Last lecture (4)

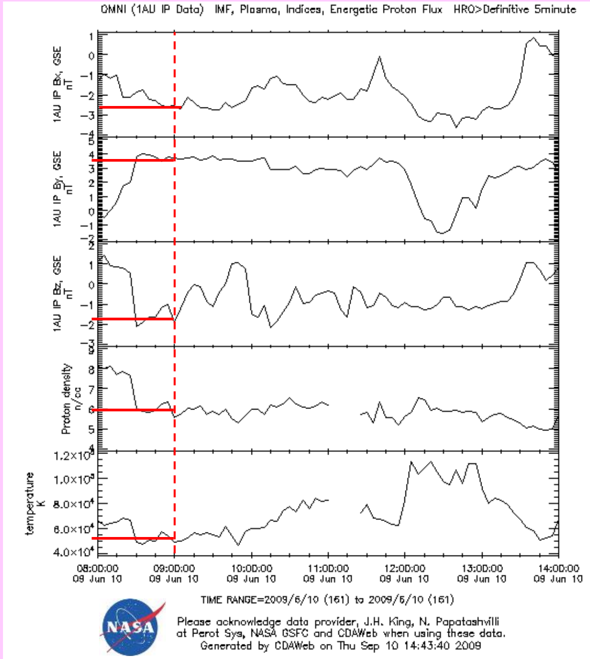
- Ionosphere
  - ionospheric layers
  - index of refraction

Today's lecture (5)

- Ionosphere
  - reflection of radio waves
  - particle drift motion in magnetized plasma
  - electrical conductivity in magnetized plasma
- Magnetosphere, introduction

Today

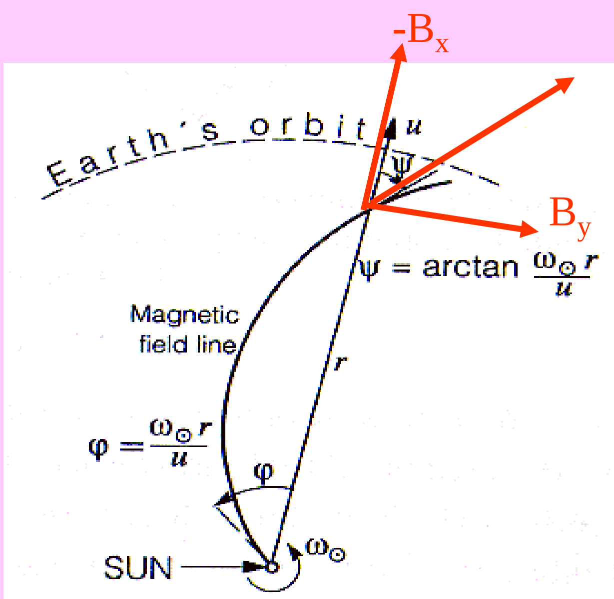
Activity	Date	Time	Room	Subject	Litterature
L1	28/8	15-17	Q21	Course description, Introduction, The Sun 1	**CGF** Ch 1.1,1.2, 1.4, 5, (p 110-113), 6.3
L2	29/8	13-15	Q2	The Sun 2, Plasma physics 1	**CGF** Ch 1.3, 5 (p 114-121)
L3	4/9	10-12	E2	Solar wind, The ionosphere and atmosphere 1, Plasma physics 2	**CGF** Ch 6.1, 2, 3.1-3.2, 3.5, **LL** Ch III, Extra material
T1	6/9	8-10	Q21	Mini-group work 1	
L4	6/9	15-17	Q2	The ionosphere 2, Plasma physics 3	**CGF** Ch 3.4, 3.7, 3.8
T2	10/9	15-17	Q21	Mini-group work 2	
L5	11/9	10-12	E3	The Earth's magnetosphere 1, Plasma physics 4	**CGF** 4-1-4.3, **LL** Ch I, II, IV.A
T3	17/9	8-10	Q21	Mini-group work 3	
L6	18/9	13-15	Q33	The Earth's magnetosphere 2, Other magnetospheres	**CGF** Ch 4.6-4.9, **LL** Ch V.
L7	19/9	13-15	Q2	Aurora, Measurement methods in space plasmas and data analysis 1	**CGF** Ch 4.5, 10**, LL** Ch VI, Extra material
T4	24/9	8-10	Q2	Mini-group work 4	
L8	24/9	15-17	V3	Space weather and geomagnetic storms	**CGF** Ch 4.4, **LL** Ch IV.B-C, VII.A-C
T5	2/10	8-10	Q31	Mini-group work 5	
L9	2/10	13-15	Q2	Alfvén waves, Interstellar and intergalactic plasma, Cosmic radiation	**CGF** Ch 7-9, Extra material
T6	8/10	15-17	Q21		
L10	9/10	10-12	Q2	Guest Lecture by Swedish astronaut Christer Fuglesang	
Written examination	16/10	14-19	L21, L22, L31		



Mini-groupwork 2

a)

$$\psi = \arctan \frac{\omega_{sun} r}{u_{sw}} \quad \Rightarrow \quad u_{sw} = \frac{\omega_{sun} r}{\tan \psi}$$



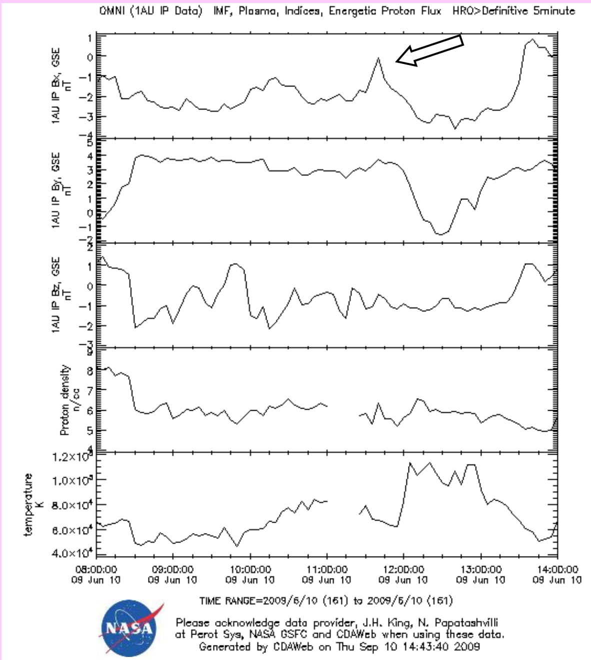
$\omega_{sun} = 2\pi/T = 2.9\cdot10^{-6}$ s^{-1} (T = 25 days at equator)

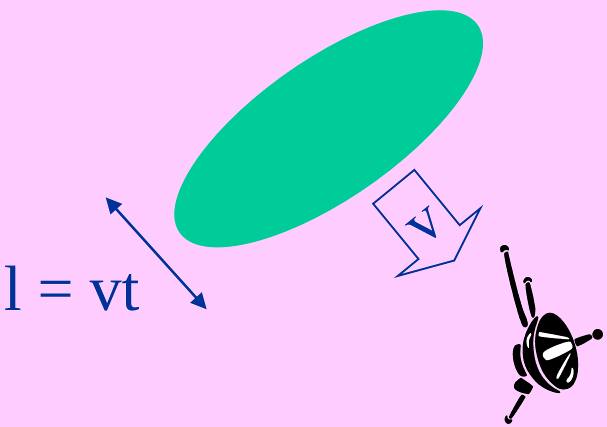
r = 1 A.U.

$\tan \psi = |B_y/B_x| \approx 3.6/2.6$ (from figure) ($\psi = 41°$)

With these figures I get $u_{sw} = 313$ km/s

Mini-groupwork 2





b)

The magnetic Reynolds number is calculated by using typical plasma flow velocities v_c and typical length scales of magnetic field variations l_c

Use solar wind velocity obtained in a) for typical flow velocity. To obtain l_c, multiply the time t it takes the magnetic field structure (indicated in the figure), to pass over the satellite and use $l_c, = vt$. I get $l_c = 2.8 \cdot 10^8$ m.

Using a temperature of $5 \cdot 10^4$ K, we can evaluate the conductivity, remembering that the temperature should be given in eV. We get the conversion from

$$W = \frac{3}{2} k_B T$$

which gives the result that 1 eV corresponds to a temperature of 7729 K. We then get $T = 6.5$ eV, and

$$\sigma = 3.1 \cdot 10^4 \text{ S/m}$$

Putting in the numbers I get

$$R_m = \mu_0 \, \sigma \, v_c \, l_c \approx 9.8 \cdot 10^{14} >> 1$$

So the solar wind magnetic field is frozen into the plasma to a very good approximation.

Energy - temperature

Average energy of molecule/atom:

$$E = \frac{3}{2} k_B T \quad \Rightarrow$$

$$T = \frac{2E}{3k_B}$$

$$1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ J} \Rightarrow$$

$$T = \frac{2E}{3k_B} = \frac{2 \cdot 1.6 \cdot 10^{-19} \text{ J}}{3 \cdot 1.38 \cdot 10^{-23} \dfrac{\text{J}}{\text{K}}} = 7729 \text{ K}$$

But beware!

In plasma physics, usually:

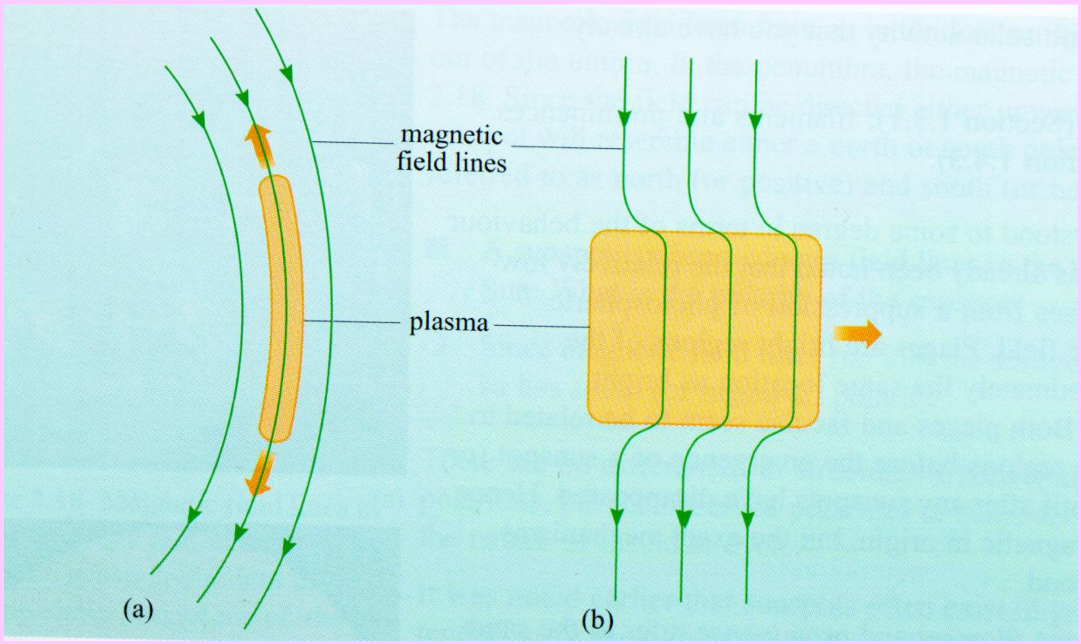
$$E = \cancel{\frac{3}{2}} k_B T \quad \Rightarrow$$

$$T = \frac{E}{k_B}$$

$$1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ J} \Rightarrow$$

$$E = k_B T = \frac{1.6 \cdot 10^{-19} \,\mathrm{J}}{1.38 \cdot 10^{-23} \,\dfrac{\mathrm{J}}{\mathrm{K}}} = 11594 \,\mathrm{K}$$

Does the plasma follow the magnetic field (a) or the other way around (b)?



Depends on relative energy density (pressure)

$$p_{pl} = nk_B T$$

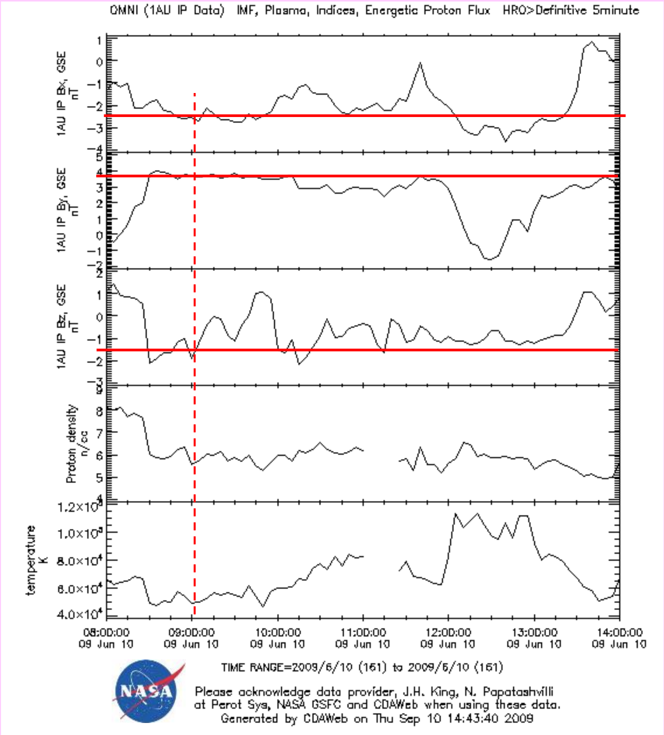
$$p_B = \frac{B^2}{2\mu_0}$$

$$\beta = \frac{p_{pl}}{p_B}$$

$\beta << 1$ $\beta >> 1$

Mini-groupwork 2

c)



$$\rho = n_e m_p = 6.1\cdot10^{6}\cdot1.67\cdot10^{-27} = 1.02\cdot10^{-20}$$

Then the kinetic energy density is (v = 313 km/s):

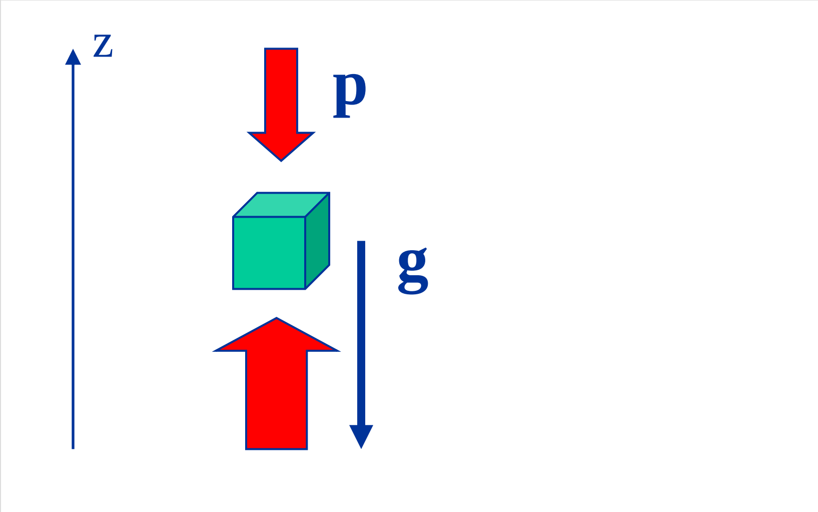
$$\rho v^2/2 = 5.0\cdot10^{-10}\ \mathrm{Jm}^{-3}$$

The magnetic energy density is (using values of figure)

$$\frac{B^2}{2\mu_0} = \frac{\left(B_x^2 + B_y^2 + B_z^2\right)^2}{2\mu_0} = (2.6^2 + 3.6^2 + 1.7^2)\cdot(10^{-9})^2/2\mu_0 = 9\cdot10^{-12}\ \mathrm{Jm}^{-3}$$

The ratio between the kinetic and magnetic energy densities is approximately **50,** thus the plasma motion determines the magnetic field configuration, and not the other way around.

Atmospheric scale height



$$-\frac{dp}{dz} = g\rho_m$$ hydrostatic equilibrium for a volume element

$$p = nk_BT = \frac{\rho k_B T}{m}$$ ideal gas law

$$-\frac{k_BT}{m}\frac{d\rho_m}{dz} = g\rho_m$$ if T is constant

$$\rho_m = const \cdot e^{-z/(k_BT/gm)} = const \cdot e^{-z/H}$$

Scale height

$H = k_BT/gm$

Continuity equation

$$\frac{dn_e}{dt} = q - r$$

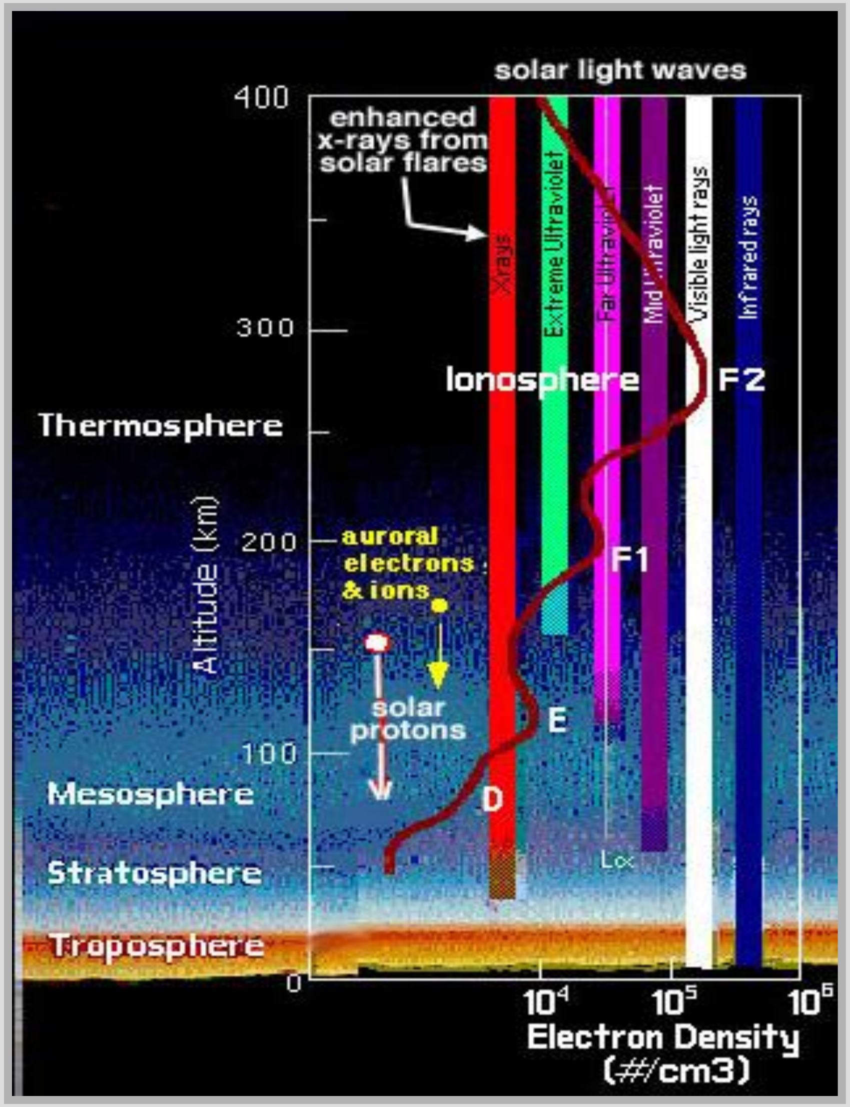
$$q = a_i I n_n$$

Ionization ($m^{-3}s^{-1}$)

Recombination ($m^{-3}s^{-1}$)

$$r = a_r n_e n_i = a_r n_e^2$$

Example: $e + O_2^+ \rightarrow O + O$ (dissociative recombination)

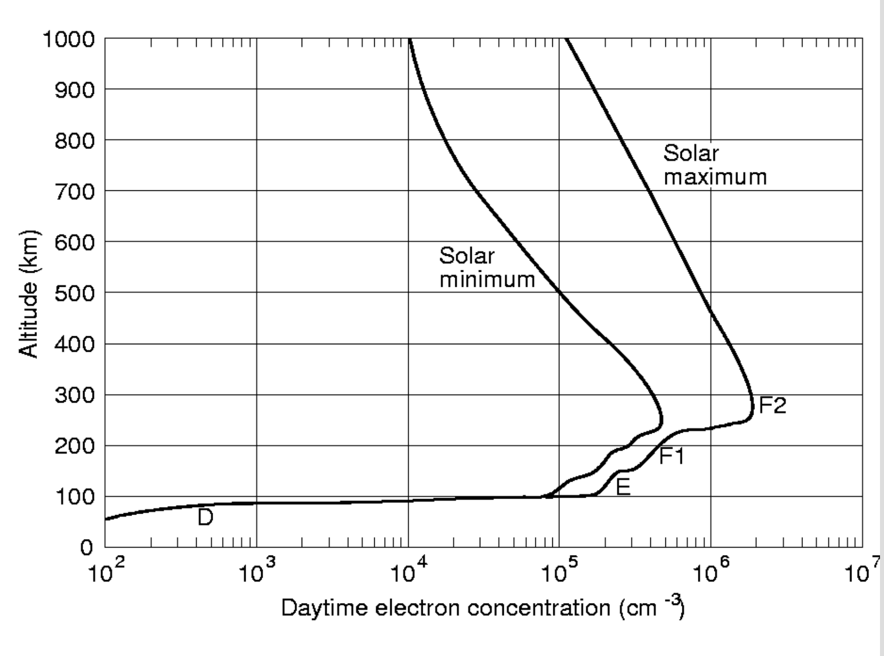


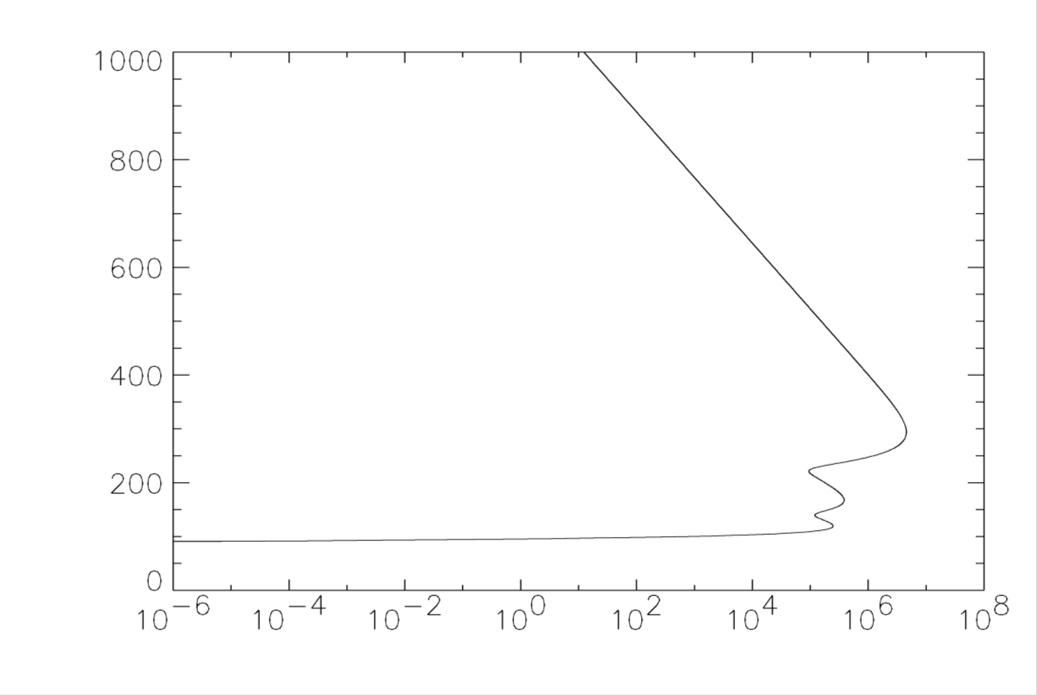
UV and X–ray radiation

$$\frac{dI}{dz} = I n_n a_a$$

Measurements

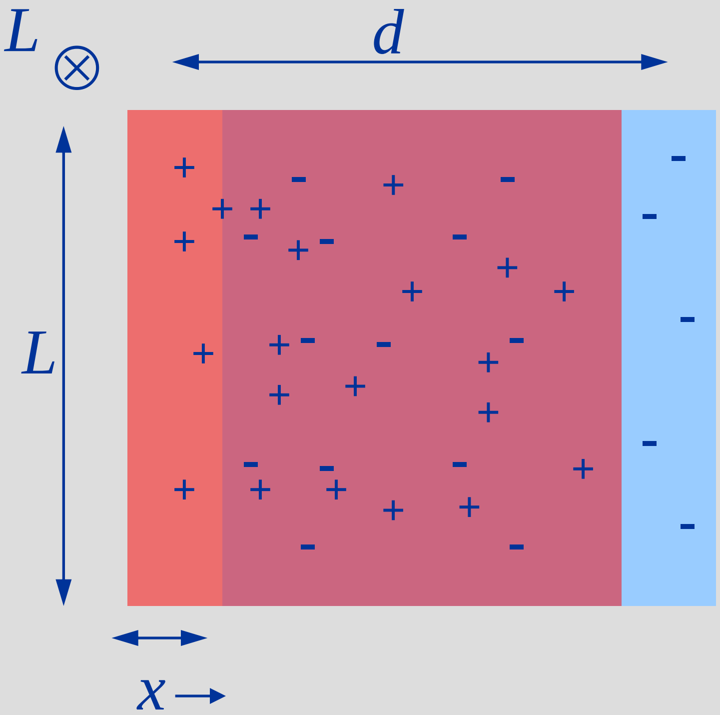
”E” + ”F1” + ”F2”





Ionospheric layers

Layer	D	E	F_1	F_2
Altitude (km)	60-85	85-140	140-200	200 - ca 1500
Nighttime electron density (cm^{-3})	$<10^2$	$2 \cdot 10^3$	—	$2 - 5 \cdot 10^5$
Daytime electron density (cm^{-3})	10^3	$1 - 2 \cdot 10^5$	$2 - 5 \cdot 10^5$	$0.5 - 2 \cdot 10^6$
Ion species	NO^+ O_2^+	NO^+ O_2^+	NO^+ O_2^+ O^+	O^+ He^+ H^+
Cause of ionization	Lyman α (1215 Å) + cosmic radiation	Lyman β (1025 Å) X-rays	UV	UV



$$-\frac{n_e e^2 x}{\varepsilon_0 m_e} = \frac{d^2 x}{dt^2}$$

$$x = \sin(\omega_{pe} t)$$

$$F = m_e a \qquad E = \frac{\sigma}{\varepsilon_0}$$

$$F = -eE \qquad \sigma = e n_e x$$

$$\boxed{\omega_{pe} \equiv \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}}$$

Index of refraction for electromagnetic waves in a plasma (corrected)

(1) $$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

(2) $$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

(3) $$\mathbf{j} = -e n_e \mathbf{v}_e$$

(4) $$m_e \frac{\partial \mathbf{v}_e}{\partial t} = -e\mathbf{E}$$

Assume all quantities vary sinusoidally, with frequency ω, *e.g.*:

$$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

(1) $$\Rightarrow \quad \nabla \times (\nabla \times \mathbf{E}) = -\nabla \times \frac{\partial \mathbf{B}}{\partial t}$$

(2) $$\Rightarrow \quad \nabla \times \frac{\partial \mathbf{B}}{\partial t} = \mu_0 \frac{\partial \mathbf{j}}{\partial t} + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\therefore \quad \nabla \times (\nabla \times \mathbf{E}) = +\mu_0 \frac{\partial}{\partial t}(e n_e \mathbf{v}_e) - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\Rightarrow$$

$$\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = +\mu_0 e n_e \frac{\partial \mathbf{v}_e}{\partial t} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\Rightarrow$$

Index of refraction for electromagnetic waves in a plasma (corrected)

$$i\mathbf{k}\cancel{\left(i\mathbf{k}\cdot\mathbf{E}\right)}-k^2\mathbf{E}=\mu_0\left(-i\omega\right)en_e\mathbf{v}_e-\frac{1}{c^2}\left(-i\omega\right)^2\mathbf{E}$$

Does not represent E.M. wave

(4) $\Rightarrow$

$$k^2\mathbf{E}=-\mu_0 en_e\frac{e\mathbf{E}}{m_e}+\frac{1}{c^2}\omega^2\mathbf{E}$$

$\Rightarrow$

$$c^2k^2=-c^2\frac{\mu_0 n_e e^2}{m_e}+\omega^2=\frac{-1}{\cancel{\mu_0}\varepsilon_0}\frac{\cancel{\mu_0}n_e e^2}{m_e}+\omega^2$$

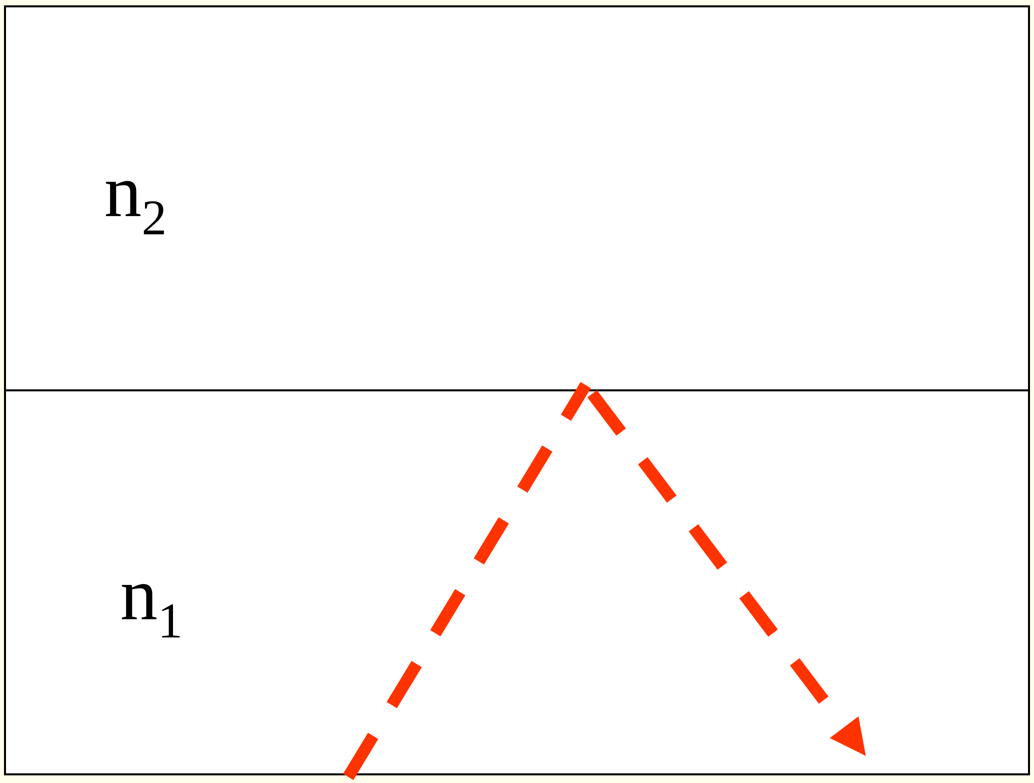
$$\therefore\quad \omega^2=c^2k^2+\omega_p^2$$

$$n^2=\frac{c^2}{v_{ph}^2}=\frac{c^2k^2}{\omega^2}=\frac{\omega^2-\omega_p^2}{\omega^2}=1-\frac{\omega_p^2}{\omega^2}$$

$\therefore$

$$\boxed{n=\sqrt{1-\frac{\omega_p^2}{\omega^2}}=\sqrt{1-\frac{f_p^2}{f^2}}}$$

Reflection of radio waves



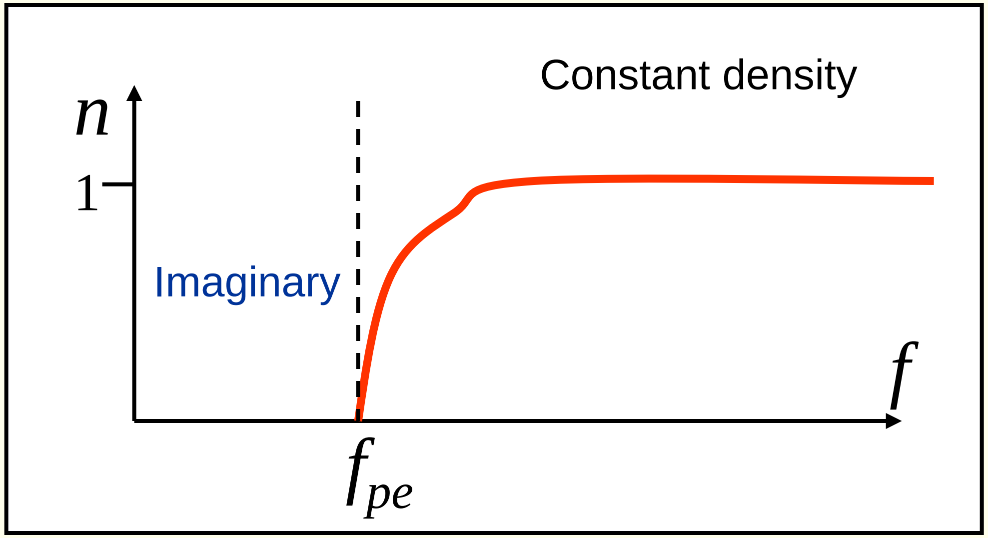
Total reflection at a sharp boundary (or large gradient) if

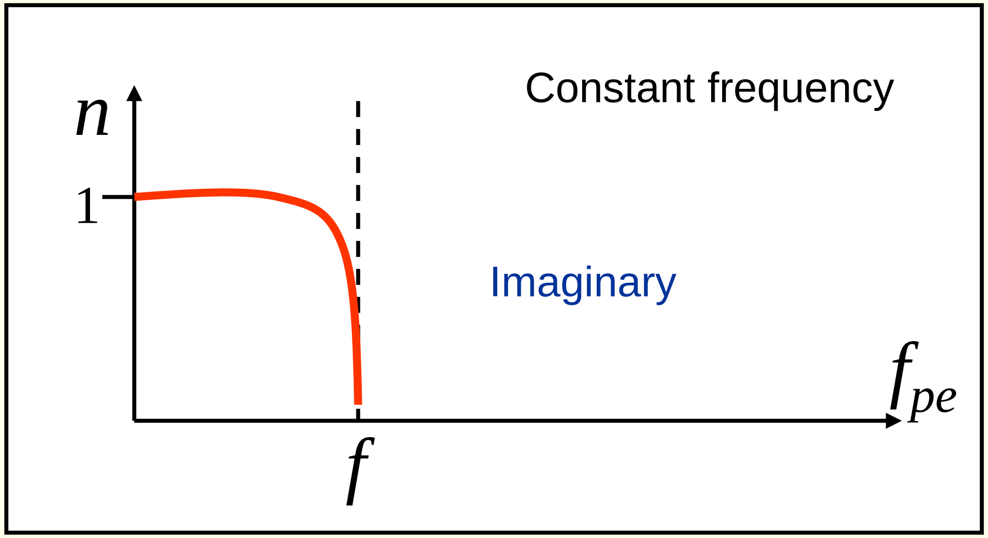
$$n_2 < n_1$$

Refraction index for plasma

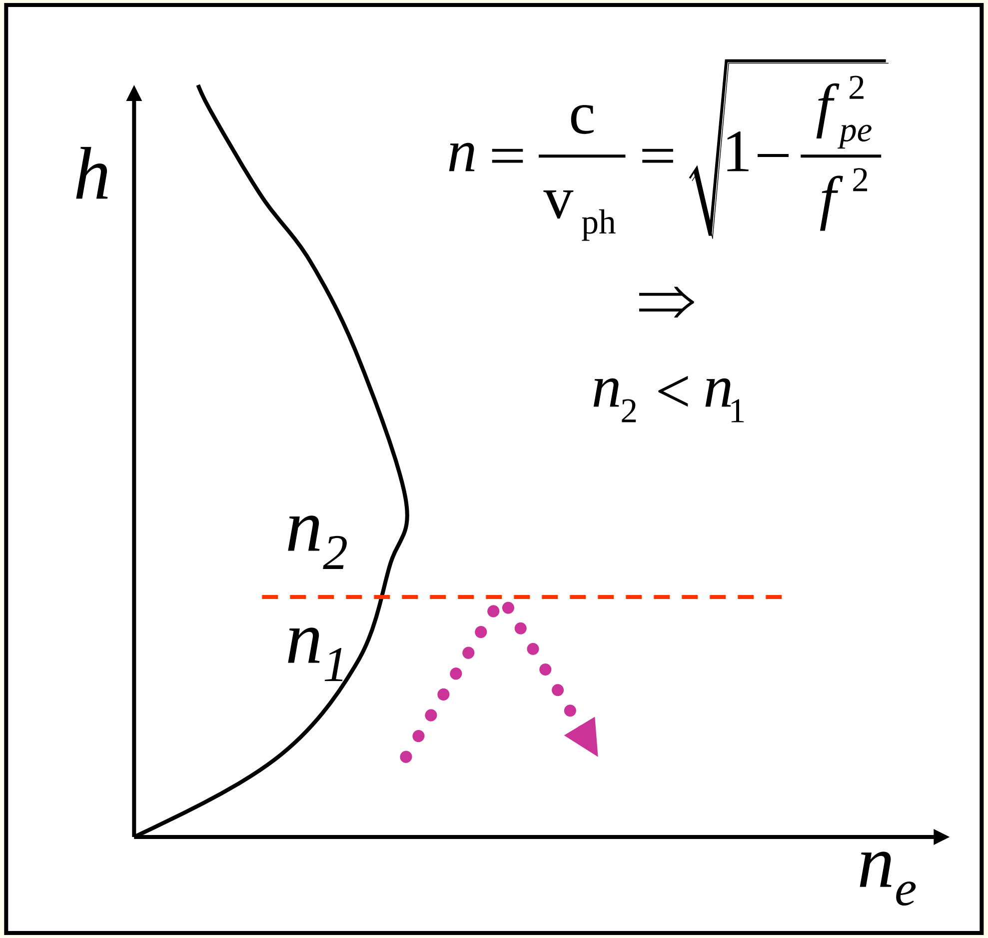
$$n = \frac{c}{v_{ph}} = \sqrt{1 - \frac{f_{pe}^2}{f^2}}$$

$$\omega_{pe} \equiv \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}$$





Where does the total reflection take place?

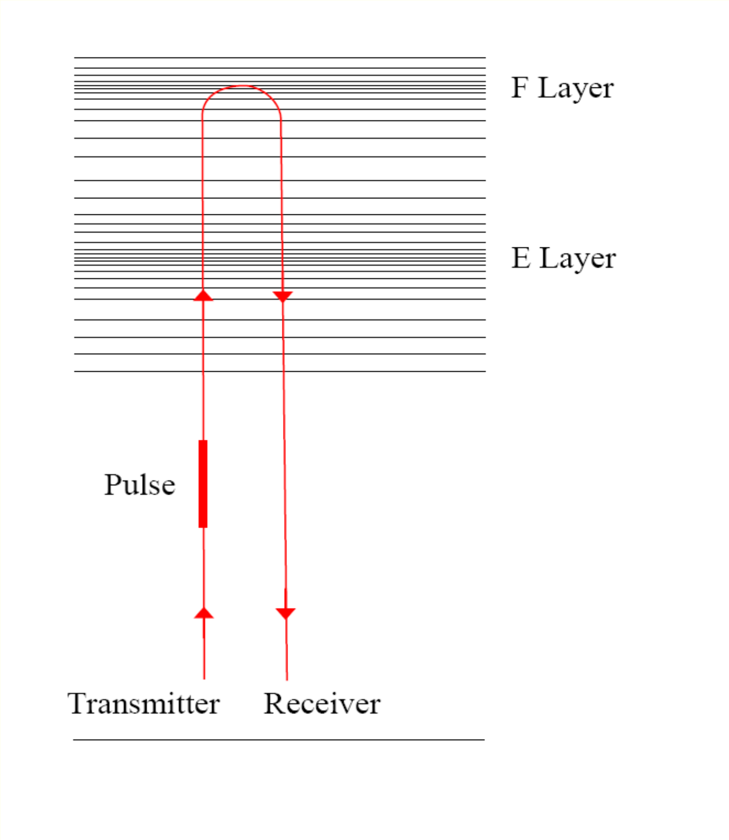


Large gradient when

$$f \approx f_{pe}$$

Higher frequencies $\rightarrow$ higher $f_{pe}\,(n_e)$

Ionosonde



The pulse will be reflected where

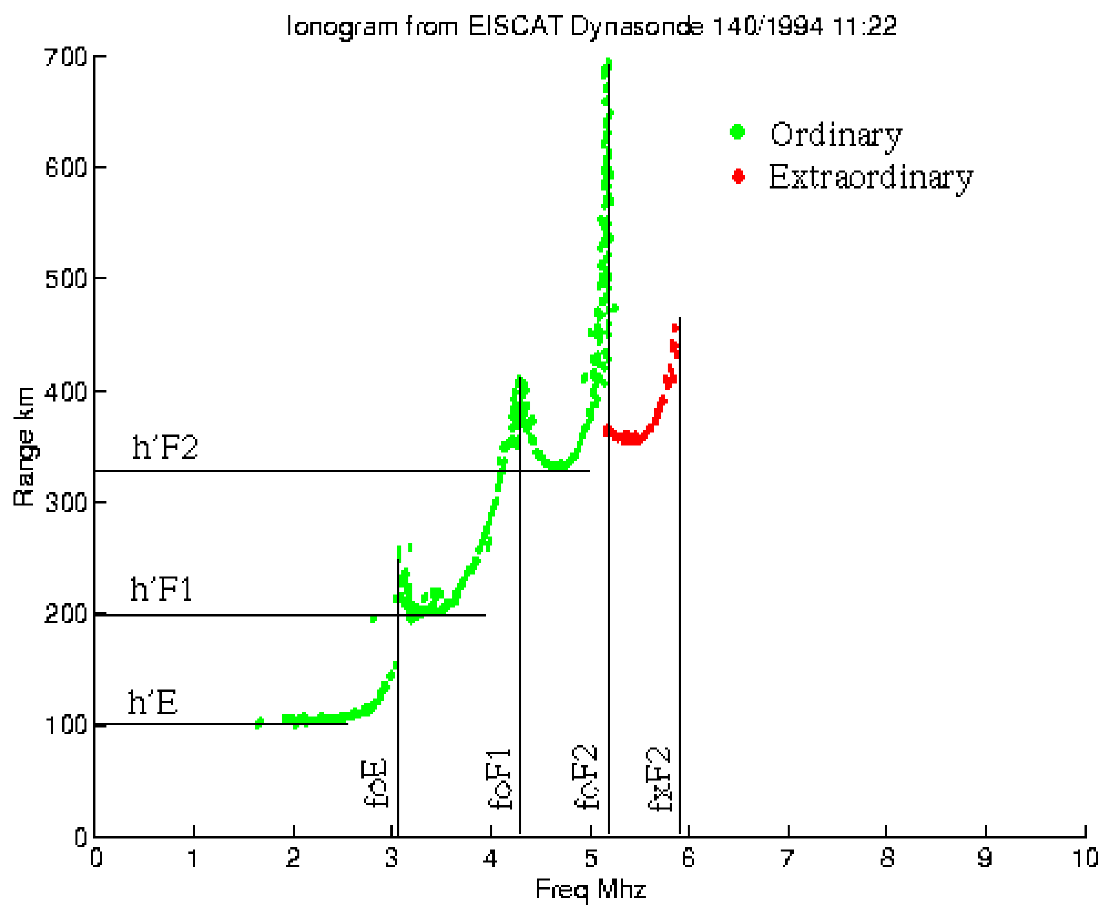
$$f = f_{pe}$$

The altitude will be determined by

$$2h = ct$$

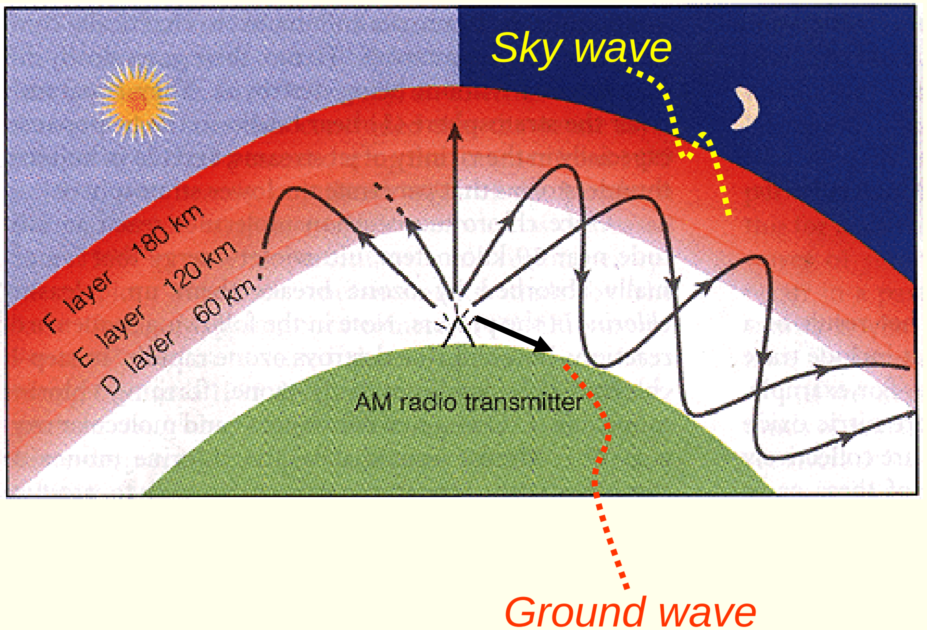
Where t is the time between when the pulse is sent out and the registered again.

Ionogram



Ionogram from EISCAT Dynasonde 140/1994 11:22
Ordinary
Extraordinary
Range km
Freq Mhz
h'F2
h'F1
h'E
foE
foF1
foF2
fxF2
0
100
200
300
400
500
600
700
1
2
3
4
5
6
7
8
9
10

Reflection of radio waves



F2-layer during night:

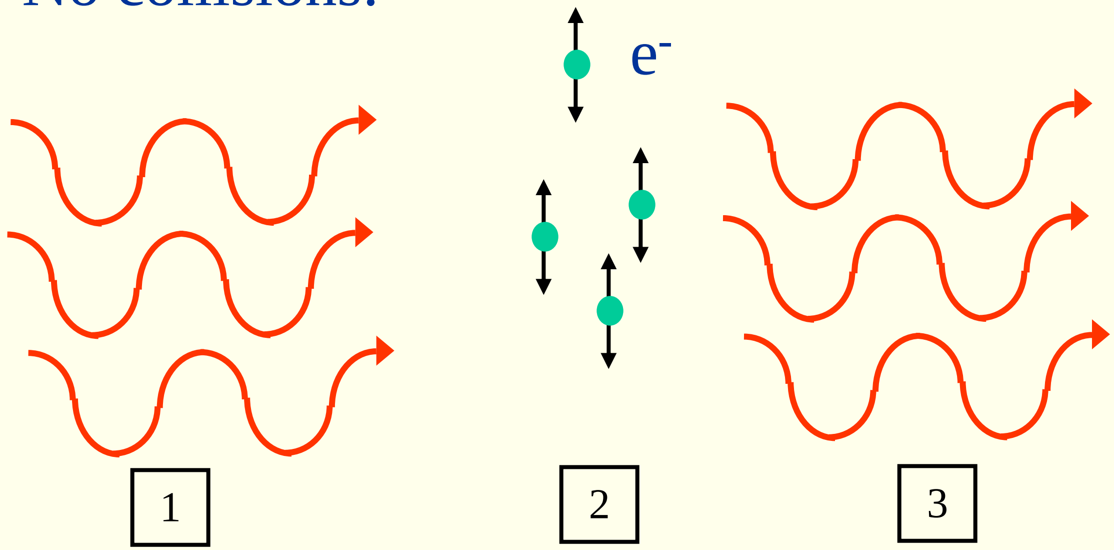
$$n_e = 5 \cdot 10^{11}\ \mathrm{m}^{-3} \quad \Rightarrow$$

$$f_{pe} = 10^7\ \mathrm{Hz} = 10\ \mathrm{MHz}$$

= HF/short wave

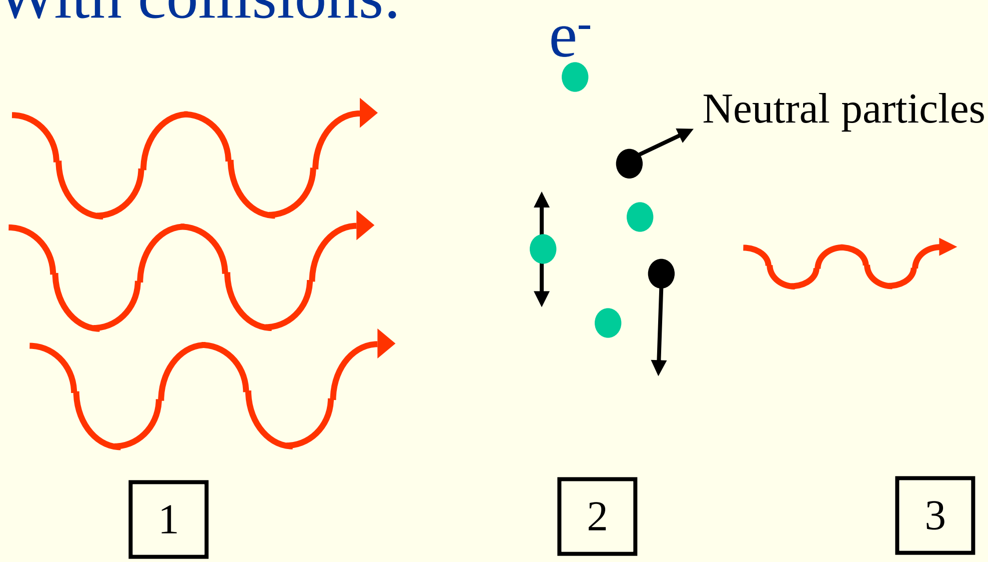
Absorption of radio waves

No collisions:



Absorption of radio waves

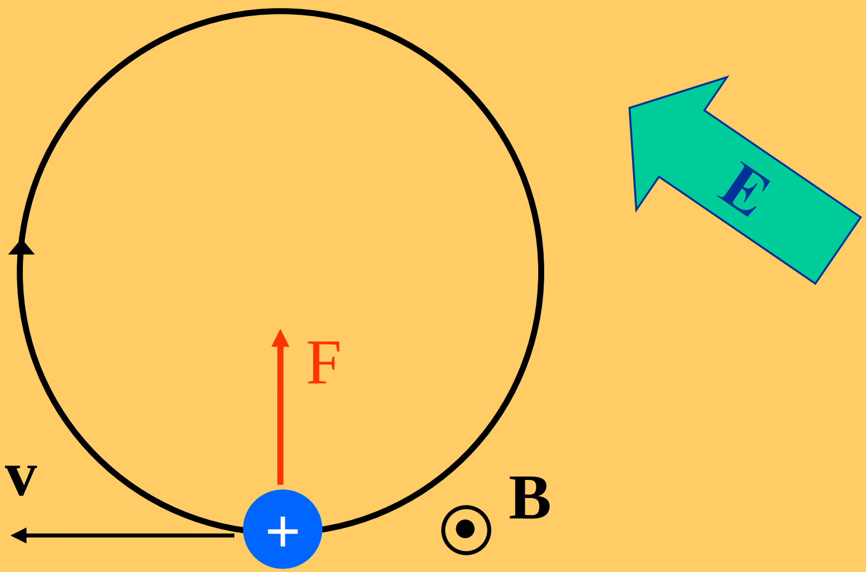
With collisions:



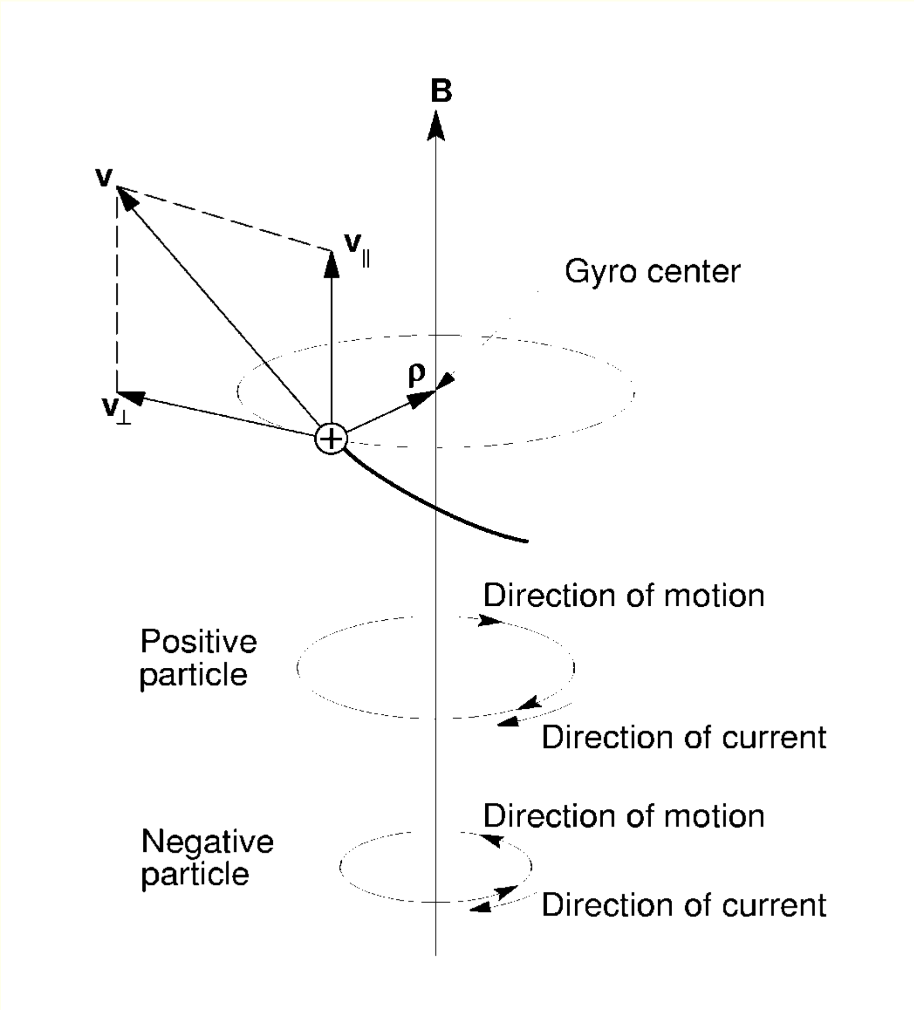
Think about this:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

What happens if you add an electric field $\mathbf{E}$?



Particle motion in magnetic field



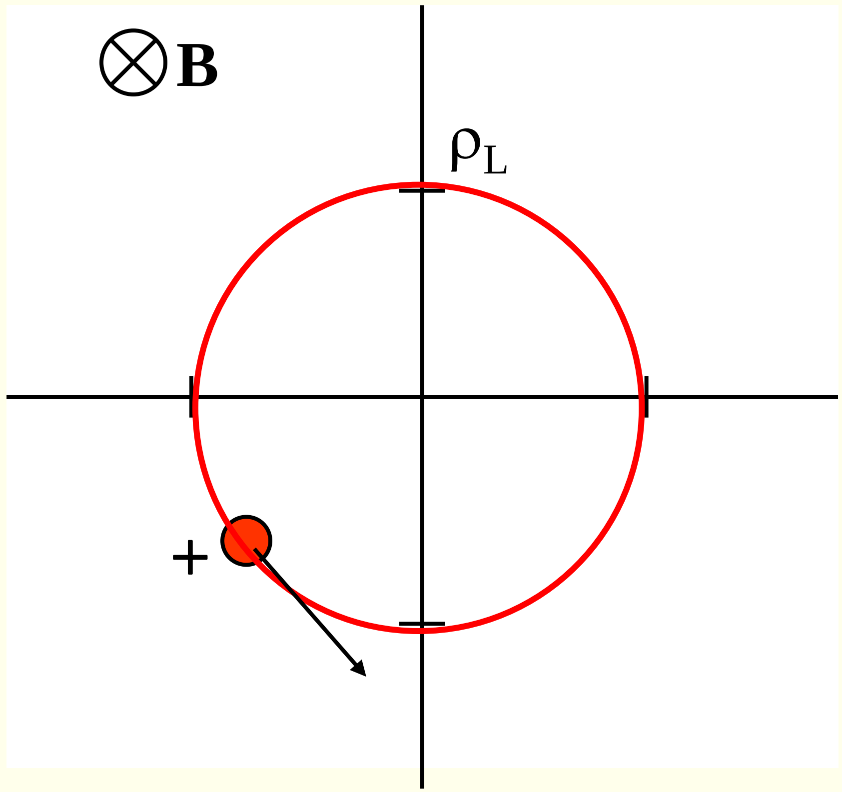
gyro radius

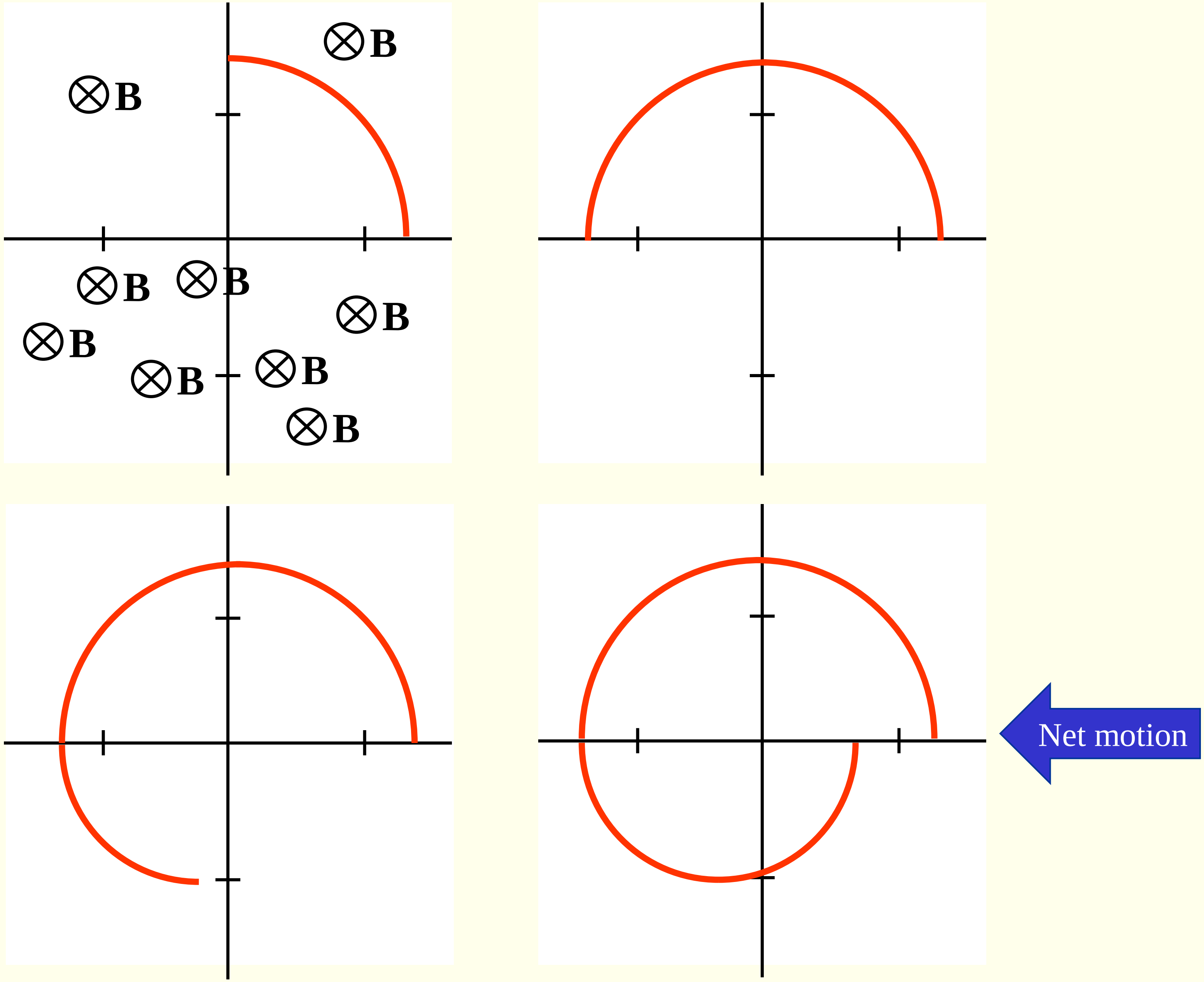
$$\rho = \frac{mv_\perp}{qB}$$

gyro frequency

$$\omega_g = \frac{qB}{m}$$

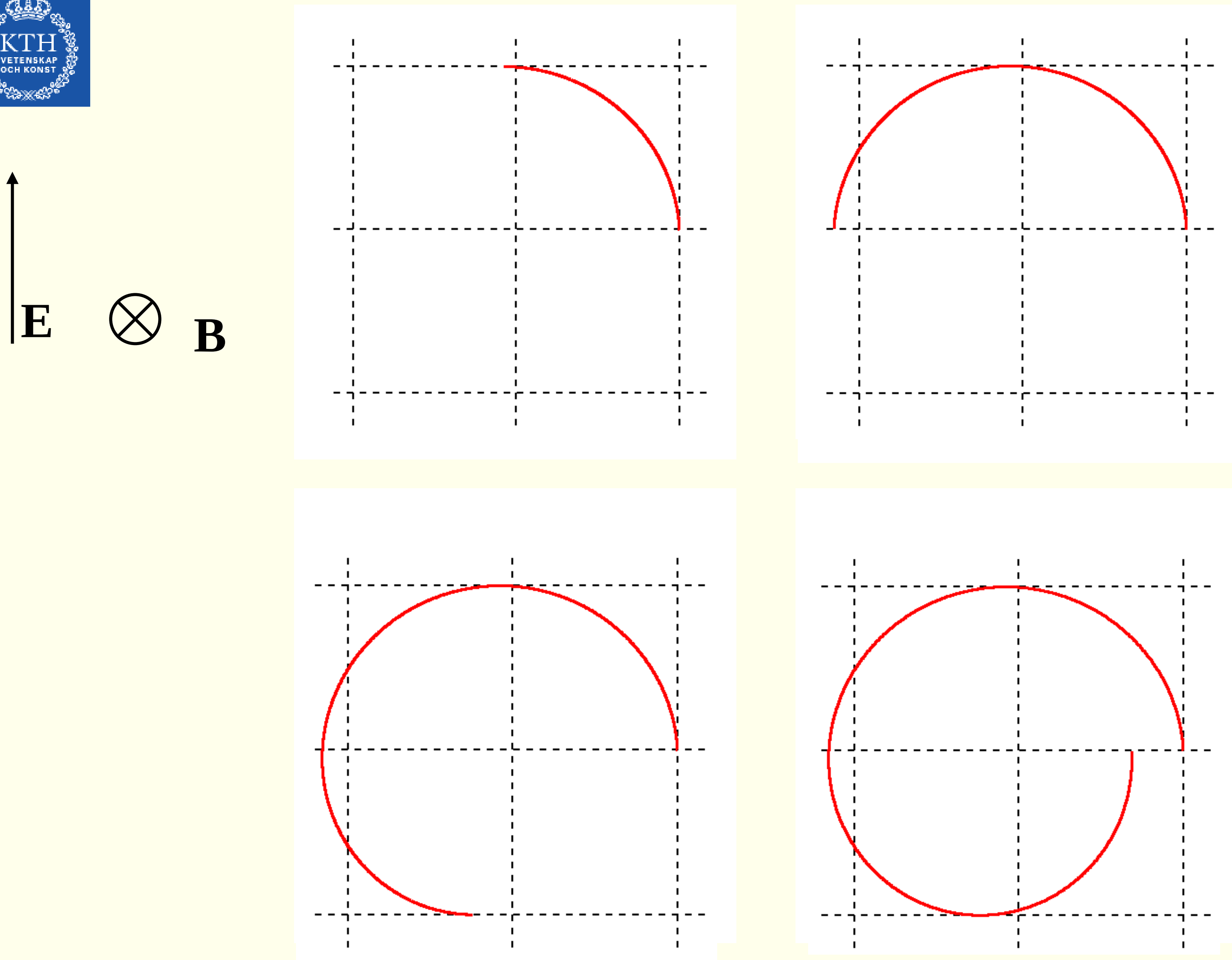
Drift motion





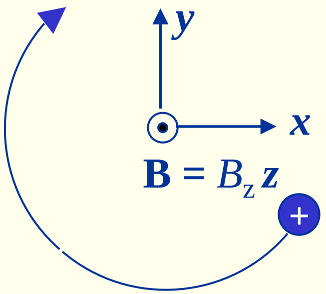
∇B

$$\rho = \frac{mv_\perp}{qB}$$


E
B

Drift motion

Consider a charged particle in a magnetic field.



Assume an electric field in the x-z plane:

$$\mathbf{E} = \left(E_x, 0, E_z\right)$$

$$m\frac{d\mathbf{v}}{dt} = q\left(\mathbf{v}\times\mathbf{B}+\mathbf{E}\right) \Rightarrow$$

$$\begin{cases} m\dfrac{dv_x}{dt} = qv_y B + qE_x \\ m\dfrac{dv_y}{dt} = -qv_x B \\ m\dfrac{dv_z}{dt} = qE_z \end{cases}$$

Constant acceleration along z

$$\Rightarrow$$

$$\begin{cases} \dfrac{d^2v_x}{dt^2} = \dfrac{qB}{m}\dfrac{dv_y}{dt} = \omega_g \dfrac{dv_y}{dt} = -\omega_g^2 v_x \\ \dfrac{d^2v_y}{dt^2} = -\dfrac{qB}{m}\dfrac{dv_x}{dt} = -\omega_g \dfrac{dv_x}{dt} = -\omega_g^2 v_y - \dfrac{q^2B}{m^2}E_x \end{cases}$$

$$\Rightarrow$$

Drift motion

$$\begin{cases} \dfrac{d^2 v_x}{dt^2} = \dfrac{qB}{m}\dfrac{dv_y}{dt} = \omega_g \dfrac{dv_y}{dt} = -\omega_g^2 v_x \\ \dfrac{d^2 v_y}{dt^2} = -\dfrac{qB}{m}\dfrac{dv_x}{dt} = -\omega_g \dfrac{dv_x}{dt} = -\omega_g^2 v_y - \dfrac{q^2 B}{m^2} E_x \end{cases}$$

$$\therefore$$

$$\begin{cases} \dfrac{d^2 v_x}{dt^2} - \omega_g^2 v_x \\ \dfrac{d^2\left(v_y + \dfrac{E_x}{B}\right)}{dt^2} = -\omega_g^2 \left(v_y + \dfrac{E_x}{B}\right) \end{cases}$$

$$\Rightarrow$$

$$\begin{cases} v_x = v_\perp e^{i\omega_g t + \delta_x} \\ v_y = -\dfrac{E_x}{B} + v_\perp e^{i\omega_g t + \delta_y} \end{cases}$$

Average over a gyro period:

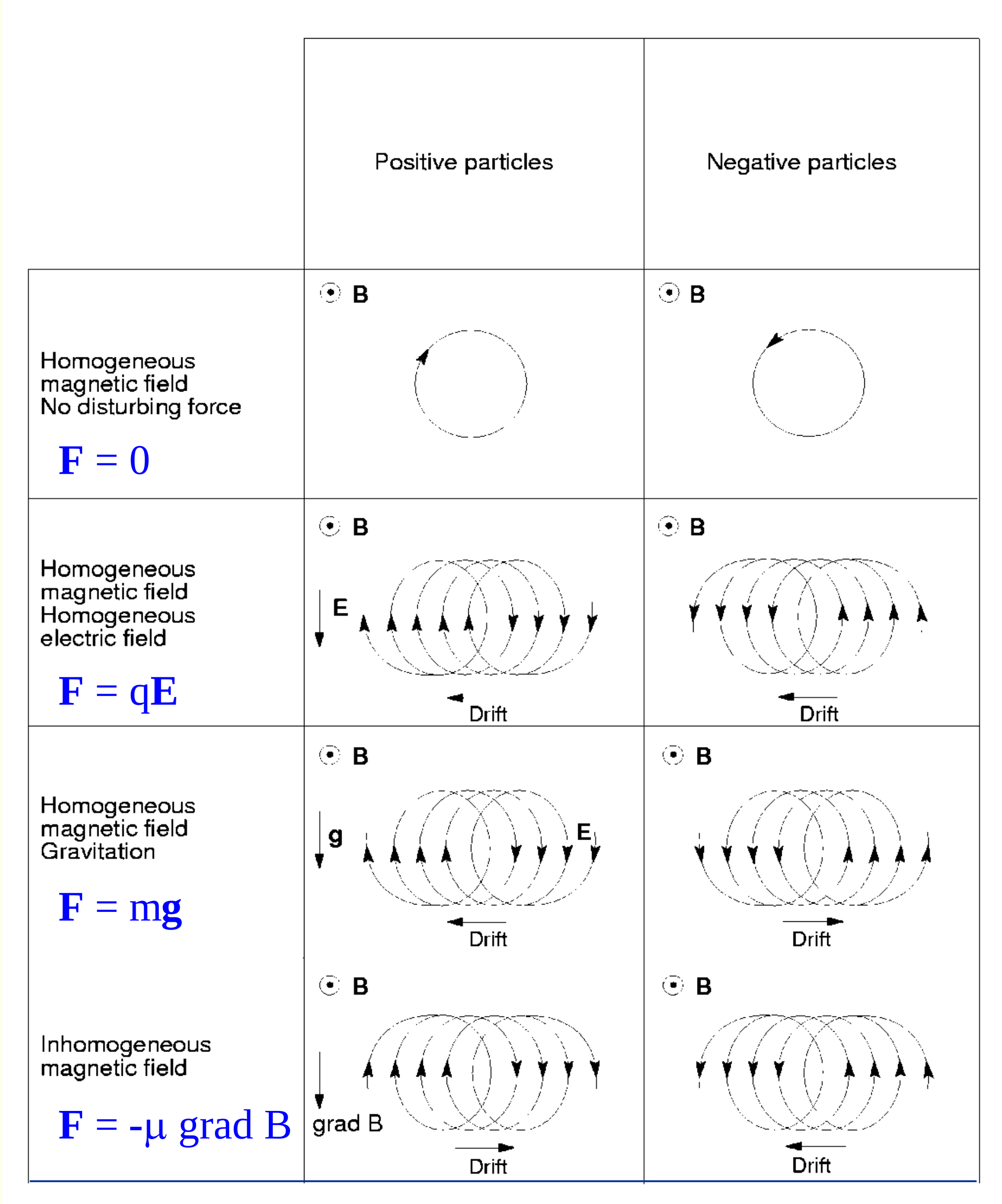
$$v_{drift,y} = -\frac{E_x}{B} = -\frac{E_x B_z}{B^2} = \frac{(\mathbf{E}\times\mathbf{B})_y}{B^2}$$

In general:

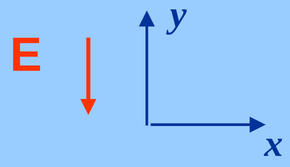
$$\boxed{\mathbf{v}_{drift} = \frac{\mathbf{E}\times\mathbf{B}}{B^2} = \frac{q\mathbf{E}\times\mathbf{B}}{qB^2} = \frac{\mathbf{F}\times\mathbf{B}}{qB^2}}$$

Drift motion

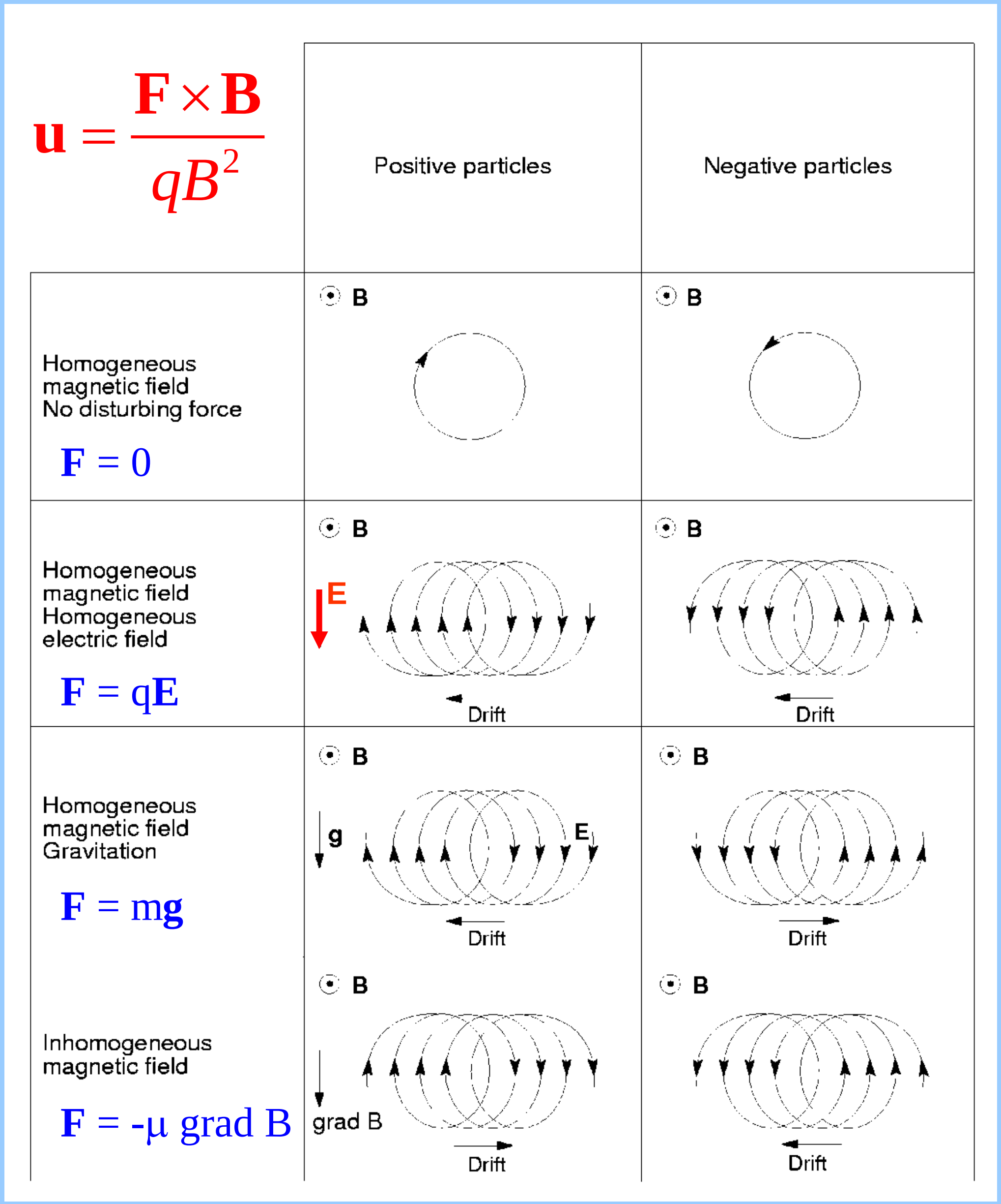
$$\mathbf{u}_{drift} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$



Suppose you apply an electric field **E** in the direction showed in the figure, and that one electron and one ion (charge –*e* and *e*) is present. What will the resulting current be?



$$\mathbf{I} \equiv e\mathbf{u}_\mathrm{i} - e\mathbf{u}_\mathrm{e}$$



Yellow $\mathbf{I} = -e\dfrac{E}{B}\hat{\mathbf{x}}$

Blue $\mathbf{I} = 0$

Red $\mathbf{I} = \dfrac{1}{2}e\dfrac{E}{B}\hat{\mathbf{x}} - \dfrac{1}{2}e\dfrac{E}{B}\hat{\mathbf{y}}$

Green $\mathbf{I} = e\dfrac{E}{B}\hat{\mathbf{y}}$

$$\mathbf{I} \equiv e\mathbf{u_i} - e\mathbf{u_e}$$

$$\mathbf{u_i} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2} = \frac{e\mathbf{E} \times \mathbf{B}}{eB^2} = -\hat{\mathbf{x}}\frac{EB}{B^2} = -\hat{\mathbf{x}}\frac{E}{B}$$

$$\mathbf{u_e} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2} = \frac{-e\mathbf{E} \times \mathbf{B}}{-eB^2} = -\hat{\mathbf{x}}\frac{EB}{B^2} = -\hat{\mathbf{x}}\frac{E}{B}$$

$$\mathbf{I} \equiv e\mathbf{u_i} - e\mathbf{u_e} = e\left(\mathbf{u_i} - \mathbf{u_e}\right) = 0$$

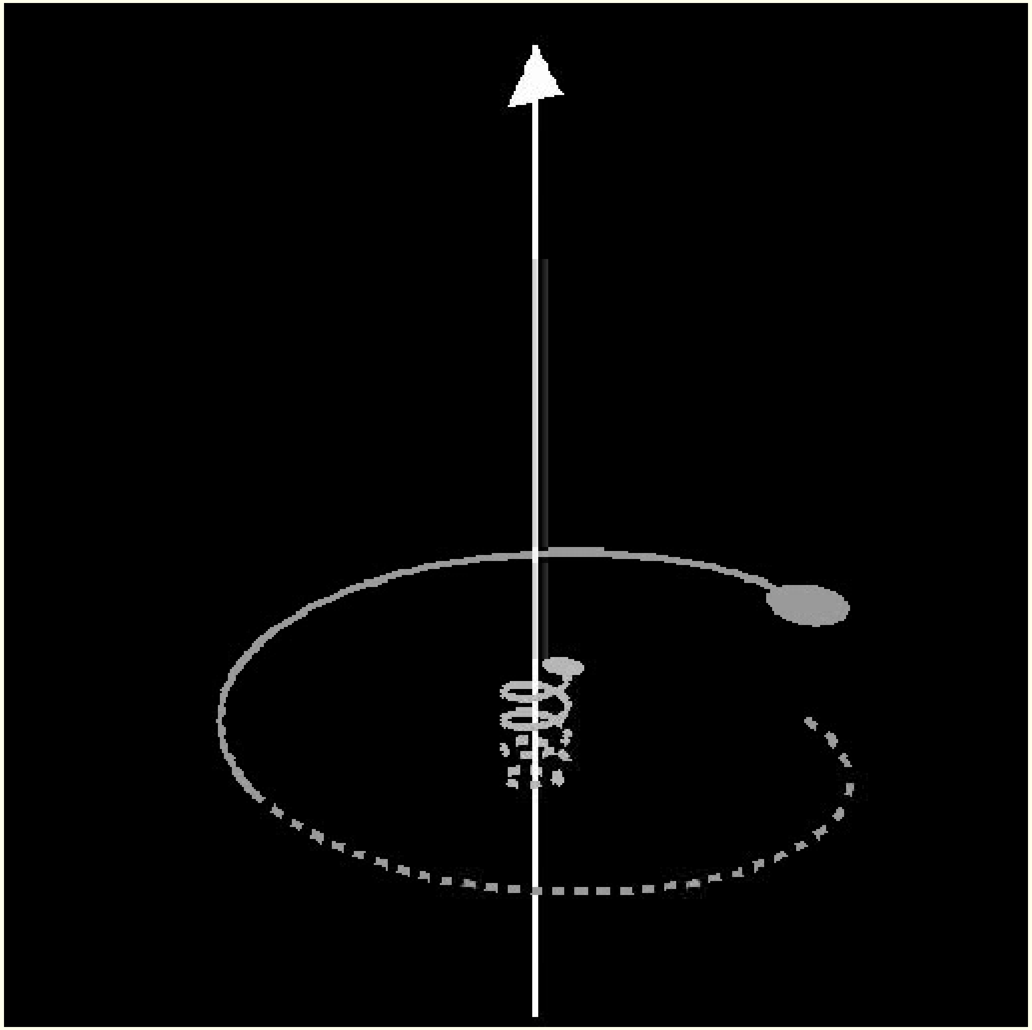
Blue

So, if there is no current when you apply an electric field, is the conductivity of the ionospheric plasma zero ?

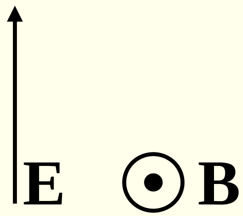
What is the electron density at 100 km?

What is the neutral density at 100 km?

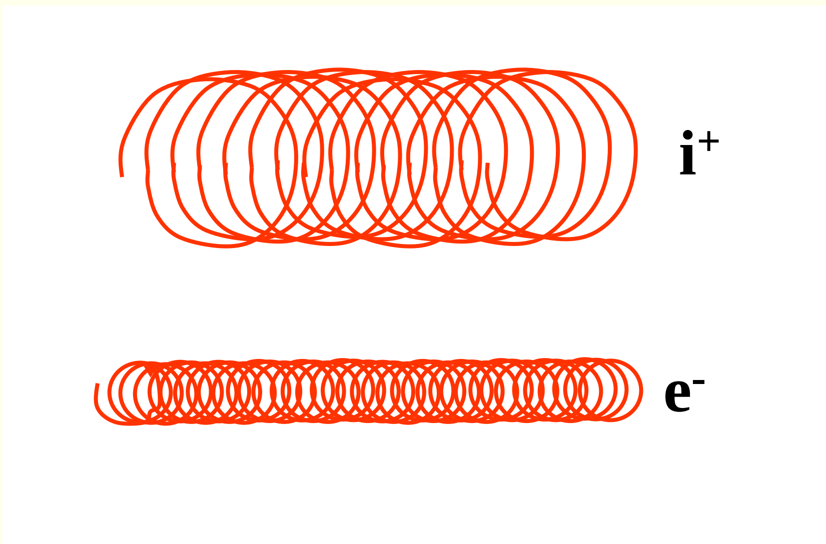
Gyro motion



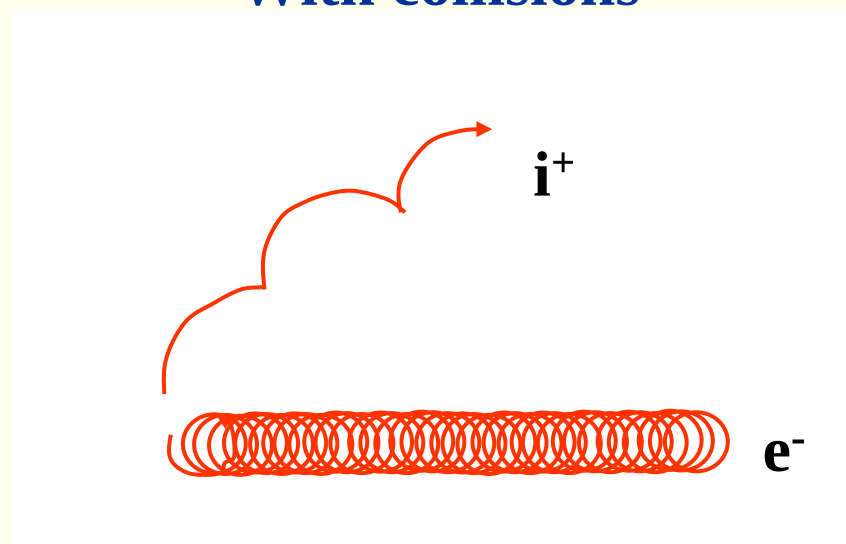
ExB-drift



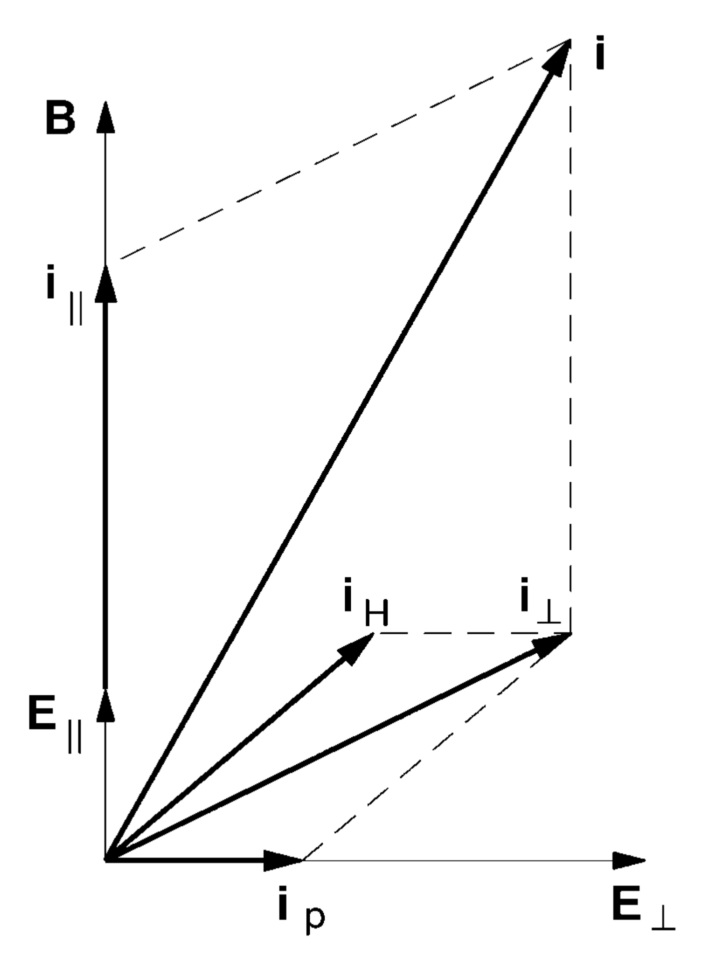
Without collisions



With collisions

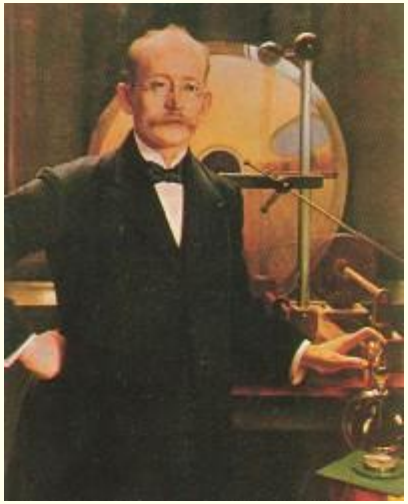


Electric conductivity in a magnetized plasma



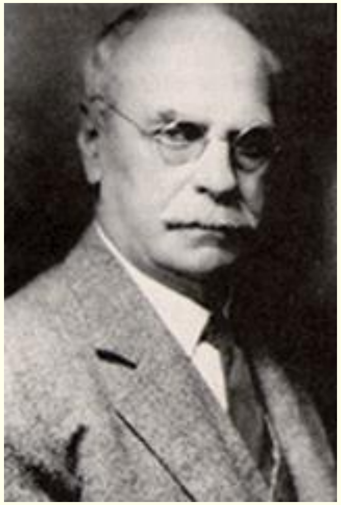
- $i_{//}$ = parallel current
- i_P = Pedersen current
- i_H = Hall current

Birkeland, Hall, Pedersen



Kristian Birkeland
1867-1917

Norwegian
scientist



Edwin Hall
1855-1938

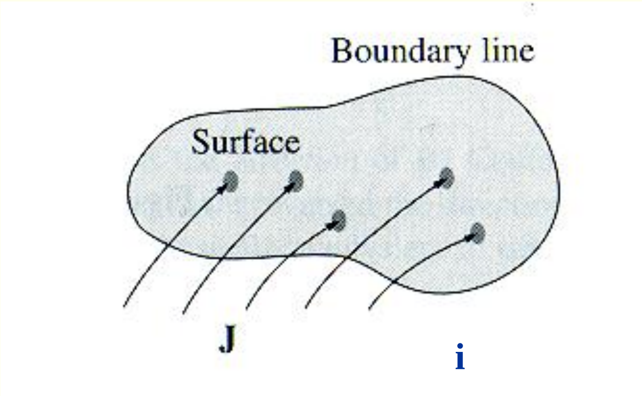
American
physicist

Peder Oluf Pedersen
1874-1941

Danish engineer
and physicist

Current density

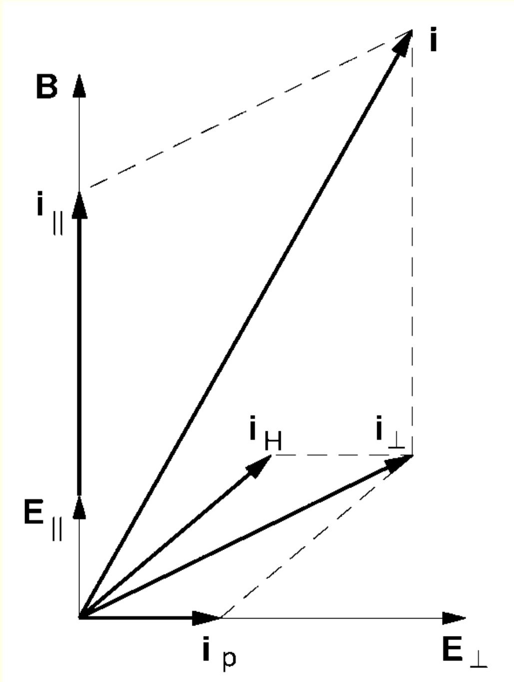
The current density **j** is a vector field with dimension [**i**] = Am^{-2}.



The total current I through the surface S is

$$I = \int_S \mathbf{j} \cdot d\mathbf{S}$$

Electric conductivity in a magnetized plasma II



$$\sigma_P = \sigma_e \frac{1}{1+\omega_{ge}^2\tau_e^2} + \sigma_i \frac{1}{1+\omega_{gi}^2\tau_i^2}$$

$$\sigma_H = \sigma_e \frac{\omega_{ge}\tau_e}{1+\omega_{ge}^2\tau_e^2} - \sigma_i \frac{\omega_{gi}\tau_i}{1+\omega_{gi}^2\tau_i^2}$$

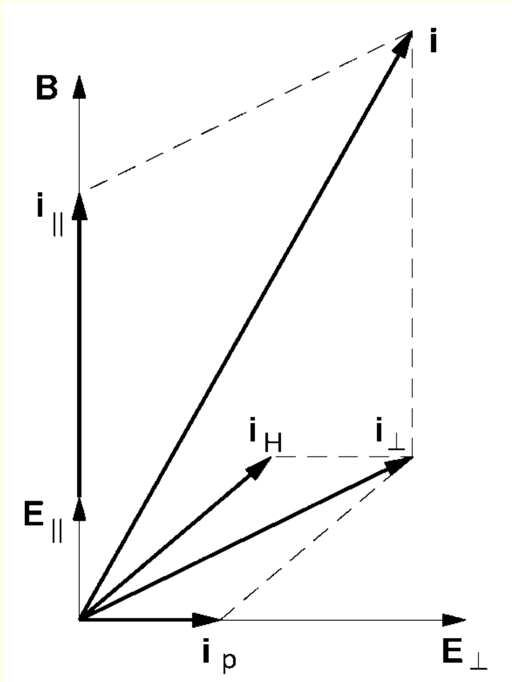
$$\sigma_{//} = \sigma_e + \sigma_i$$

$$\sigma_e = e^2 n\tau_e / m_e \qquad \sigma_i = e^2 n\tau_i / m_i$$

$$i_{//} = \sigma_{//} E_{//}$$

$$\left.\begin{array}{l} i_P = \sigma_P E_\perp \\ i_H = \sigma_H E_\perp \end{array}\right\} \quad \text{or} \quad \mathbf{i}_\perp = \sigma_P \mathbf{E}_\perp + \sigma_H \frac{\mathbf{B}\times\mathbf{E}_\perp}{B}$$

Electric conductivity in a magnetized plasma II

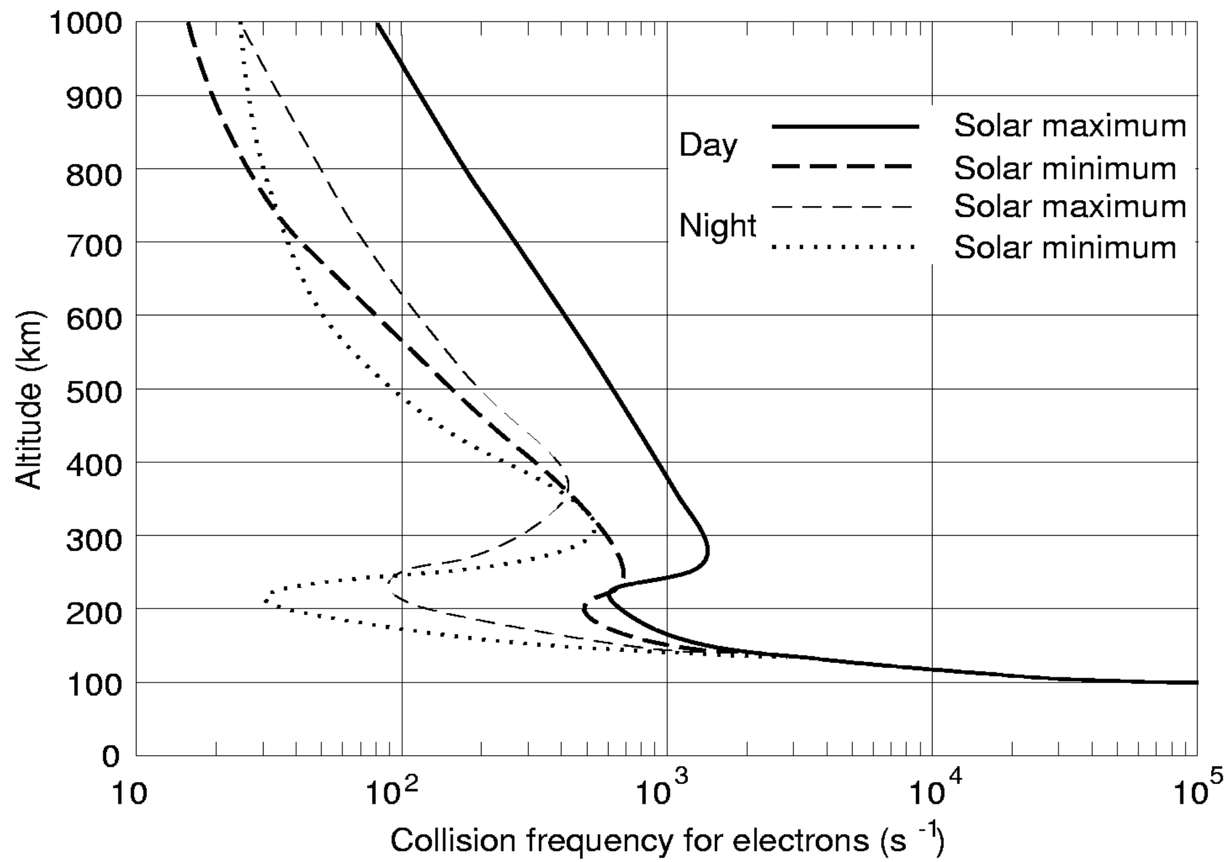


$$\mathbf{i} = \boldsymbol{\sigma} \cdot \mathbf{E}$$

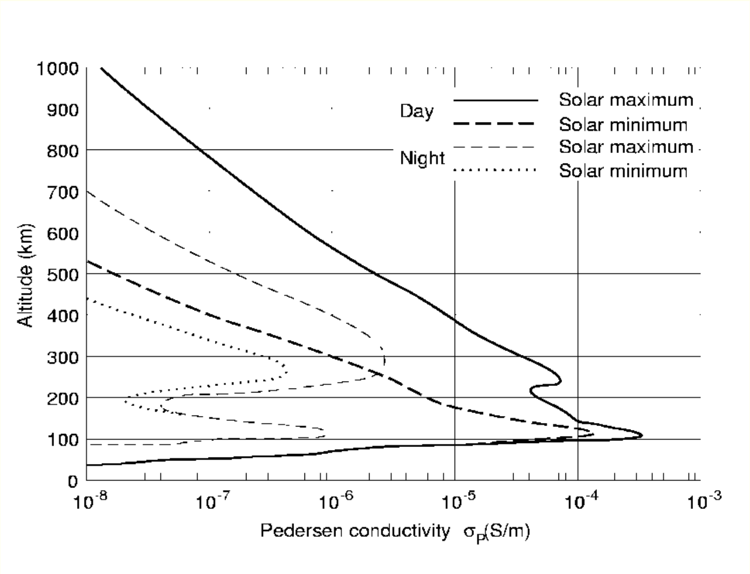
$$\boldsymbol{\sigma} = \underbrace{\begin{pmatrix} \sigma_P & \sigma_H & 0 \\ -\sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_{//} \end{pmatrix}}_{\textit{conductivity tensor}}$$

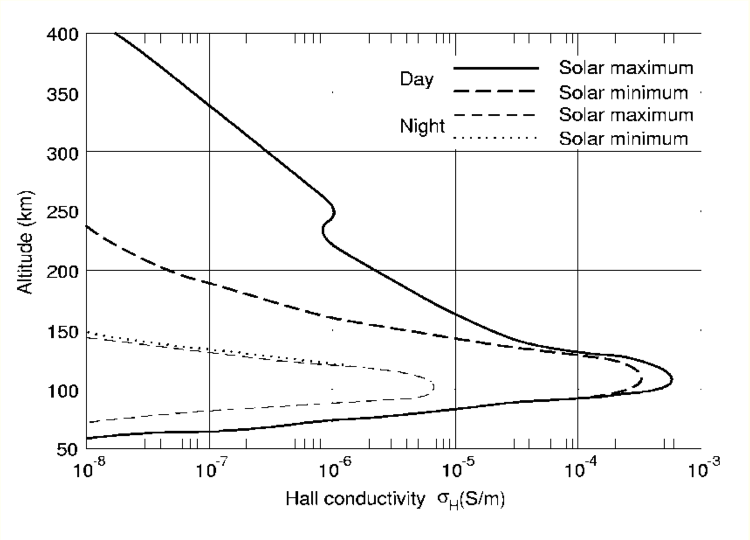
May be formulated as a tensor equation

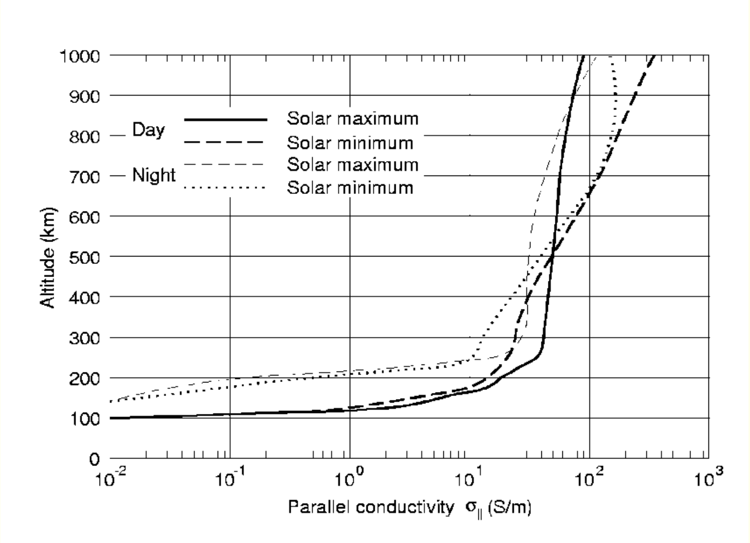
Collisional frequency



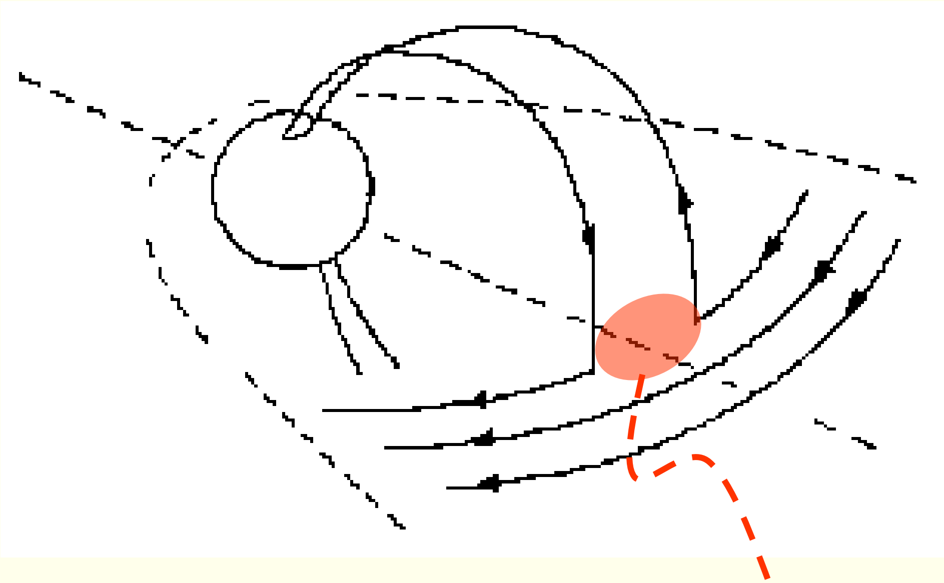
Ionospheric conductivities







Consequence: Birkeland currents



When the conductivity out in the magnetosphere is low, it is easier for the current to close through the ionosphere via currents parallel to the geomagnetic field. Such currents are called *Birkeland* currents.

Exemple: Electric field 700 km above the aurora.

$$\mathbf{E} = E_x\hat{\mathbf{x}} + E_y\hat{\mathbf{y}}$$

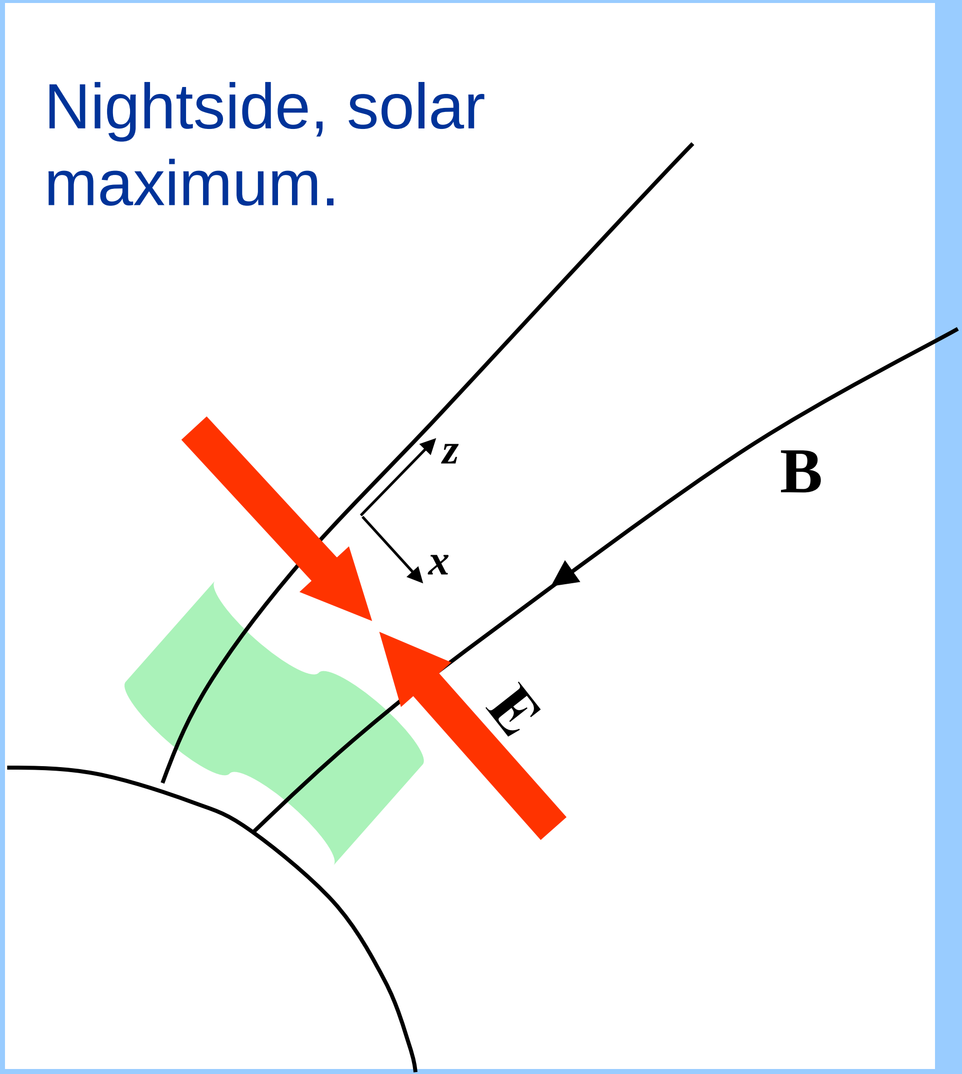
$$E_x = 1\,\mathrm{Vm}^{-1}$$

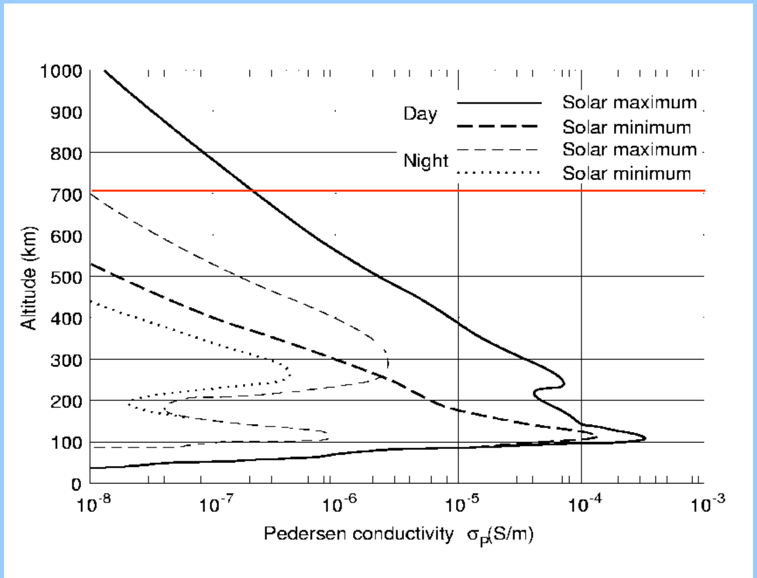
$$E_z = 1\,\mu\mathrm{Vm}^{-1}$$

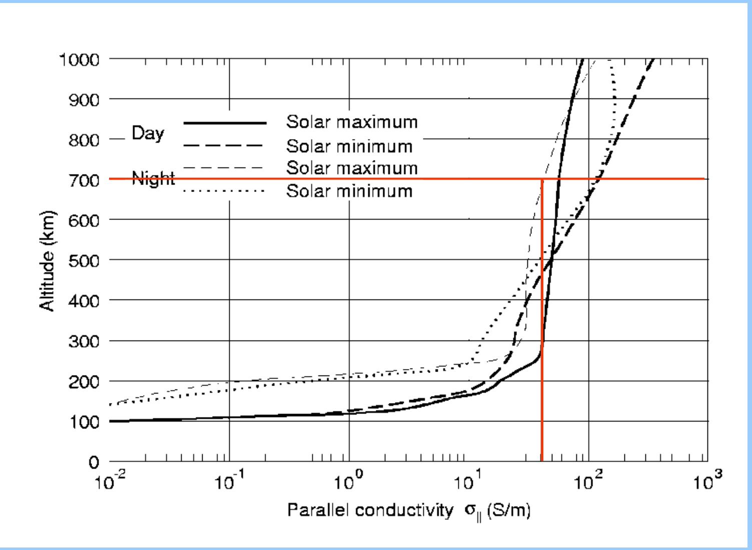
$$\left.\begin{array}{l} j_P = j_x = 0.01\ \mu\mathrm{Am}^{-2} \\ j_{//} = j_z = 40\ \mu\mathrm{Am}^{-2} \end{array}\right\}$$ Yellow

$$\left.\begin{array}{l} j_P = j_x = 10.0\ \mu\mathrm{Am}^{-2} \\ j_{//} = j_z = 4.0\ \mu\mathrm{Am}^{-2} \end{array}\right\}$$ Red

$$\left.\begin{array}{l} j_P = j_x = 1.0\ \mu\mathrm{Am}^{-2} \\ j_{//} = j_z = 40\ \mathrm{mAm}^{-2} \end{array}\right\}$$ Blue







$$\sigma_P \approx 1 \cdot 10^{-8}\ \mathrm{Sm}^{-1}$$

$$\sigma_{//} \approx 40\ \mathrm{Sm}^{-1}$$

$$j_P = j_x = \sigma_P E_x = 1 \cdot 10^{-8}\ \mathrm{Am}^{-2} = 0.01\,\mu\mathrm{Am}^{-2}$$

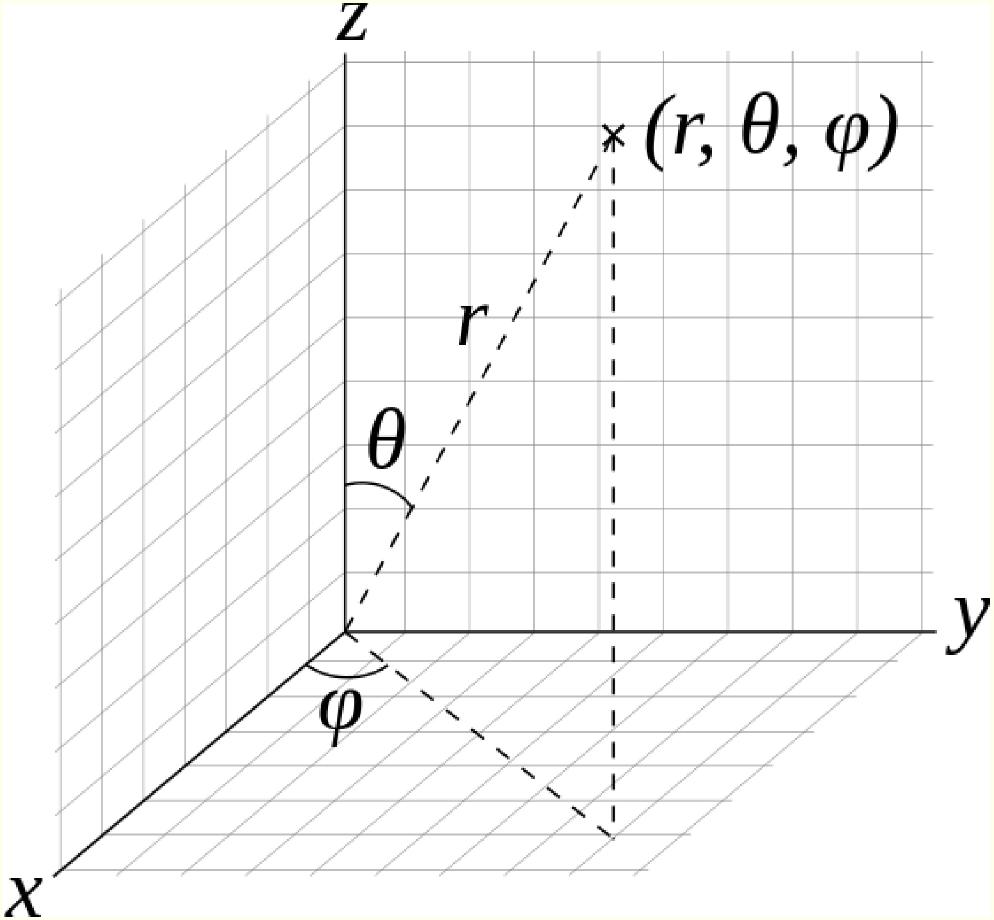
$$j_{//} = j_z = \sigma_{//} E_z = 40 \cdot 10^{-6}\ \mathrm{Am}^{-2} = 40\,\mu\mathrm{Am}^{-2}$$

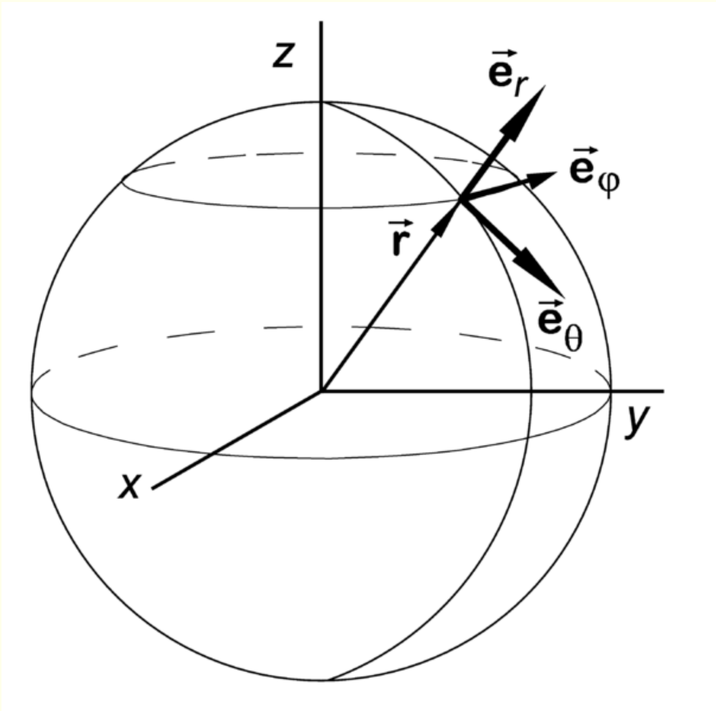
Yellow

How do we define ”the magnetosphere”?

The region in space where the magnetic field is dominated by the geomagnetic field.

Polar (spherical) coordinates





$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \arccos\left(\frac{z}{r}\right)$$

$$\varphi = \arctan\left(\frac{y}{x}\right)$$

$$x = r\,\sin\theta\,\cos\varphi$$

$$y = r\,\sin\theta\,\sin\varphi$$

$$z = r\,\cos\theta$$

Geomagnetic field

Approximated by a dipole close to Earth.

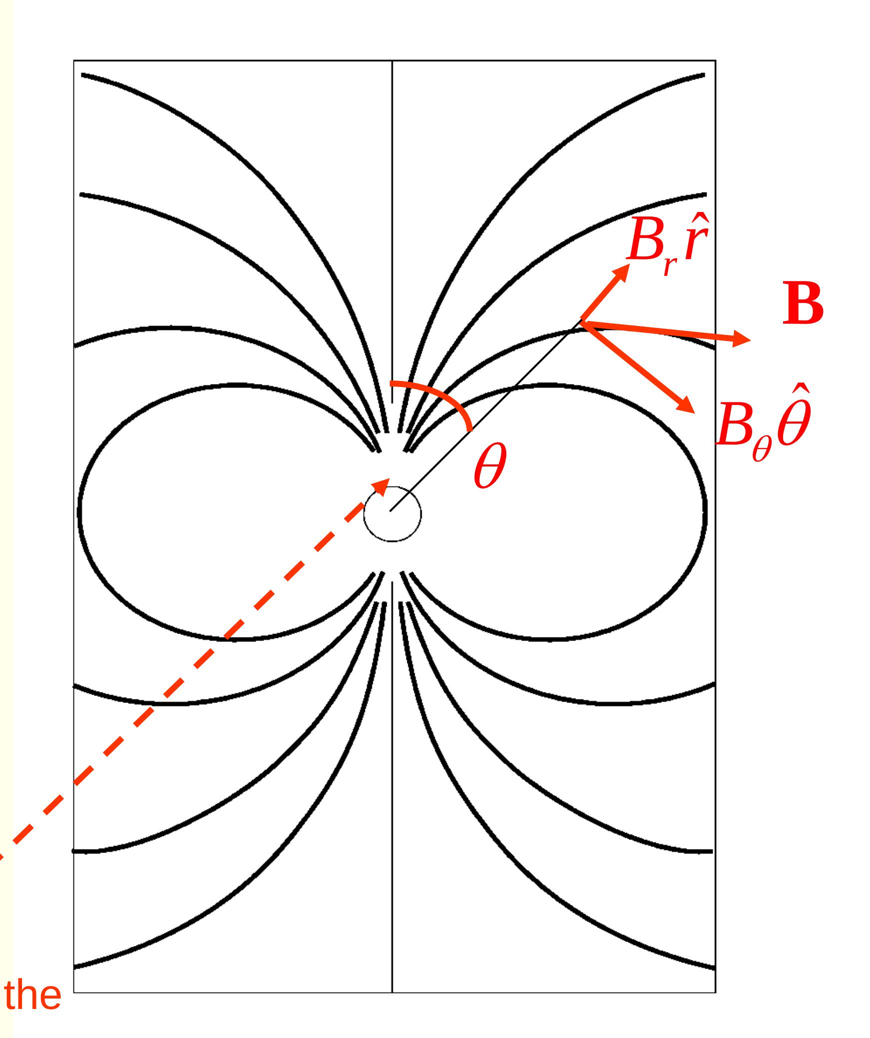
$$B_r = B_p \left(\frac{R_E}{r}\right)^3 \cos\theta$$

$$B_\theta = \frac{B_p}{2} \left(\frac{R_E}{r}\right)^3 \sin\theta$$

$$a = \frac{2\pi R_E^3 B_p}{\mu_0}$$

magnetic dipole moment

Magnetic field at the "north pole"



Geomagnetic field

Alternative formulation of dipole field

$$B_r = B_p \left(\frac{R_E}{r}\right)^3 \cos\theta \qquad B_r = \frac{\mu_0 a}{2\pi} \frac{1}{r^3} \cos\theta$$

$$B_\theta = \frac{B_p}{2} \left(\frac{R_E}{r}\right)^3 \sin\theta \qquad B_\theta = \frac{\mu_0 a}{2\pi} \cdot \frac{1}{2} \cdot \frac{1}{r^3} \sin\theta$$

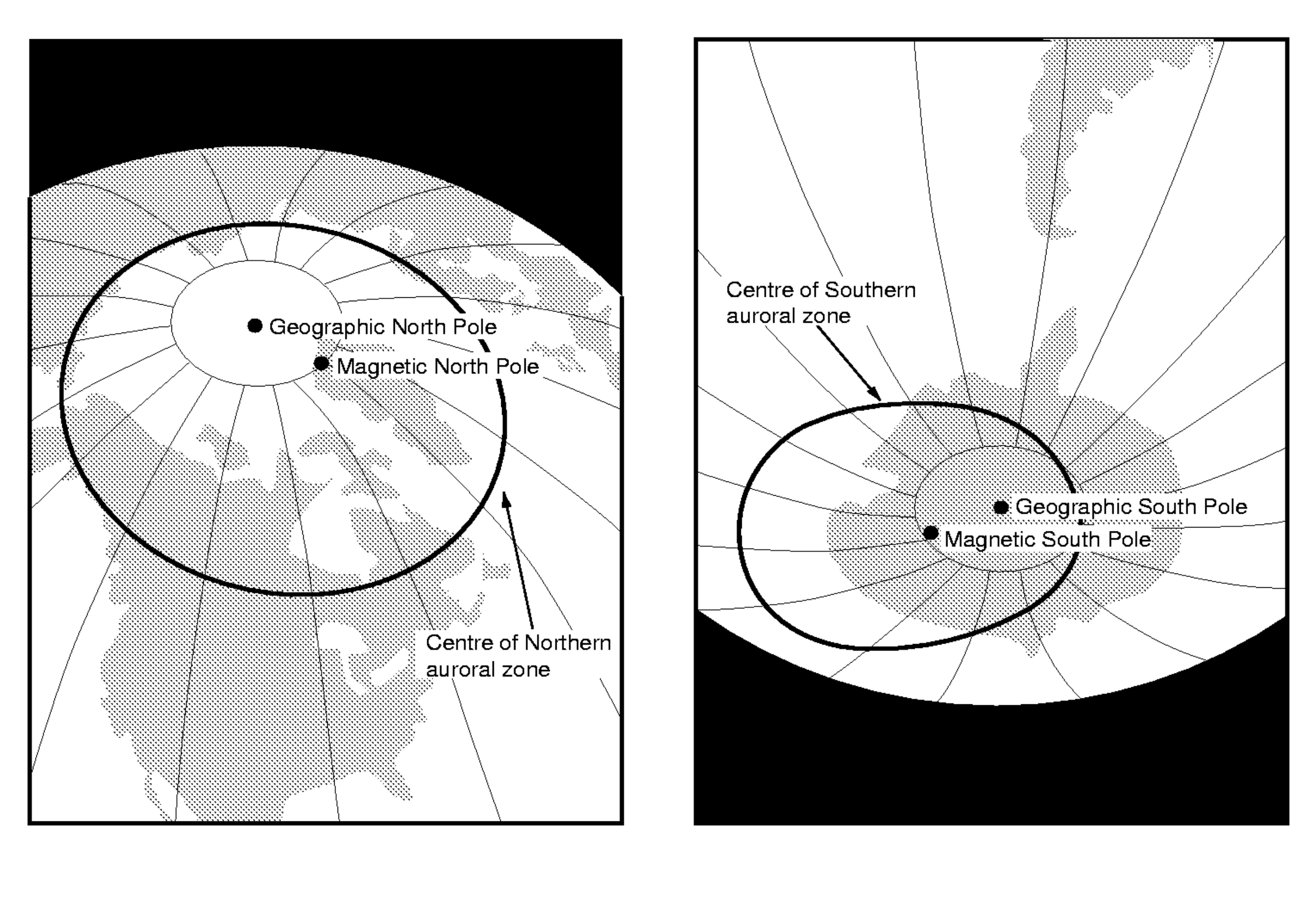
$$a = \frac{2\pi R_E^3 B_p}{\mu_0}$$

magnetic dipole moment

Geomagnetic field

- Angle between dipole axis and spin axis: $\approx 11°$
- The geographic north pole is a magnetic south pole, and vice versa.
- $B_{equator} = 31\ \mu T$,

  $B_{pole} = 62\ \mu T$



Geomagnetic field

Modified by solar wind into tail-like configuration