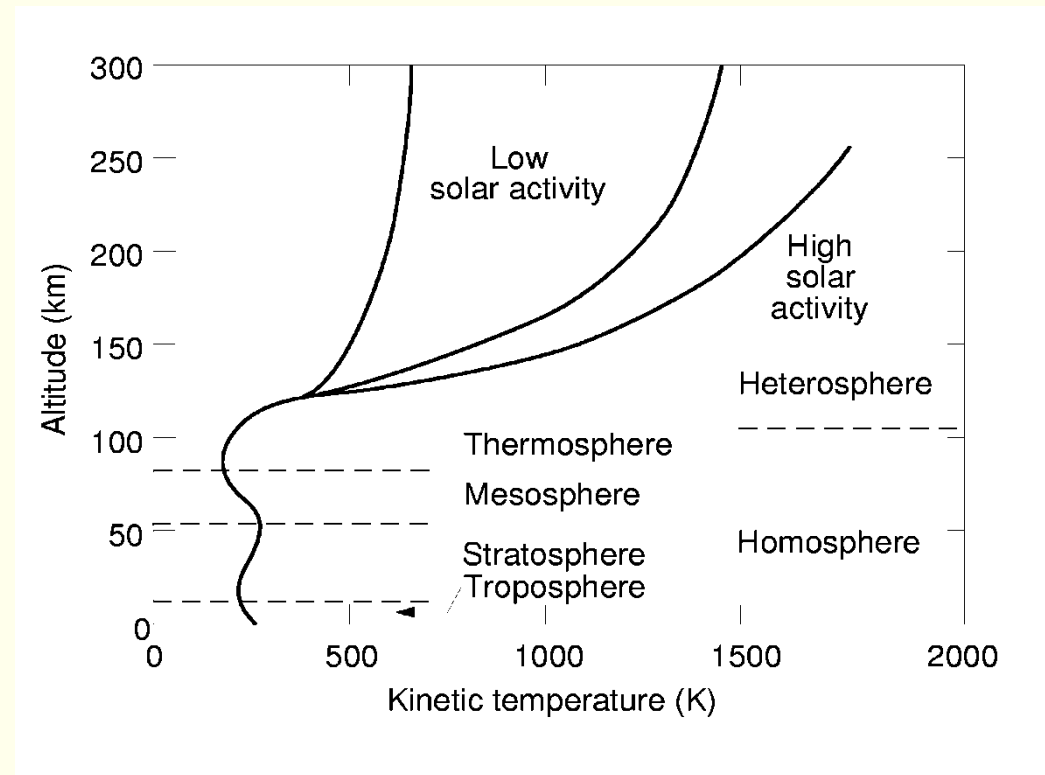
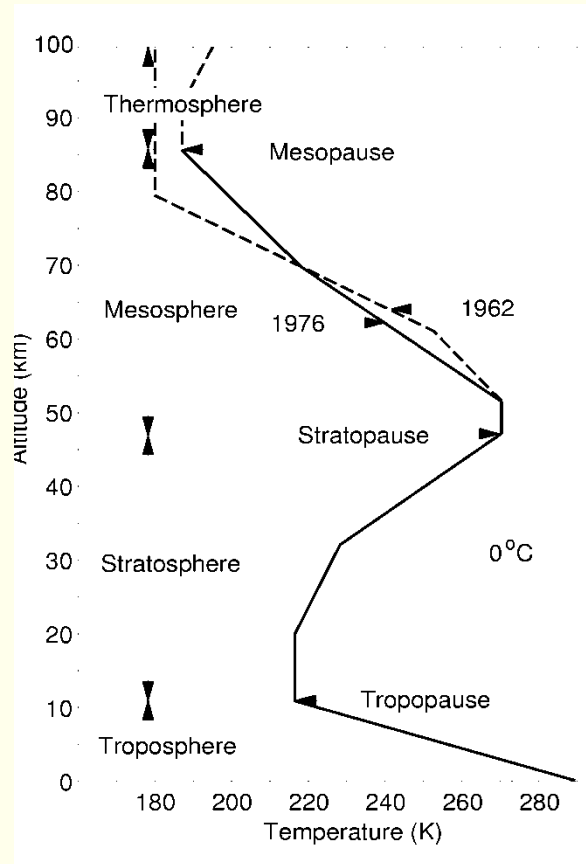


$$H = k_B T / g m = (1.38 \cdot 10^{-23} \cdot 290) / (9.81 \cdot 14 \cdot 2 \cdot 1.67 \cdot 10^{-27}) =$$
$$= 8724 \text{ m} \approx 9 \text{ km}$$

Green

What did we neglect
when we derived the
scale height?

Temperature profile



A graph showing the variation of atmospheric density with altitude. The y-axis represents Altitude (km) from 0 to 1000. The x-axis represents Number density (m^{-3}) on a logarithmic scale from 10^8 to 10^{24} . The graph is divided into the Homosphere (below ~100 km) and Heterosphere (above ~100 km) by a horizontal line labeled 'Turbopause'. Several curves are plotted: O_1 (solid line), H_1 (dashed line), N_2 (dash-dot line), O_2 (dotted line), Ar (dotted line), and Total (solid line). A blue arrow points to the O_1 curve at high altitude, and a red arrow points to the H_1 curve at high altitude.

Turbulent mixing – one scale height



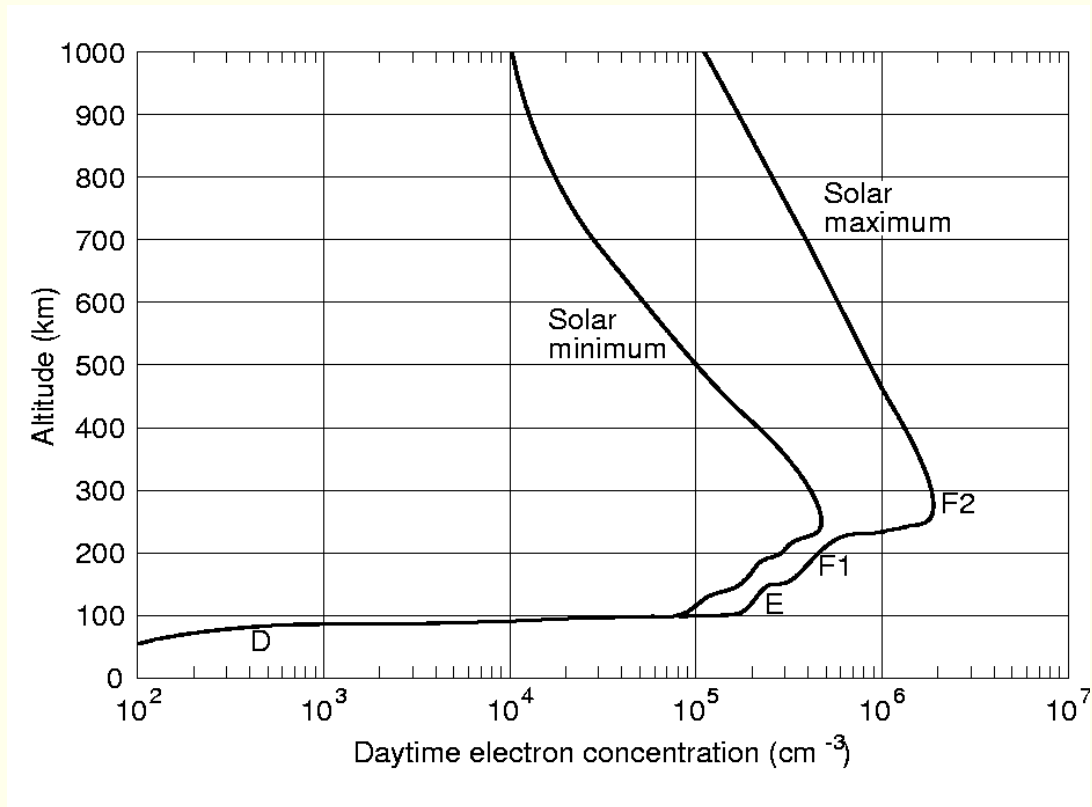
Ionosphere

- The ionized, electrically conducting part of the upper atmosphere
- The ionosphere is a **plasma**

History

- Stewart, 1882: Explained variations in the geomagnetic field
- Kenelly & Heavyside, 1902: explained Marconis transatlantic radio communication experiments
- Appleton & Barnett: experimental proof

Altitude distribution of electron density (n_e)



Continuity equation = conservation of ?

$$\frac{\partial n_e}{\partial t} = q - r - \nabla \cdot (n_e \mathbf{v}_e)$$

Ionization ($\text{m}^{-3}\text{s}^{-1}$)

Recombination ($\text{m}^{-3}\text{s}^{-1}$)

Flow ($\text{m}^{-3}\text{s}^{-1}$)

Continuity equation

$$\frac{dn_e}{dt} = q - r$$

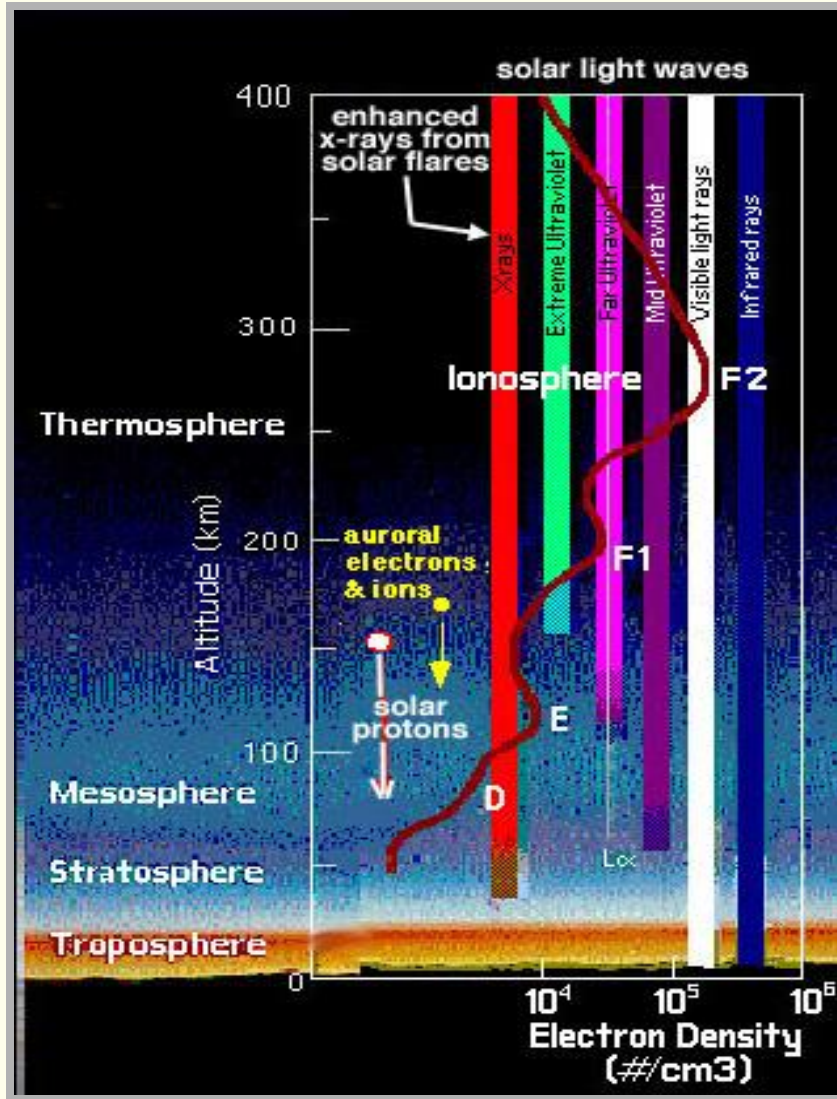
$$q = a_i I n_n$$

Ionization ($\text{m}^{-3}\text{s}^{-1}$)

Recombination ($\text{m}^{-3}\text{s}^{-1}$)

$$r = a_r n_e n_i = a_r n_e^2$$

Example: $e + \text{O}_2^+ \rightarrow \text{O} + \text{O}$ (dissociative recombination)



UV and X-ray radiation

$$\frac{dI}{dz} = In_n a_a$$



Derive Chapman layer



What does it look like in reality?

- Temperature not constant
- Many different wavelengths in solar radiation
- Several different molecules and atoms in neutral atmosphere. Composition also depends on altitude.

"E-region" - simple model calculation

O₂ dominating species, 10 nm X-ray radiation

$$a_a = 9.3 \times 10^{-23}$$

$$a_j = 9.3 \times 10^{-23}$$

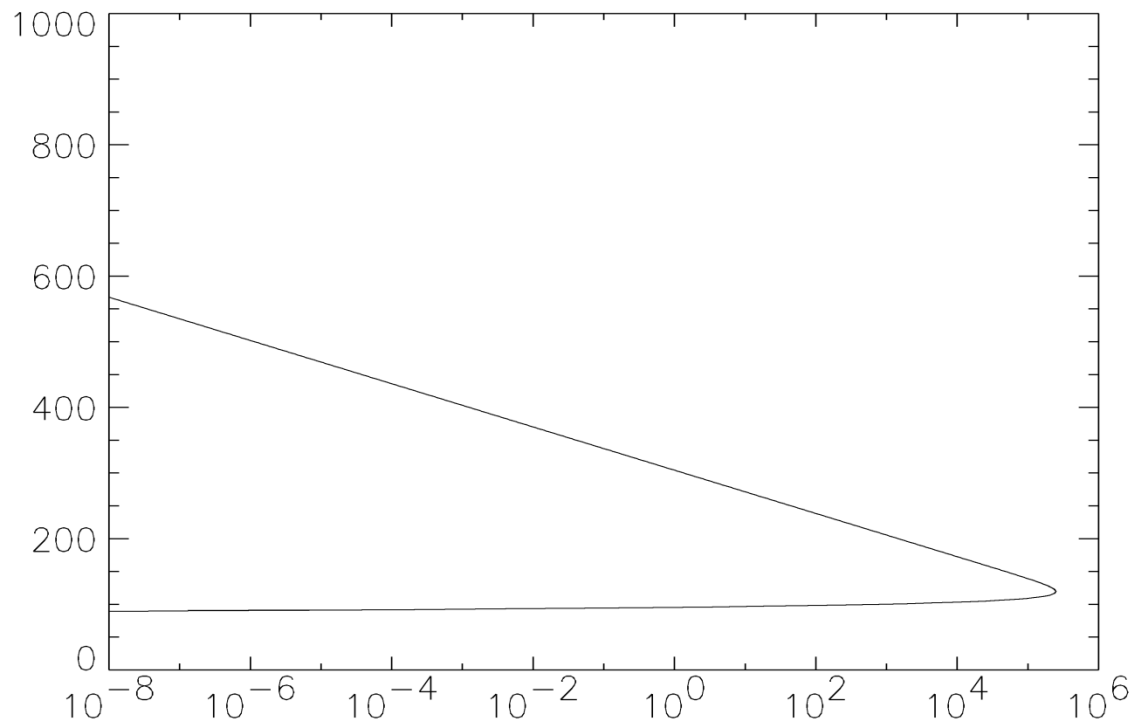
$$a_r = 3.0 \times 10^{-14}$$

$$T = 270$$

$$m = 16 \cdot 2 \cdot \text{amu}$$

$$n_0 = 2.7 \times 10^{25} \text{ m}^{-3}$$

$$I_0 = 3.6 \times 10^{13} \text{ photons/m}^2/\text{s}$$



"F1-region" - simple model calculation

O₂ dominating species, 30 nm UV radiation

$$a_a = 9.3 \times 10^{-23}$$

$$a_j = 9.3 \times 10^{-23}$$

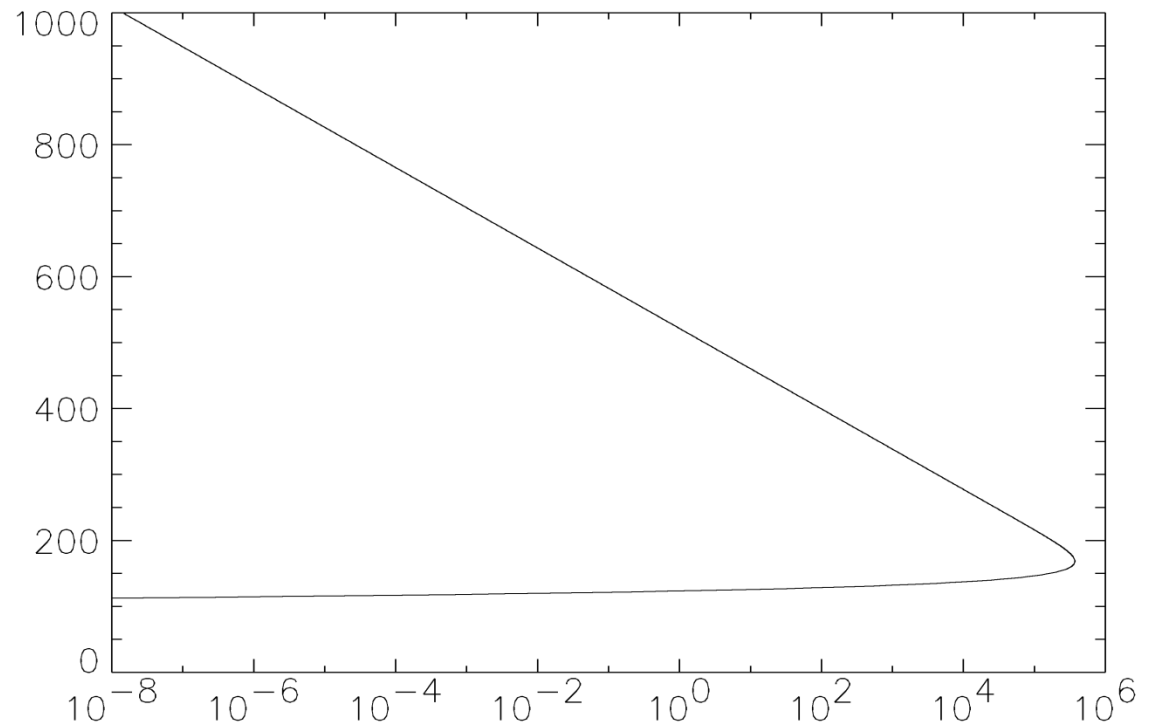
$$a_r = 3.0 \times 10^{-14}$$

$$T = 500$$

$$m = 16 \cdot 2 \cdot \text{amu}$$

$$n_0 = 2.7 \times 10^{25} \text{ m}^{-3}$$

$$I_0 = 1.5 \times 10^{14} \text{ photons/m}^2/\text{s}$$



"F2-region" - simple model calculation

O dominating species, 30 nm UV radiation

$$a_a = 9.3 \times 10^{-23}$$

$$a_j = 9.3 \times 10^{-23}$$

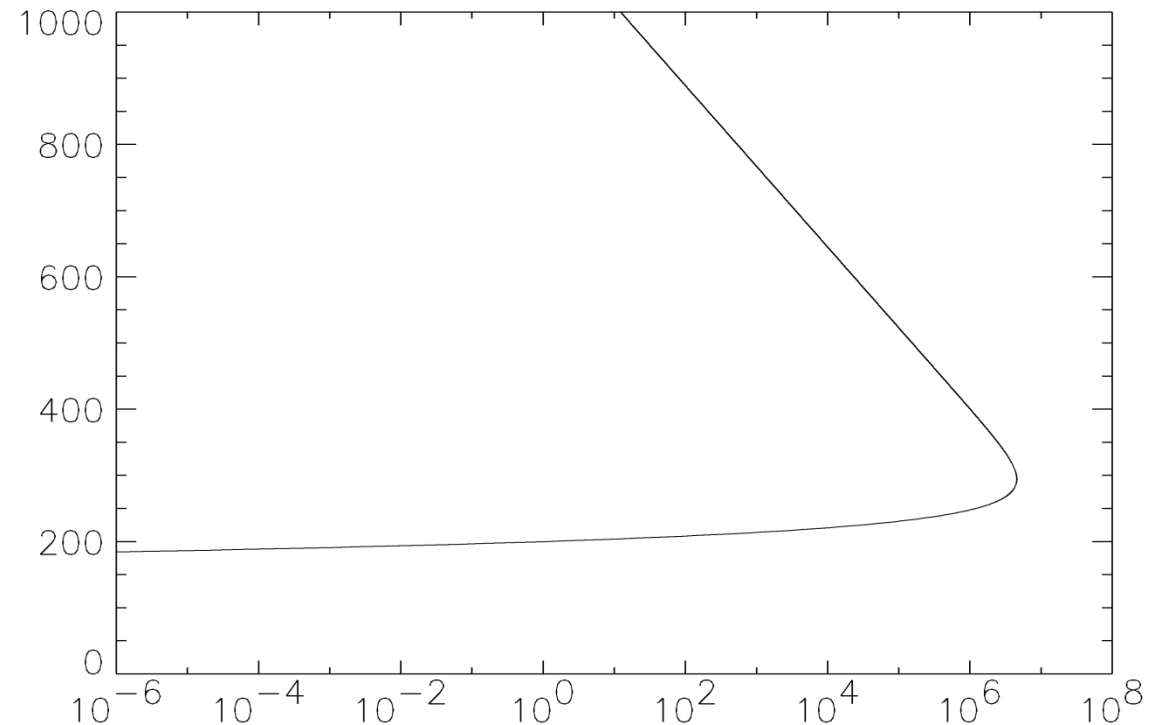
$$a_r = 1.0 \times 10^{-16}$$

$$T = 500$$

$$m = 16 \text{ amu}$$

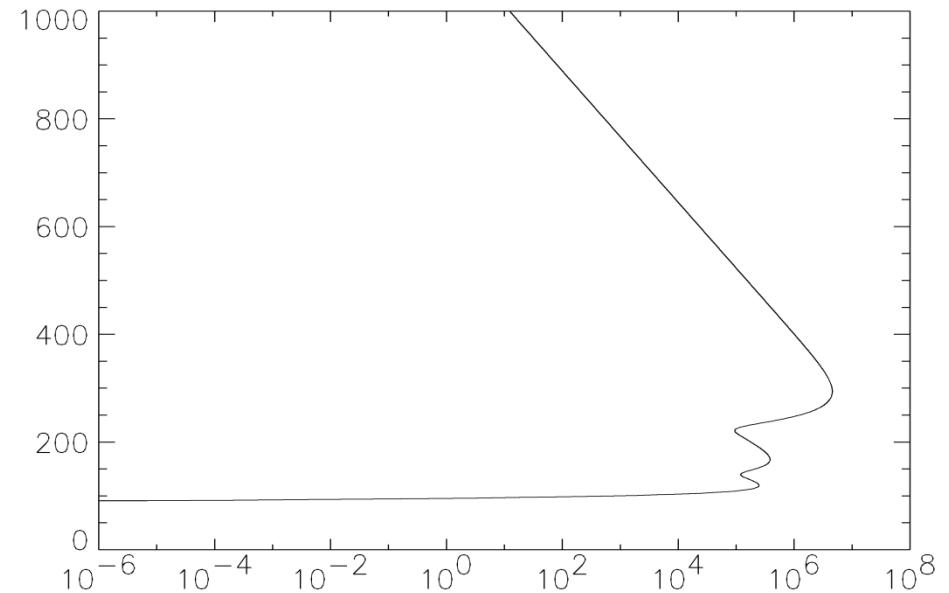
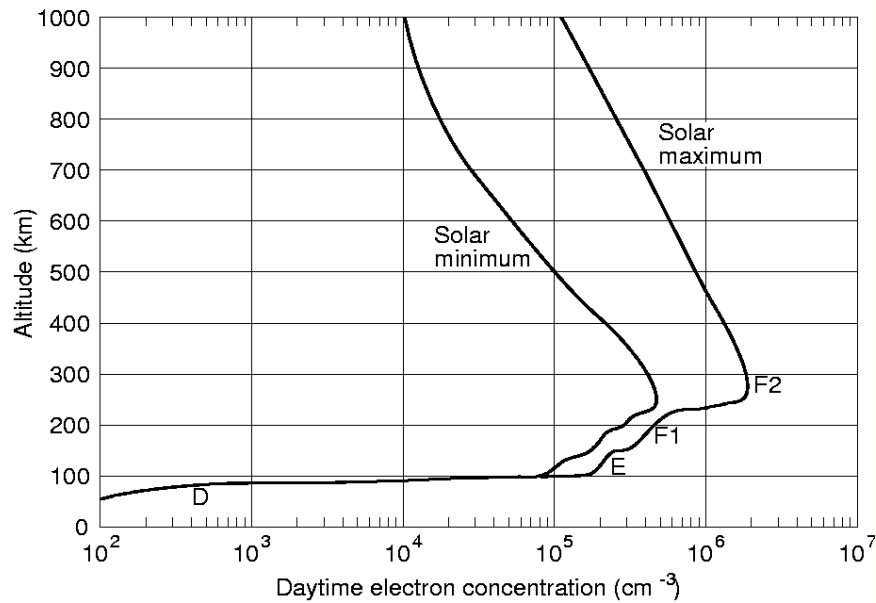
$$n_0 = 2.7 \times 10^{25} \text{ m}^{-3}$$

$$I_0 = 1.5 \times 10^{14} \text{ photons/m}^2/\text{s}$$



Measurements

"E" + "F1" + "F2"



Ionospheric layers

Layer	D	E	F ₁	F ₂
Altitude (km)	60-85	85-140	140-200	200 - ca 1500
Nighttime electron density (cm ⁻³)	<10 ²	2 · 10 ³	—	2 - 5 · 10 ⁵
Daytime electron density (cm ⁻³)	10 ³	1 - 2 · 10 ⁵	2 - 5 · 10 ⁵	0.5 - 2 · 10 ⁶
Ion species	NO ⁺ O ₂ ⁺	NO ⁺ O ₂ ⁺	NO ⁺ O ₂ ⁺ O ⁺	O ⁺ He ⁺ H ⁺
Cause of ionization	Lyman _α (1215 Å) + cosmic radiation	Lyman _β (1025 Å) X-rays	UV	UV



Propagation of radio waves in the ionosphere

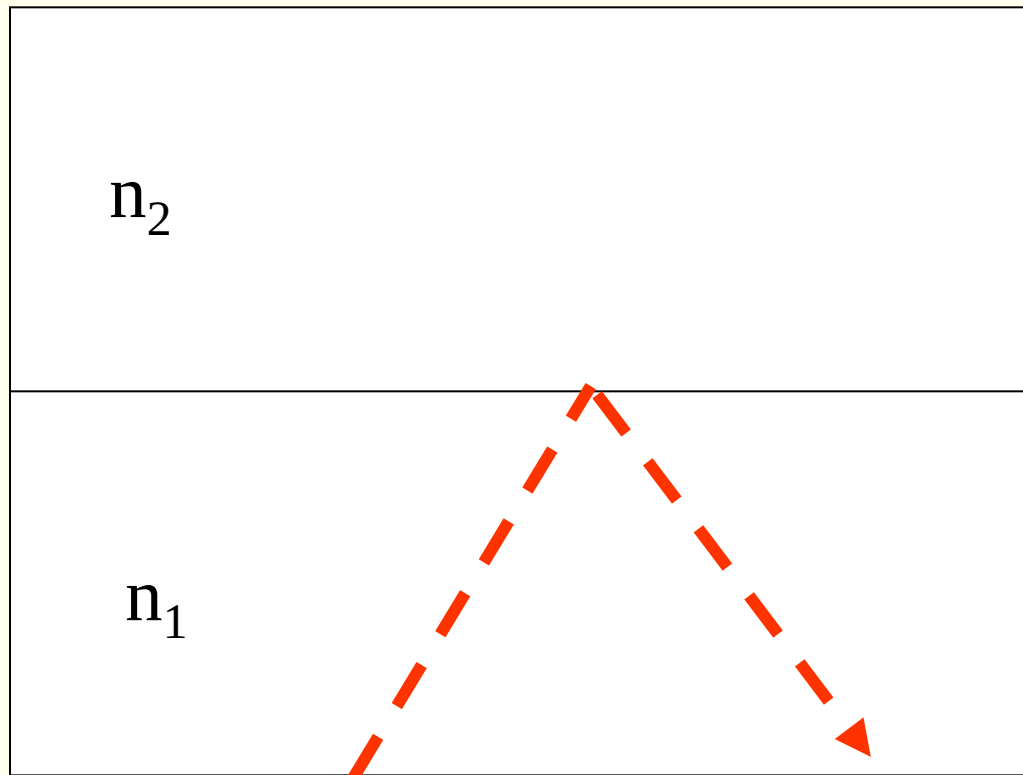
1. Absorption/damping

Takes place in the D-region due to high collision frequency. (Collisions with neutral atoms.)

2. Reflection

takes place in the F-region due to large gradients in the refraction index.

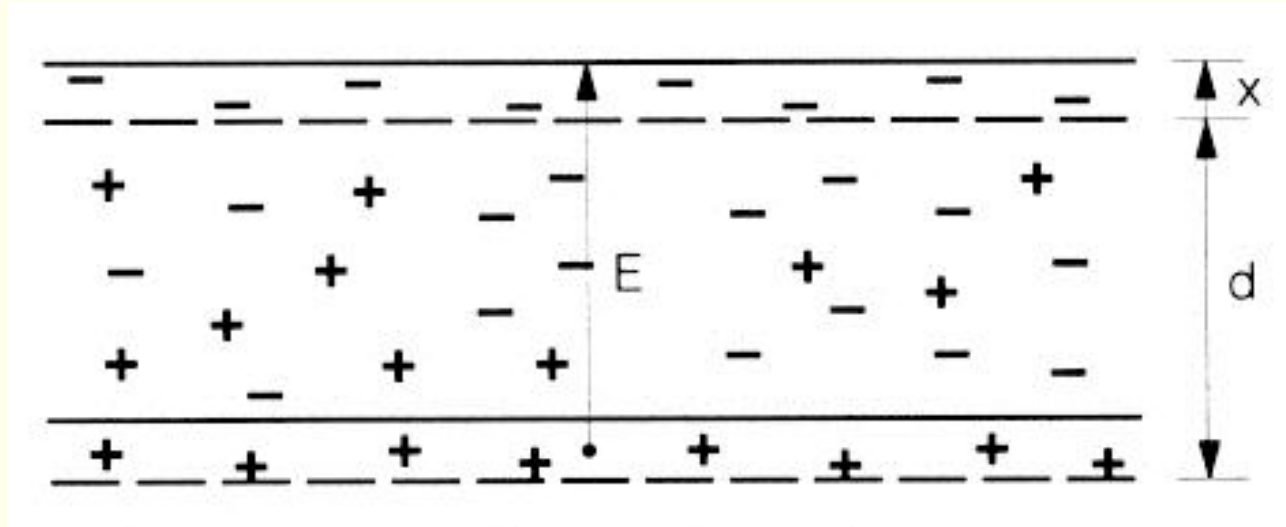
Reflection of radio waves



Total reflection at a sharp boundary (or large gradient) if

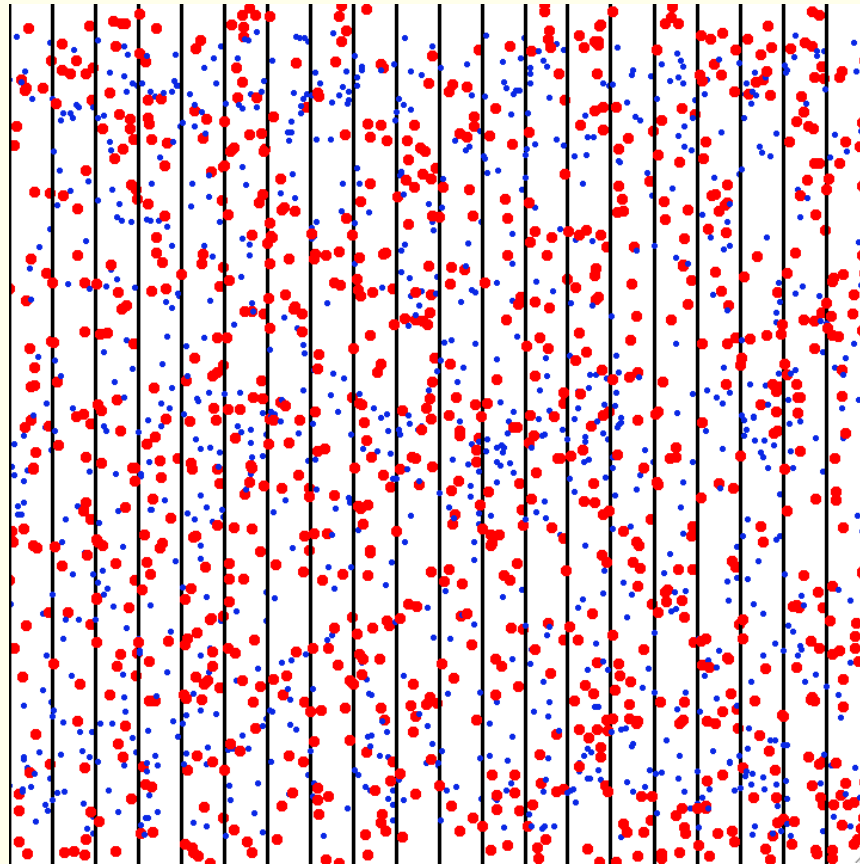
$$n_2 < n_1$$

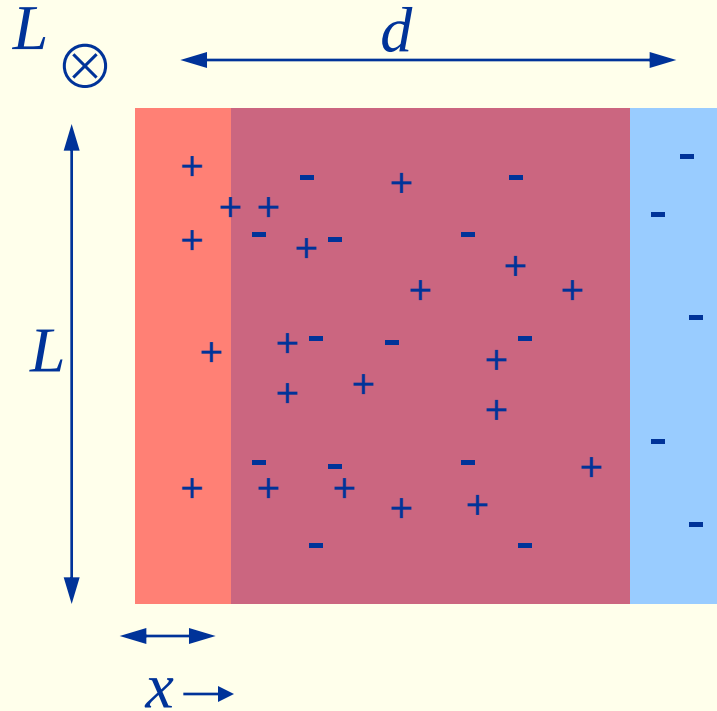
Plasma frequency



Charge imbalance creates an electric field which tends to even out the imbalance.

Plasma oscillations parallel to B





Newtons law on an individual electron inside the slab:

$$F = m_e a$$

$$F = -eE$$

$$E = \frac{\sigma}{\epsilon_0}$$

Surface charge density

$$\sigma = -en_e x$$

$$E = \frac{\sigma}{\epsilon_0}$$

$$\sigma = en_e x$$

$$\Rightarrow -\frac{n_e e^2 \chi}{\epsilon_0 m_e} = \frac{d^2 \chi}{dt^2}$$

$$x = \sin(\omega_{pe} t)$$

$$\omega_{pe} \equiv \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}$$

What is the plasma frequency f_{pe} at the daytime E-region, close to solar minimum? (see Fälthammar p 28)

$$f_{pe} = \frac{\omega_{pe}}{2\pi} \equiv \frac{1}{2\pi} \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}$$

Blue

7 kHz

Yellow

400 MHz

Green

3 MHz

Red

2 GHz

$$f = \frac{\omega_{pe}}{2\pi} \equiv \frac{1}{2\pi} \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}} = \frac{1}{2\pi} \sqrt{\frac{(1.6 \cdot 10^{-19})^2}{8.854 \cdot 10^{-12} \cdot 0.91 \cdot 10^{-30}}} \sqrt{n_e} =$$

$$8.97 \sqrt{n_e} = 8.97 \sqrt{10^5 \cdot 10^6} = 2.8 \cdot 10^6 \text{ Hz} = 2.8 \text{ MHz}$$

Green

Index of refraction for electromagnetic waves in a plasma

$$(1) \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$(2) \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$(3) \quad \mathbf{j} = -en_e \mathbf{v}_e$$

$$(4) \quad m_e \frac{\partial \mathbf{v}_e}{\partial t} = -e\mathbf{E}$$

Assume all quantities vary sinusoidally, with frequency ω , e.g.:

$$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$$

$$(1) \Rightarrow \nabla \times (\nabla \times \mathbf{E}) = -\nabla \times \frac{\partial \mathbf{B}}{\partial t}$$

$$(2) \Rightarrow \nabla \times \frac{\partial \mathbf{B}}{\partial t} = \mu_0 \frac{\partial \mathbf{j}}{\partial t} + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\therefore \nabla \times (\nabla \times \mathbf{E}) = -\mu_0 \frac{\partial}{\partial t} (en_e \mathbf{v}_e) + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$\Rightarrow$

$$\nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\mu_0 en_e \frac{\partial \mathbf{v}_e}{\partial t} + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$\Rightarrow$

Index of refraction for electromagnetic waves in a plasma

$$\cancel{ik(ik \cdot \mathbf{E})} - k^2 \mathbf{E} = \mu_0 (-i\omega) en_e \mathbf{v}_e + \frac{1}{c^2} (-i\omega)^2 \mathbf{E}$$

Does not represent E.M. wave

(4) $\Rightarrow$

$$-k^2 \mathbf{E} = \mu_0 (-i\omega) en_e \frac{ie\mathbf{E}}{\omega m_e} + \frac{1}{c^2} (-i\omega)^2 \mathbf{E}$$

$\Rightarrow$

$$c^2 k^2 = -c^2 \frac{\mu_0 n_e e^2}{m_e} + \omega^2 = \frac{-1}{\cancel{\mu_0 \epsilon_0}} \frac{\cancel{\mu_0} n_e e^2}{m_e} + \omega^2$$

$$\therefore \omega^2 = c^2 k^2 + \omega_p^2$$

$$n^2 = \frac{c^2}{v_{ph}^2} = \frac{c^2 k^2}{\omega^2} = \frac{\omega^2 - \omega_p^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}$$

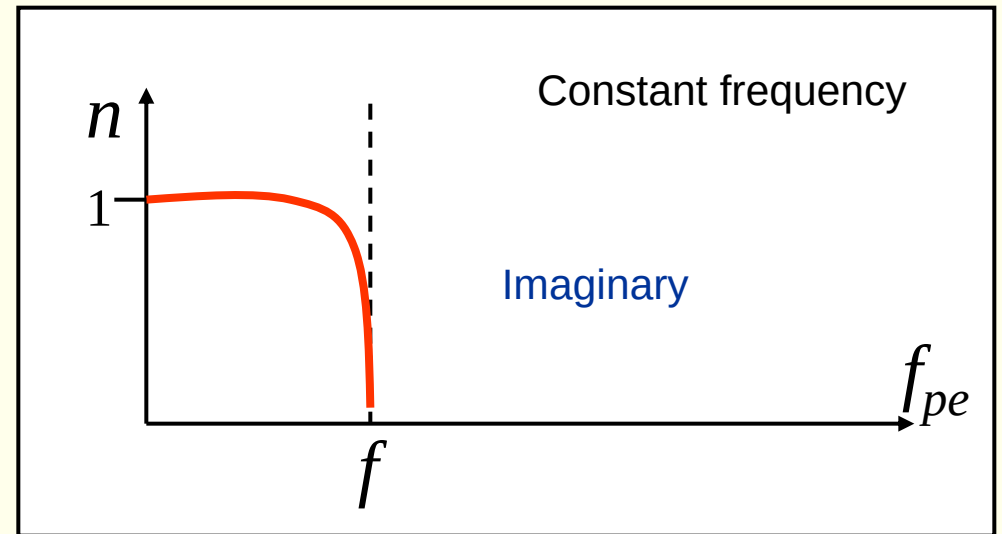
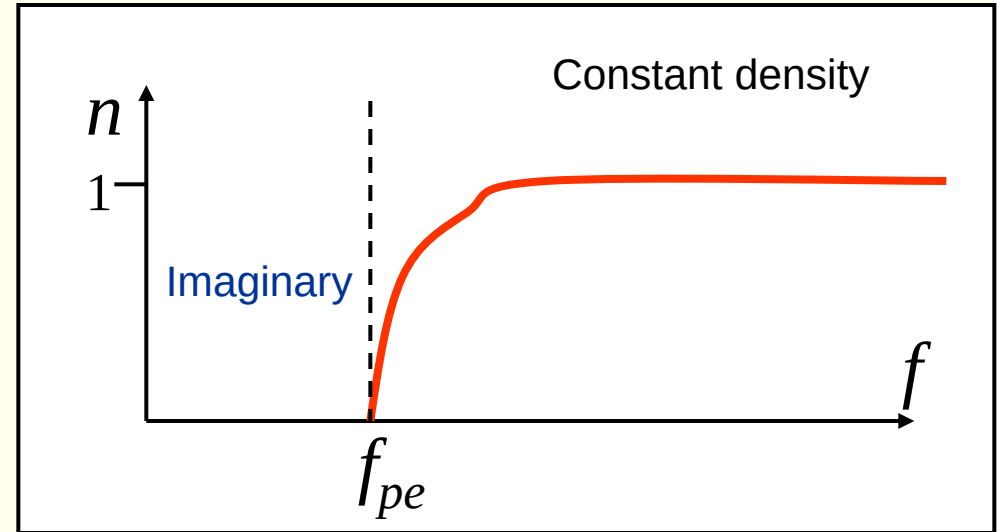
$\therefore$

$$n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} = \sqrt{1 - \frac{f_p^2}{f^2}}$$

Refraction index for plasma

$$n = \frac{c}{v_{ph}} = \sqrt{1 - \frac{f_{pe}^2}{f^2}}$$

$$\omega_{pe} \equiv \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}$$

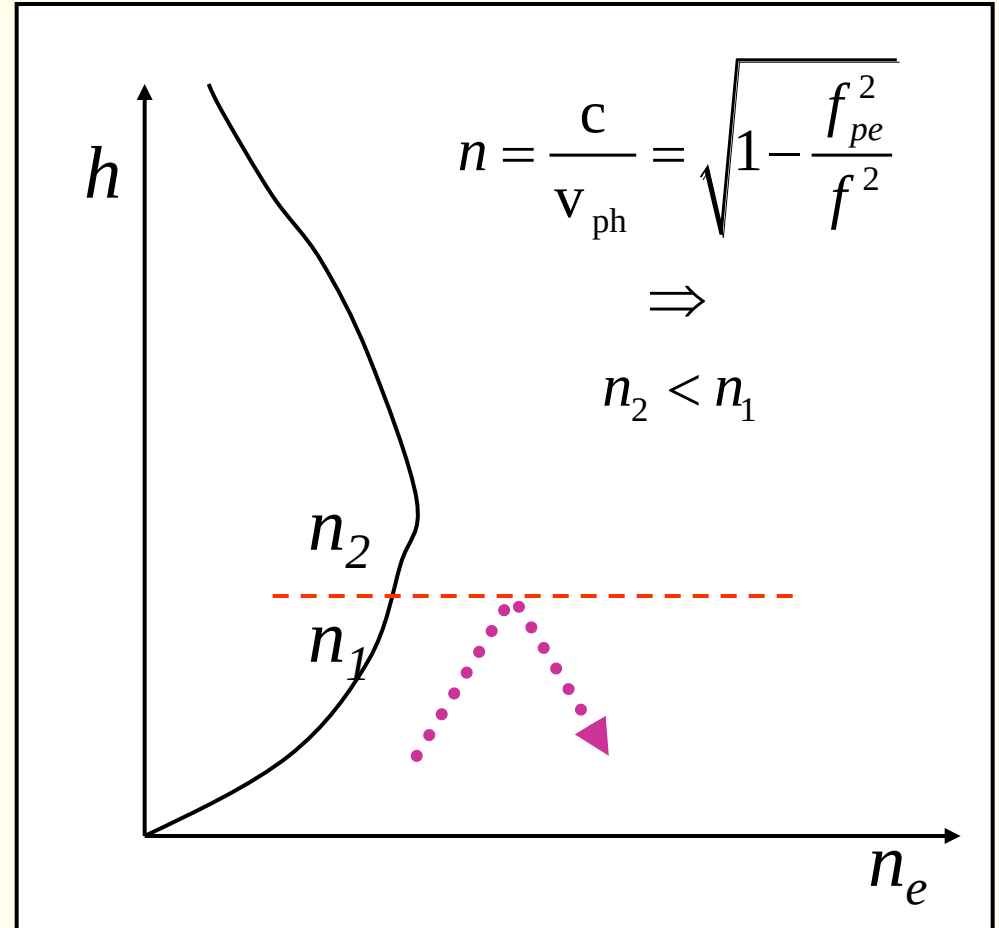


Where does the total reflection take place?

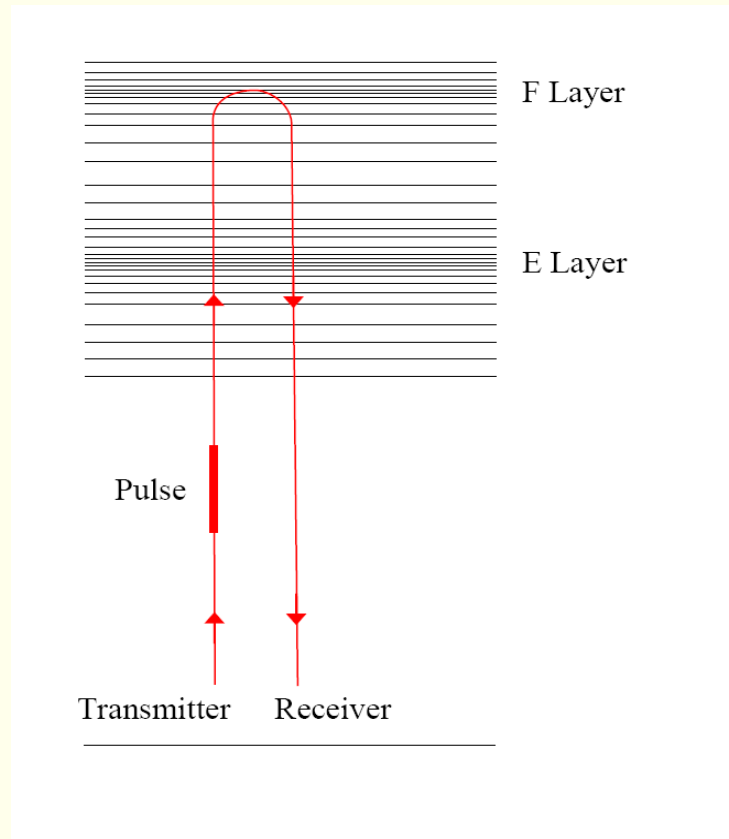
Large gradient when

$$f \approx f_{pe}$$

Higher frequencies $\rightarrow$ higher $f_{pe}(n_e)$



Ionosonde



The pulse will be reflected where

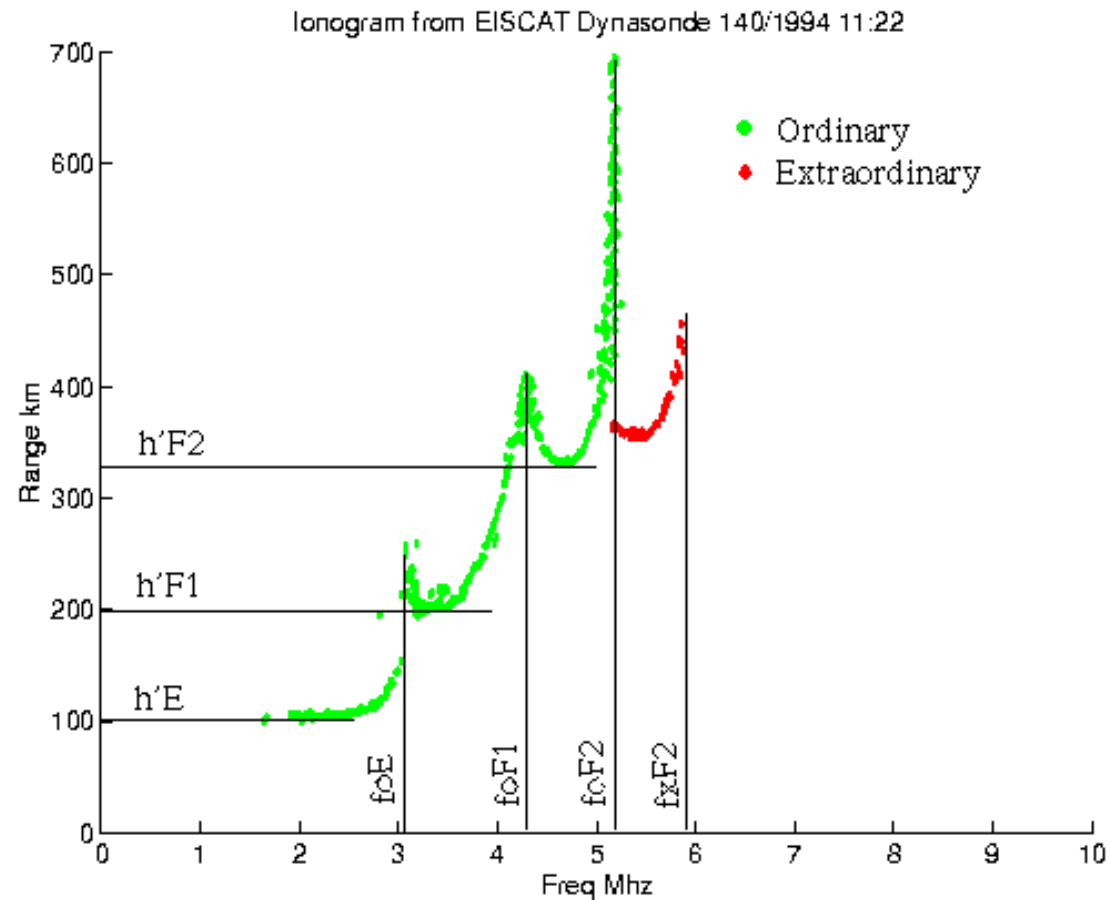
$$f = f_{pe}$$

The altitude will be determined by

$$2h = ct$$

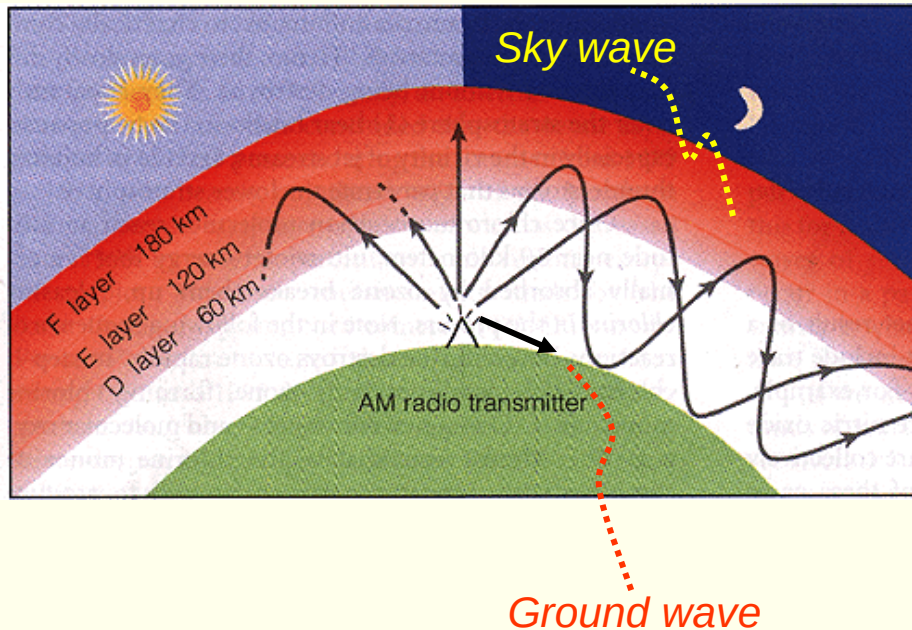
Where t is the time between when the pulse is sent out and the registered again.

Ionogram



Reflection of radio waves

F2-layer during night:



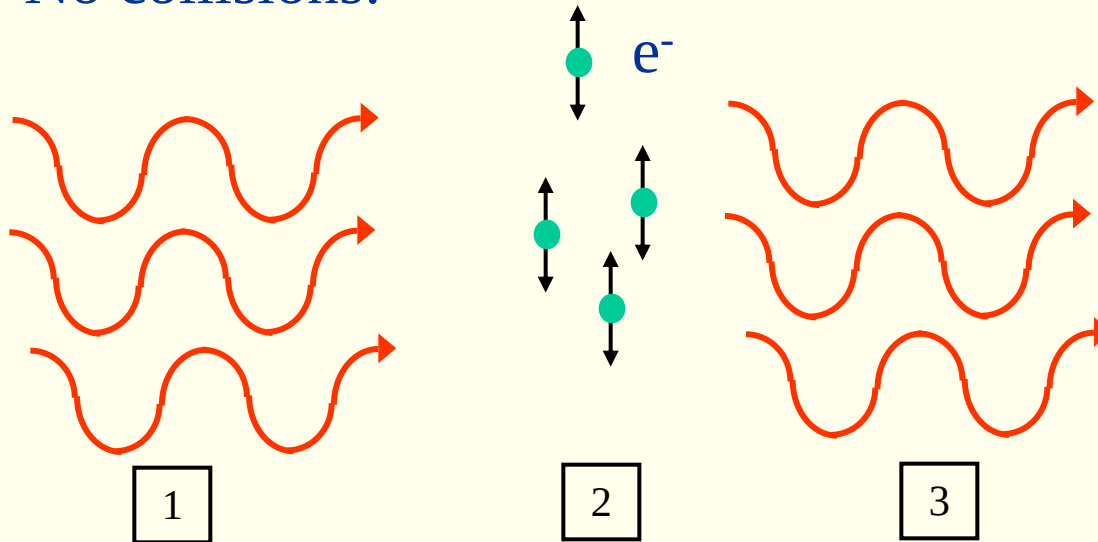
$$n_e = 5 \cdot 10^{11} \text{ m}^{-3} \Rightarrow$$

$$f_{pe} = 10^7 \text{ Hz} = 10 \text{ MHz}$$

= HF/short wave

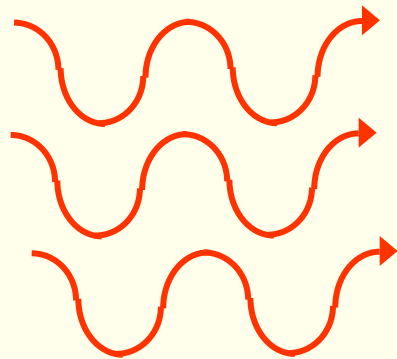
Absorption of radio waves

No collisions:

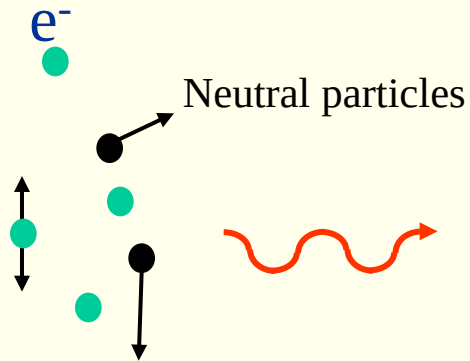


Absorption of radio waves

With collisions:



1



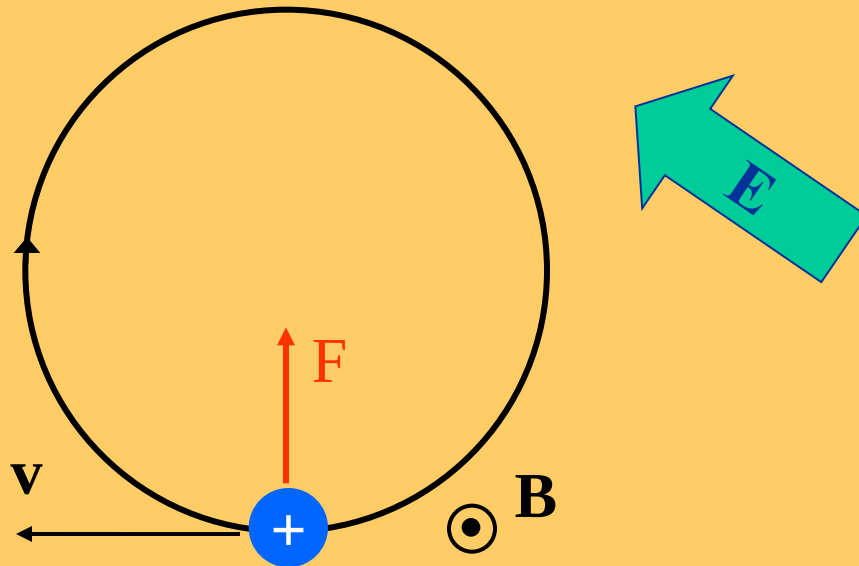
2

3

Think about this:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

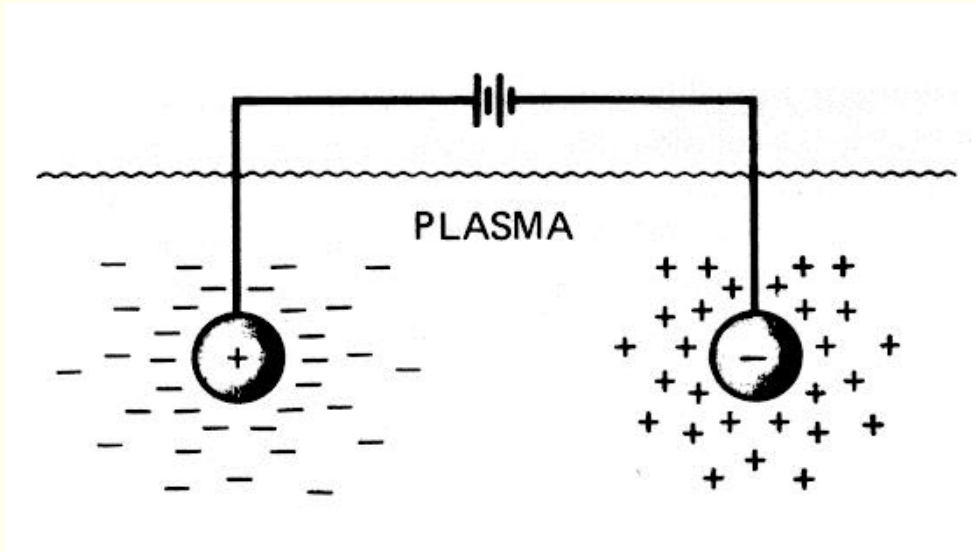
What happens if you add an electric field $\mathbf{E}$?





Last Minute!

Quasineutrality



$$\Phi = \Phi_0 e^{-x/\lambda_D}$$

Debye length

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{n_e e^2}}$$

$$\frac{\Delta n}{n} = \frac{(n_e - n_i)}{n_e} < \left(\frac{\lambda_D}{l_c} \right)^2$$

$$l_C \gg \lambda_D \Rightarrow$$

Plasma close to neutral:

$$n_e \approx n_i$$



