



# Last lecture (5)

- Particle drift velocity in magnetized plasma
- Electrical conductivity in ionosphere

# Today's lecture (6)

- Magnetosphere, introduction
- Magnetospheric size (standoff distance)
- Particle motion in the magnetosphere
- Other magnetospheres



# Today

Activity	Date	Time	Room	Subject	Litterature
L1	28/8	15-17	Q21	Course description, Introduction, The Sun 1	CGF Ch 1.1,1.2, 1.4, 5, (p 110-113), 6.3
L2	29/8	13-15	Q2	The Sun 2, Plasma physics 1	CGF Ch 1.3, 5 (p 114-121)
L3	4/9	10-12	E2	Solar wind, The ionosphere and atmosphere 1, Plasma physics 2	CGF Ch 6.1, 2, 3.1-3.2, 3.5, LL Ch III, Extra material
T1	6/9	8-10	Q21	Mini-group work 1	
L4	6/9	15-17	Q2	The ionosphere 2, Plasma physics 3	CGF Ch 3.4, 3.7, 3.8
T2	10/9	15-17	Q21	Mini-group work 2	
L5	11/9	10-12	E3	The Earth's magnetosphere 1, Plasma physics 4	CGF 4-1-4.3, LL Ch I, II, IV.A
T3	17/9	8-10	Q21	Mini-group work 3	
L6	18/9	13-15	Q33	The Earth's magnetosphere 2, Other magnetospheres	CGF Ch 4.6-4.9, LL Ch V.
L7	19/9	13-15	Q2	Aurora, Measurement methods in space plasmas and data analysis 1	CGF Ch 4.5, 10, LL Ch VI, Extra material
T4	24/9	8-10	Q2	Mini-group work 4	
L8	24/9	15-17	V3	Space weather and geomagnetic storms	CGF Ch 4.4, LL Ch IV.B-C, VII.A-C
T5	2/10	8-10	Q31	Mini-group work 5	
L9	2/10	13-15	Q2	Alfvén waves, Interstellar and intergalactic plasma, Cosmic radiation	CGF Ch 7-9, Extra material
T6	8/10	15-17	Q21		
L10	9/10	10-12	Q2	Guest Lecture by Swedish astronaut Christer Fuglesang	
Written examination	16/10	14-19	L21, L22, L31		

# **EF22445 Space Physics II**

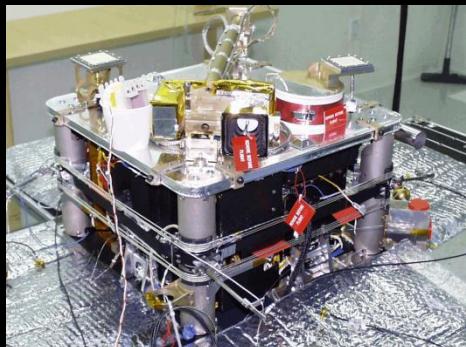
## **7.5 ECTS credits, P2**

- shocks and boundaries in space
- solar wind interaction with magnetized and unmagnetized bodies
- reconnection
- sources of magnetospheric plasma
- magnetospheric and ionospheric convection
- auroral physics
- storms and substorms
- global oscillations of the magnetosphere

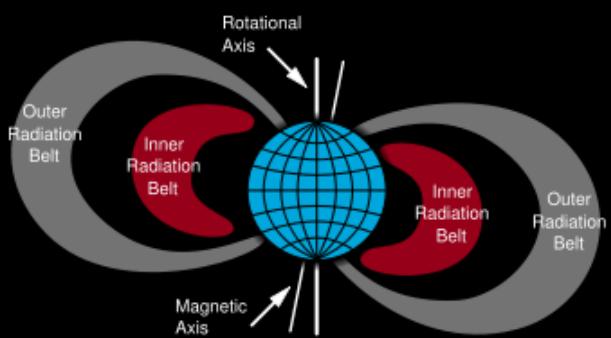
# Courses at the Alfvén Laboratory

## EF2260 SPACE ENVIRONMENT AND SPACECRAFT ENGINEERING , 6 ECTS credits, period 2

- environments spacecraft may encounter in various orbits around the Earth, and the constraints this places on spacecraft design
- basic operation principles underlying the thermal control system and the power systems in spacecraft
- measurements principles in space



The Astrid-2 satellite



*Radiation environment in near-earth space*

### Projects:

- Design power supply for spacecraft
- Study of radiation effects on electronics

# Mini-groupwork 3

a)

$$\frac{\partial n_e}{\partial t} = q - \alpha n_e^2$$

$$\frac{dn_e(t)}{dt} = 0 \Rightarrow$$

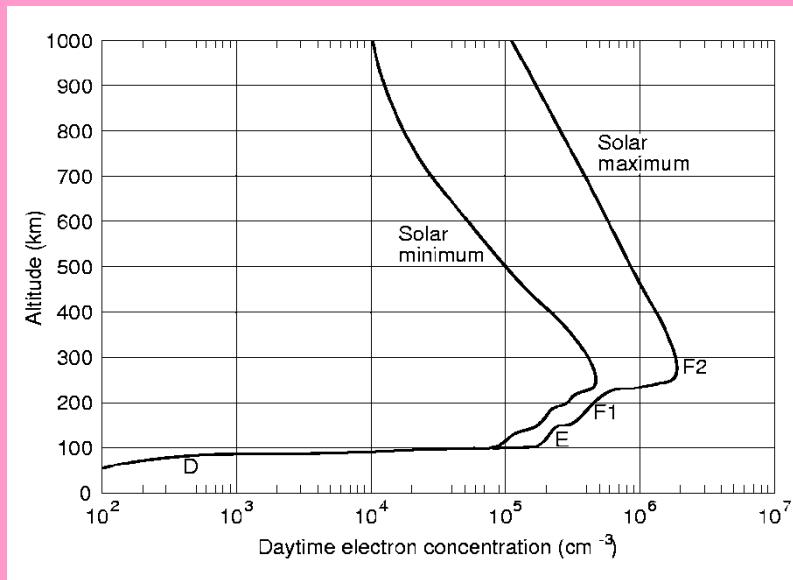
$$\alpha = \frac{q}{n_e^2}$$

$$q = 1.7 \cdot 10^4 \text{ cm}^{-3}\text{s}^{-1} = 1.7 \cdot 10^{10} \text{ m}^{-3}\text{s}^{-1}$$

$$n_e(150 \text{ km}) = 2 \cdot 10^5 \text{ cm}^{-3} = 2 \cdot 10^{11} \text{ m}^{-3}$$

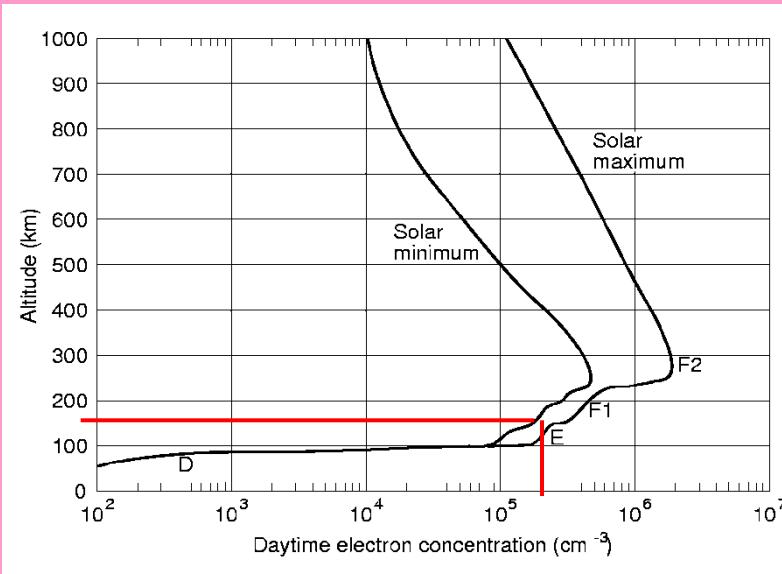
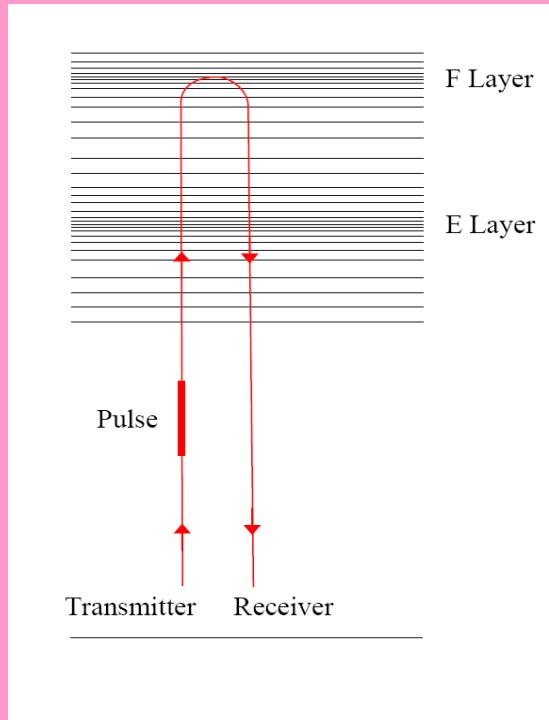
Thus

$$\alpha = 4.2 \cdot 10^{-13} \text{ m}^3\text{s}^{-1}$$



# Mini-groupwork 3

b)



$$f_p = \frac{1}{2\pi} \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}} \approx 9\sqrt{n_e}$$

$$f_p = 5 \cdot 10^6 = 9\sqrt{n_e}$$

$\Rightarrow$

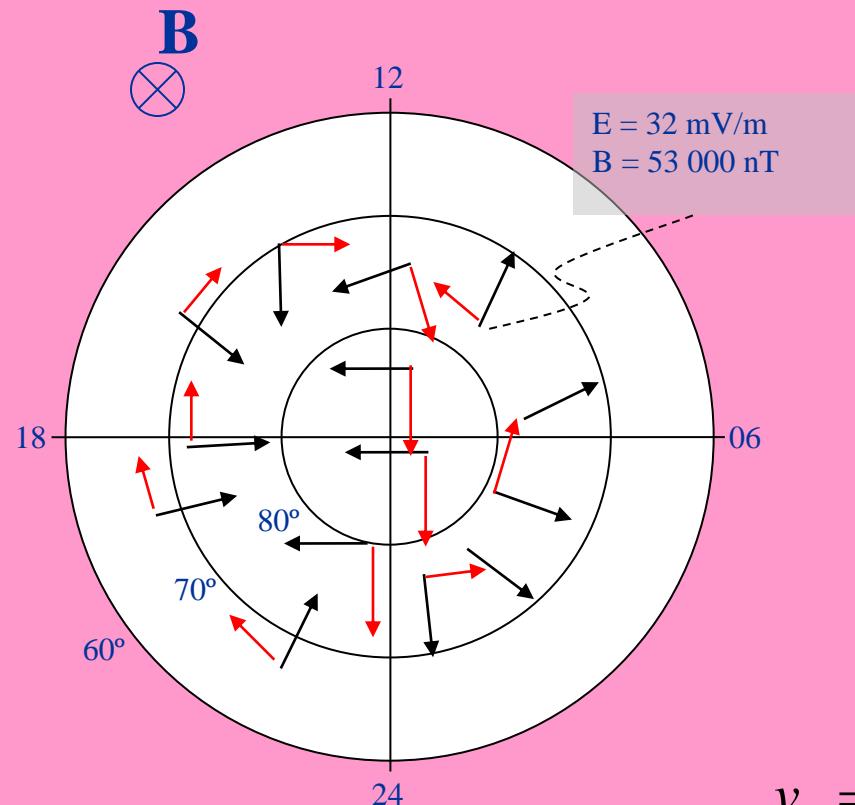
$$n_e = \left( \frac{5 \cdot 10^6}{9} \right)^2 = 3 \cdot 10^{11} \text{ m}^{-3}$$

$$h = 150 \text{ km}$$

$$t = \frac{2h}{c} = \frac{300 \cdot 10^3}{3 \cdot 10^8} = 10^{-3} \text{ s}$$

# Mini-groupwork 3

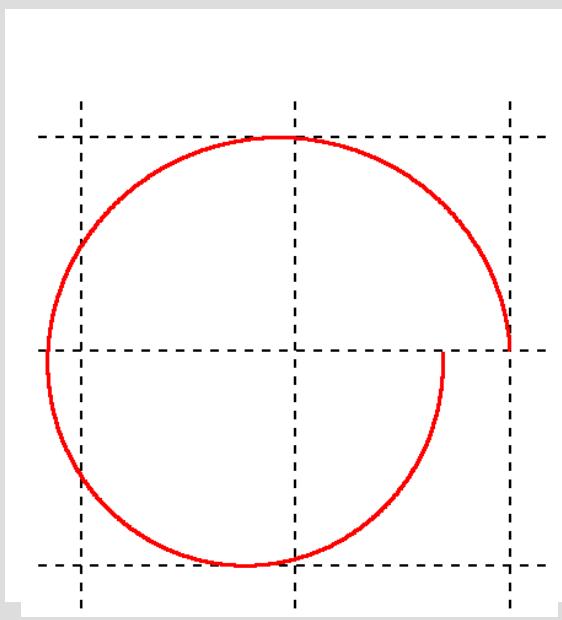
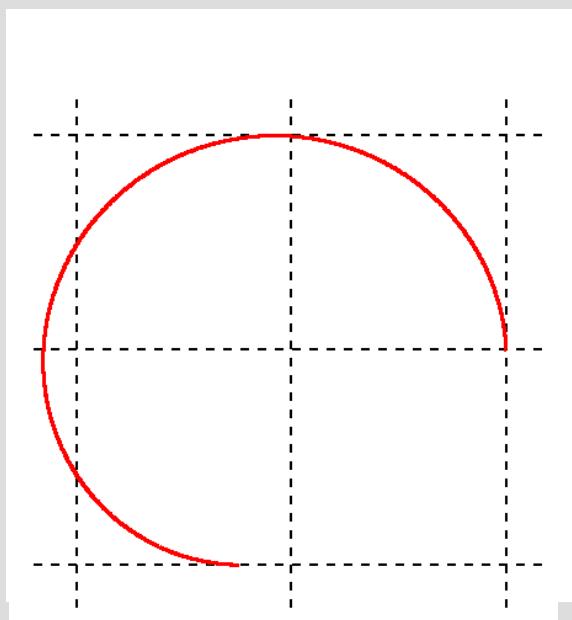
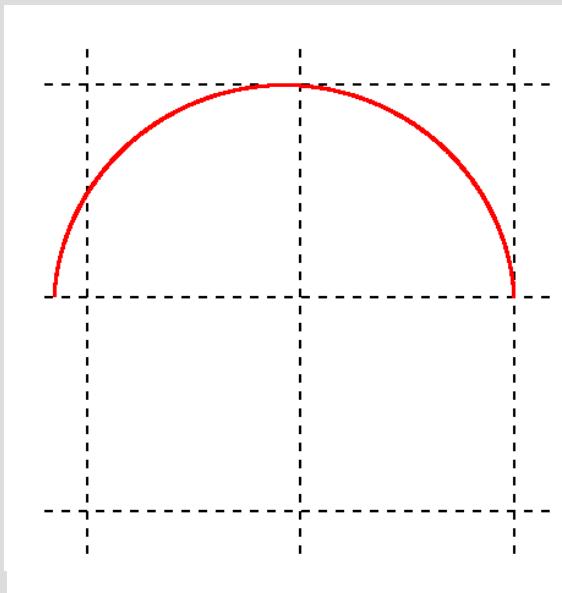
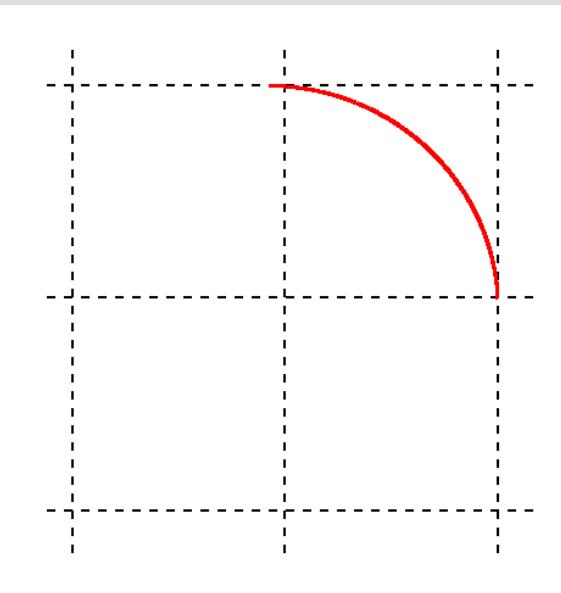
c)



$$v_d = \frac{\mathbf{E} \times \mathbf{B}}{B^2} = \frac{E}{B} = \frac{32 \cdot 10^{-3}}{53000 \cdot 10^{-9}} = 604 \text{ ms}^{-1}$$

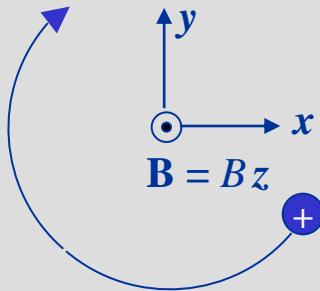


$\uparrow$   
 $E \otimes B$



# Drift motion

Consider a charged particle in a magnetic field.



Assume an electric field in the x-z plane:

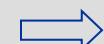
$$\mathbf{E} = (E_x, 0, E_z)$$

$$m \frac{d\mathbf{v}}{dt} = q(\mathbf{v} \times \mathbf{B} + \mathbf{E}) \implies$$

$$\begin{cases} m \frac{dv_x}{dt} = qv_y B + qE_x \\ m \frac{dv_y}{dt} = -qv_x B \\ m \frac{dv_z}{dt} = qE_z \end{cases} \quad \text{Constant acceleration along } z$$



$$\begin{cases} \frac{d^2v_x}{dt^2} = \frac{qB}{m} \frac{dv_y}{dt} = \omega_g \frac{dv_y}{dt} = -\omega_g^2 v_x \\ \frac{d^2v_y}{dt^2} = -\frac{qB}{m} \frac{dv_x}{dt} = -\omega_g \frac{dv_x}{dt} = -\omega_g^2 v_y - \frac{q^2 B}{m^2} E_x \end{cases}$$



# Drift motion

$$\begin{cases} \frac{d^2 v_x}{dt^2} = \frac{qB}{m} \frac{dv_y}{dt} = \omega_g \frac{dv_y}{dt} = -\omega_g^2 v_x \\ \frac{d^2 v_y}{dt^2} = -\frac{qB}{m} \frac{dv_x}{dt} = -\omega_g \frac{dv_x}{dt} = -\omega_g^2 v_y - \frac{q^2 B}{m^2} E_x \end{cases}$$

∴

$$\begin{cases} \frac{d^2 v_x}{dt^2} - \omega_g^2 v_x \\ \frac{d^2 \left( v_y + \frac{E_x}{B} \right)}{dt^2} = -\omega_g^2 \left( v_y + \frac{E_x}{B} \right) \end{cases}$$

$$\begin{cases} v_x = v_\perp e^{i\omega_g t + \delta_x} \\ v_y = -\frac{E_x}{B} + v_\perp e^{i\omega_g t + \delta_y} \end{cases}$$

Average over a gyro period:

$$v_{drift,y} = -\frac{E_x}{B} = -\frac{E_x B_z}{B^2} = \frac{(\mathbf{E} \times \mathbf{B})_y}{B^2}$$

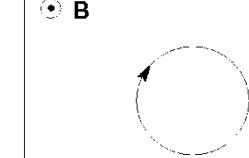
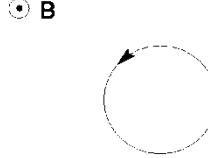
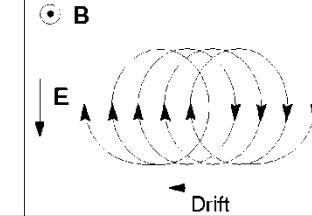
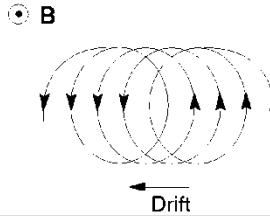
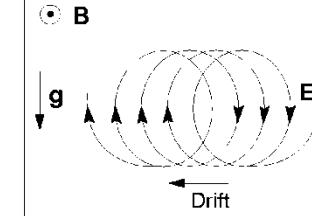
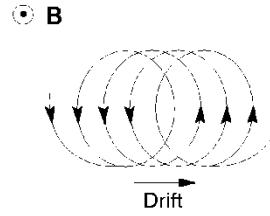
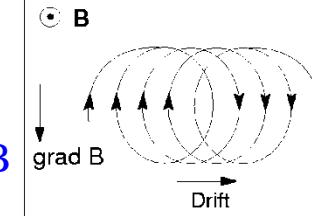
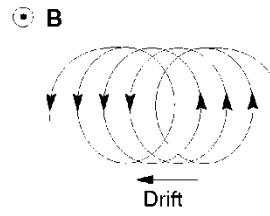
In general:

$$\mathbf{v}_{drift} = \frac{\mathbf{E} \times \mathbf{B}}{B^2} = \frac{q\mathbf{E} \times \mathbf{B}}{qB^2} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

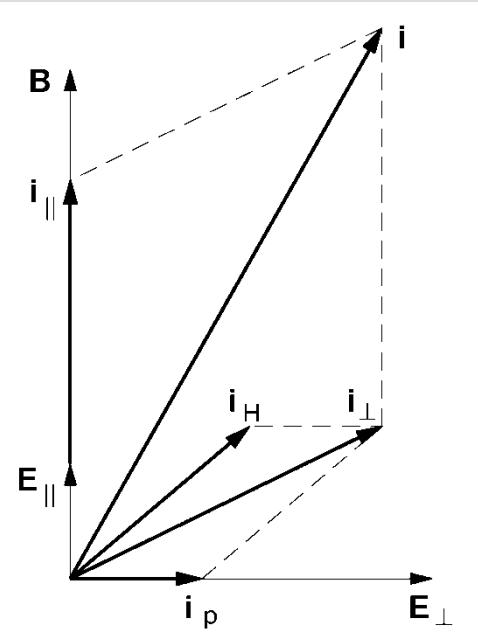


# Drift motion

$$\mathbf{u}_{drift} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

	Positive particles	Negative particles
Homogeneous magnetic field No disturbing force $\mathbf{F} = 0$		
Homogeneous magnetic field Homogeneous electric field $\mathbf{F} = q\mathbf{E}$		
Homogeneous magnetic field Gravitation $\mathbf{F} = mg$		
Inhomogeneous magnetic field $\mathbf{F} = -\mu \text{grad } \mathbf{B}$		

# Possible to get current in opposite direction to E?



$$\sigma_P = \sigma_e \frac{1}{1 + \omega_{ge}^2 \tau_e^2} + \sigma_i \frac{1}{1 + \omega_{gi}^2 \tau_i^2}$$

$$\sigma_H = \sigma_e \frac{\omega_{ge} \tau_e}{1 + \omega_{ge}^2 \tau_e^2} - \sigma_i \frac{\omega_{gi} \tau_i}{1 + \omega_{gi}^2 \tau_i^2}$$

$$\sigma_{\parallel} = \sigma_e + \sigma_P$$

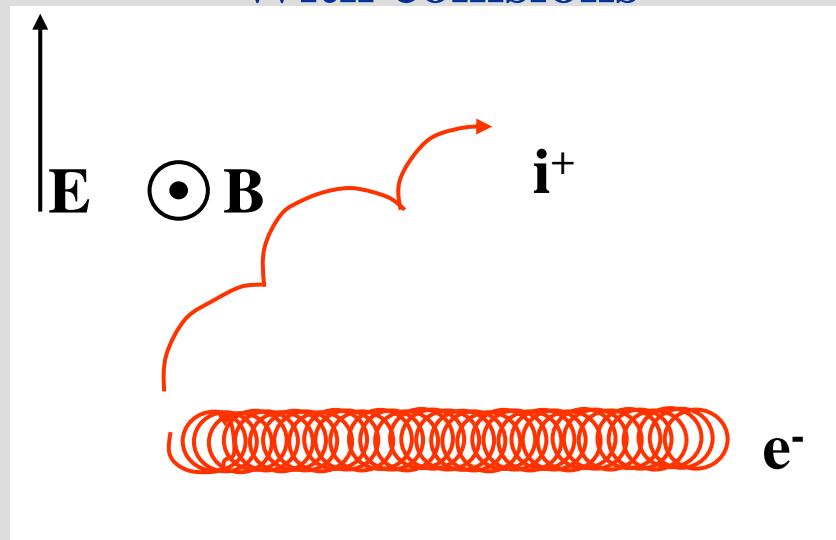
$$\sigma_e = e^2 n \tau_e / m_e$$

$$\sigma_i = e^2 n \tau_i / m_i$$

$$i_{\parallel} = \sigma_{\parallel} E_{\parallel}$$

$$i_p = \sigma_P E_{\perp} \\ i_H = \sigma_H E_{\perp}$$

**With collisions**



# Geomagnetic field

Approximated by a dipole close to Earth.

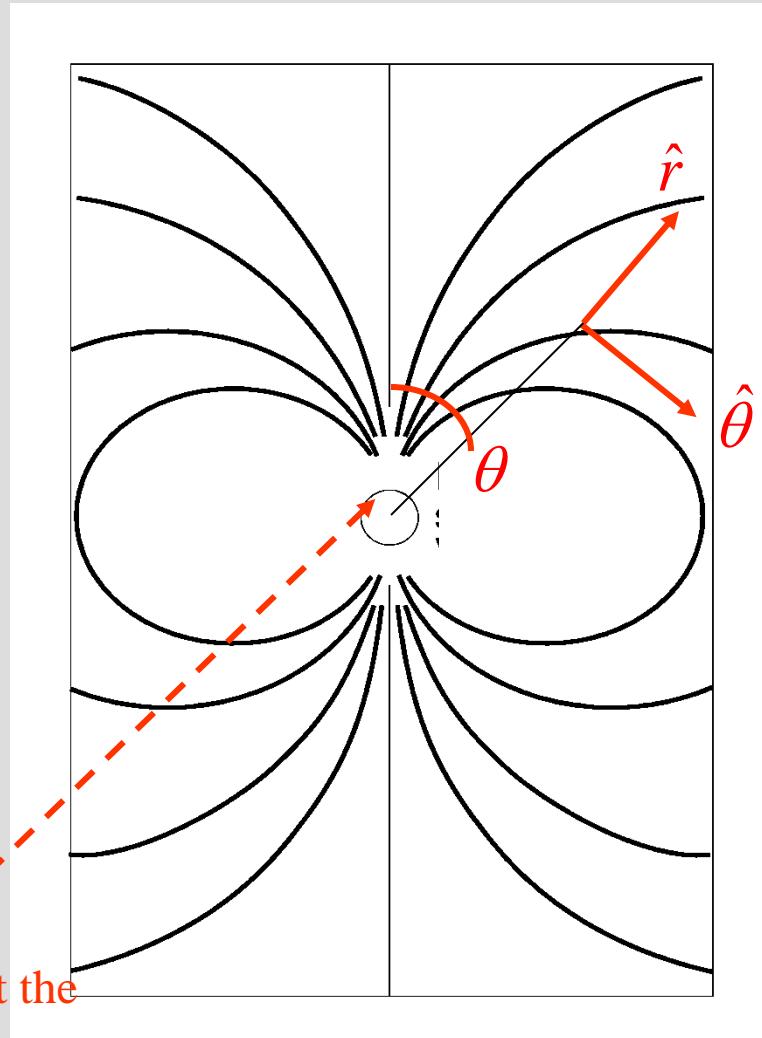
$$B_r = B_p \left( \frac{R_E}{r} \right)^3 \cos \theta$$

$$B_\theta = \frac{B_p}{2} \left( \frac{R_E}{r} \right)^3 \sin \theta$$

$$m = \frac{2\pi R_E^3 B_p}{\mu_0}$$

magnetic dipole moment

Magnetic field at the  
“north pole”



# Geomagnetic field

Alternative formulation of dipole field

$$B_r = B_p \left( \frac{R_E}{r} \right)^3 \cos \theta$$

$$B_\theta = \frac{B_p}{2} \left( \frac{R_E}{r} \right)^3 \sin \theta$$

$$B_r = \frac{\mu_0 a}{2\pi} \frac{1}{r^3} \cos \theta$$

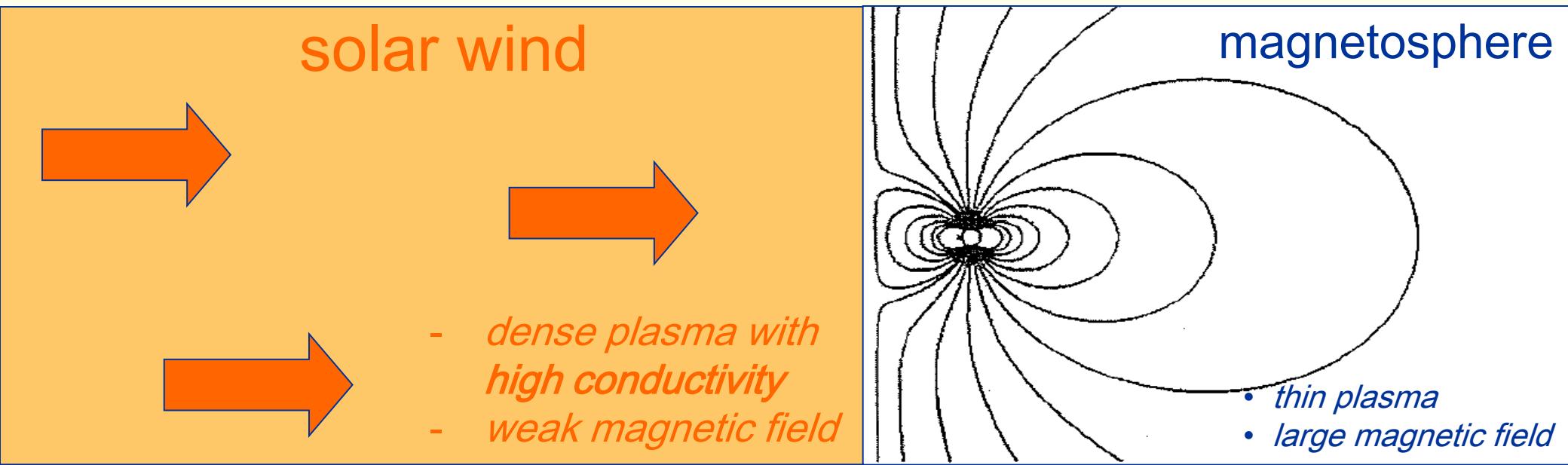
$$B_\theta = \frac{\mu_0 a}{2\pi} \cdot \frac{1}{2} \cdot \frac{1}{r^3} \sin \theta$$

$$a = \frac{2\pi R_E^3 B_p}{\mu_0}$$



magnetic dipole moment

# Stand-off distance from pressure balance



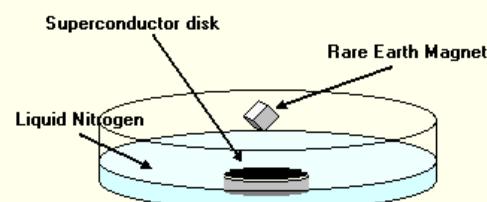
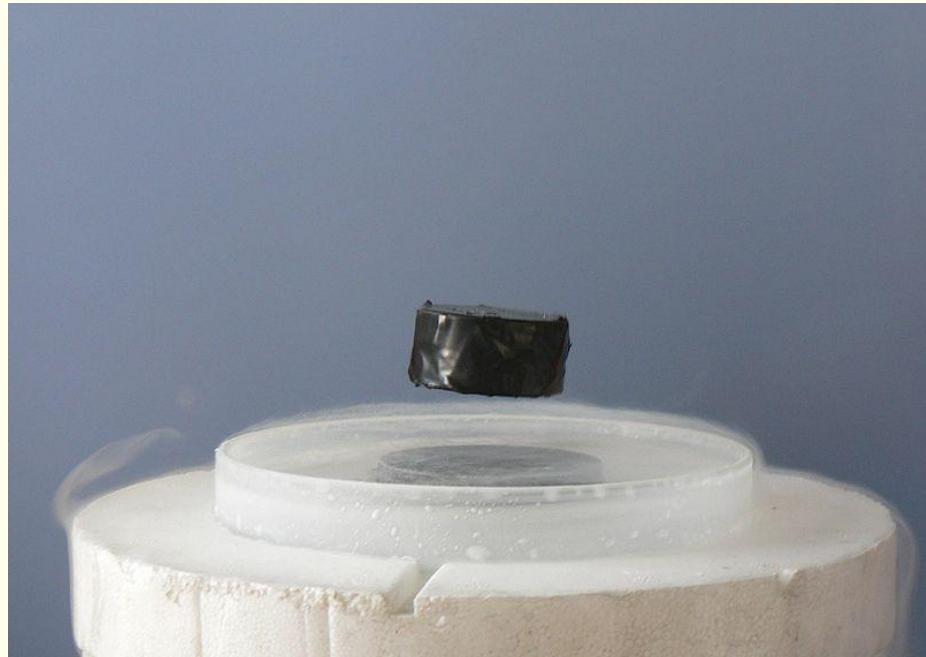
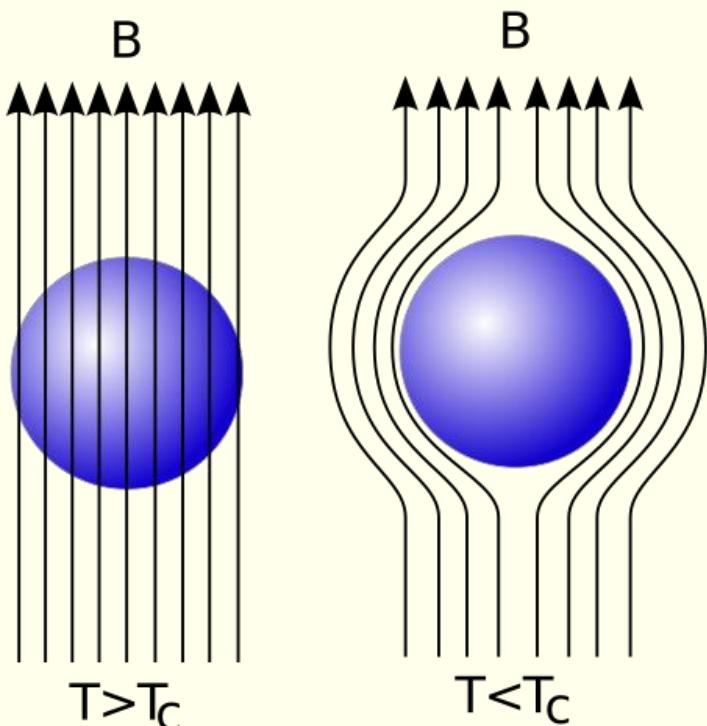
Dynamic pressure:

$$p_d = \rho_{SW} v_{SW}^2$$

Magnetic pressure:

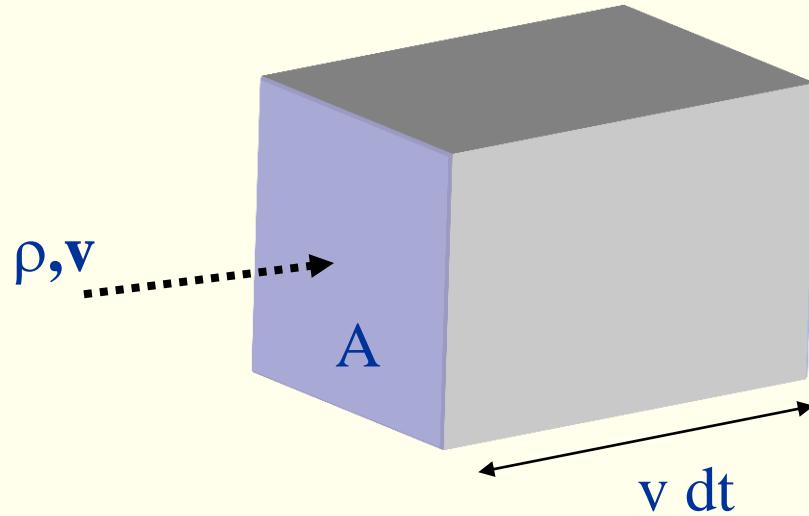
$$p_B = \frac{B^2}{2\mu_0}$$

# Meissner effect in super-conductors



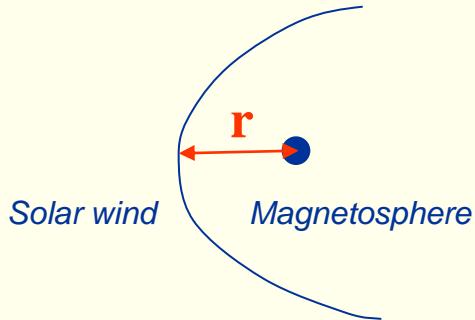
**The Meissner Effect**

# Dynamic (kinetic) pressure



$$p_d = \frac{F}{A} = \frac{d(mv)}{dt} \frac{1}{A} \approx \frac{\Delta(mv)}{\Delta t} \frac{1}{A} = \frac{\rho \cdot A v \Delta t \cdot v}{\Delta t A} = \rho v^2$$

# Magnetopause "stand-off distance"



Dynamic pressure:

$$p_d = \rho_{SW} v_{SW}^2$$

Magnetic pressure:

$$p_B = \frac{1}{2\mu_0} B^2$$

Dipole field strength  
(in equatorial plane):

$$B = \frac{\mu_0 a}{4\pi} \frac{1}{r^3}$$

$$p_d = p_B \quad \Rightarrow$$

$$\rho_{SW} v_{SW}^2 = \left[ \frac{\mu_0 a}{4\pi} \frac{1}{r^3} \right]^2 / 2\mu_0 \quad \Rightarrow$$

$$r = \left( \frac{\mu_0 a}{4\pi} \right)^{1/3} \left( 2\mu_0 \rho_{SW} v_{SW}^2 \right)^{-1/6}$$

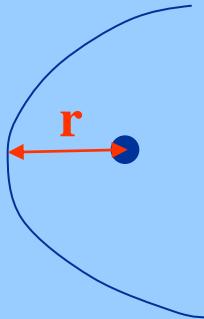
$$a = 8 \times 10^{22} \text{ Am}^2, \quad v = 500 \text{ km/s}, \quad \rho_{SW} = 10^7 \times 1.7 \times 10^{-27} \text{ kg/m}^3:$$

$$\mathbf{r} = 7 \mathbf{R}_e$$

$$(1 \mathbf{R}_e = 6378 \text{ km})$$

# Standoff distance

$$v = 500 \text{ km/s}, \quad \rho_{SW} = 10^7 \times 1.7 \times 10^{-27} \text{ kg/m}^3; \quad \mathbf{r} = 7 \mathbf{R}_e$$



$$r = \left( \frac{\mu_0 a}{4\pi} \right)^{1/3} \left( 2\mu_0 \rho_{SW} v_{SW}^2 \right)^{-1/6}$$

How will the standoff distance change if the magnetosphere is hit by a coronal mass ejection (CME)? ( $\rho = 10\rho_{SW}$ ,  $v = 1000 \text{ km/s}$ )

Blue

$$r = 1.8 \mathbf{R}_e$$

Yellow

$$r = 5.8 \mathbf{R}_e$$

Green

$$r = 3.8 \mathbf{R}_e$$

Red

$$r = 9.8 \mathbf{R}_e$$



# Standoff distance

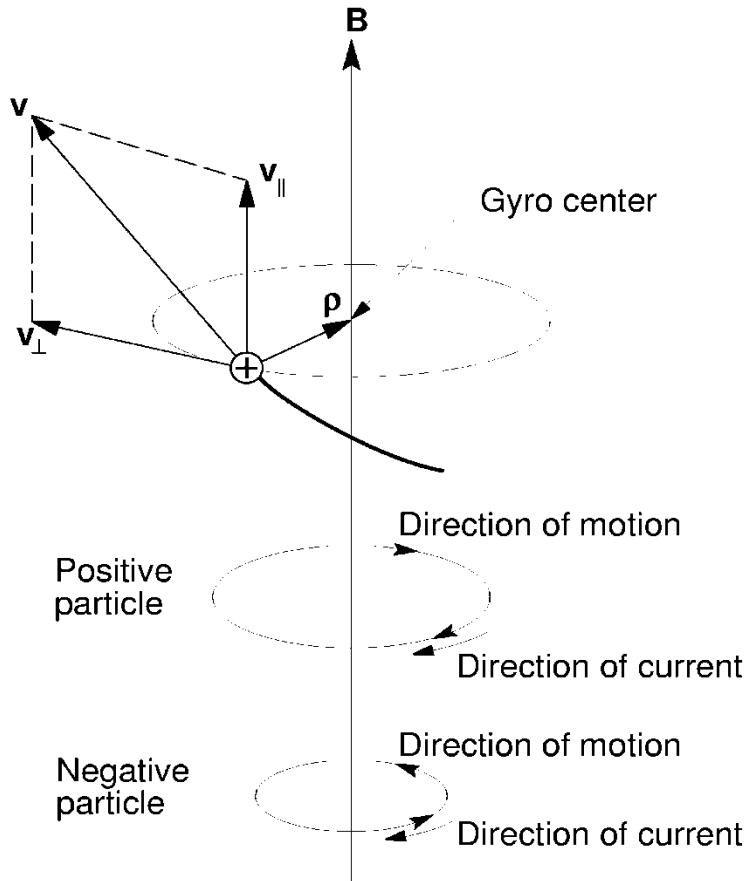
$$r = \left( \frac{\mu_0 a}{4\pi} \right)^{1/3} \left( 2\mu_0 10 \rho_{SW} (2v)_{SW}^2 \right)^{-1/6} = \left( \frac{\mu_0 a}{4\pi} \right)^{1/3} \left( 2\mu_0 \rho_{SW} v_{SW}^2 \right)^{-1/6} 40^{-1/6}$$

$$40^{-1/6} \cdot 7 = 0.54 \cdot 7 = 3.8$$

Green

$r = 3.8 R_e$

# Particle motion in magnetic field



## gyro radius

$$\rho = \frac{mv_{\perp}}{qB}$$

## gyro frequency

$$\omega_g = \frac{qB}{m}$$

## magnetic moment

$$\mu = IA = q f_g \pi \rho^2 = mv_{\perp}^2/2B$$



# Adiabatic invariant

## DEFINITION:

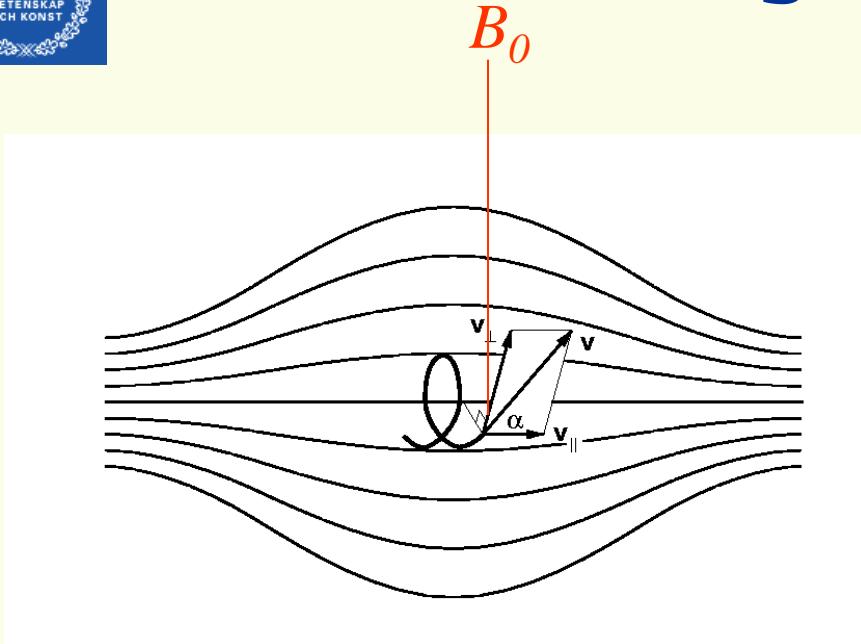
An **adiabatic invariant** is a property of a physical system which stays constant when changes are made slowly.

*By 'slowly' in the context of charged particle motion in magnetic fields, we mean much slower than the gyroperiod.*

'First adiabatic invariant' of particle drift:

$$\mu = \frac{mv_{\perp}^2}{2B}$$

# Magnetic mirror



The magnetic moment  $\mu$  is an *adiabatic invariant*.

$$\mu = \frac{mv_{\perp}^2}{2B} = \frac{mv^2 \sin^2 \alpha}{2B}$$

$mv^2/2$  constant (energy conservation) →

$$\frac{\sin^2 \alpha}{B} = \text{konst}$$

What happens with  $\alpha$  as the particle moves into the stronger magnetic field?

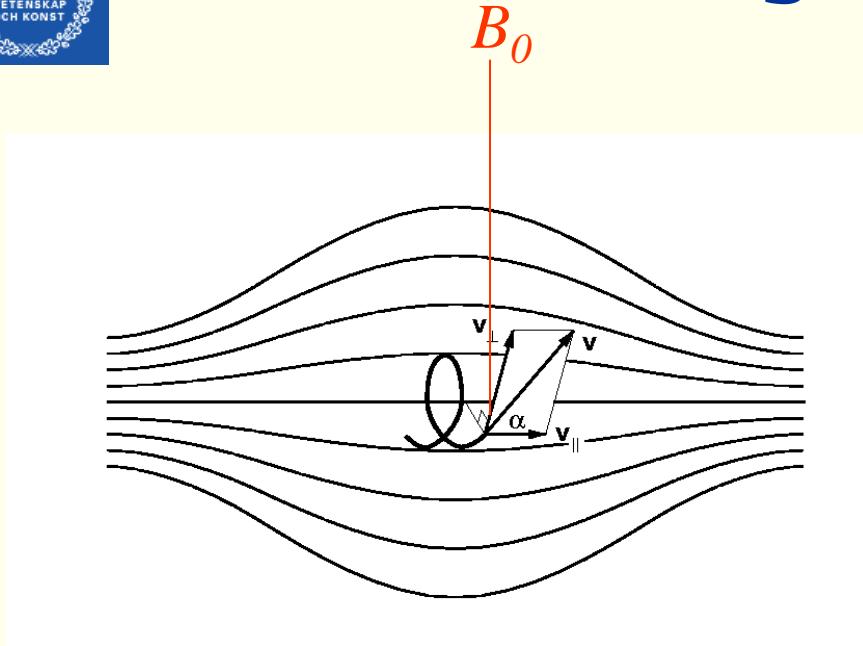
Red

$\alpha$  increases

Yellow

$\alpha$  decreases

# Magnetic mirror



The magnetic moment  $\mu$  is an *adiabatic invariant*.

$$\mu = \frac{mv_{\perp}^2}{2B} = \frac{mv^2 \sin^2 \alpha}{2B}$$

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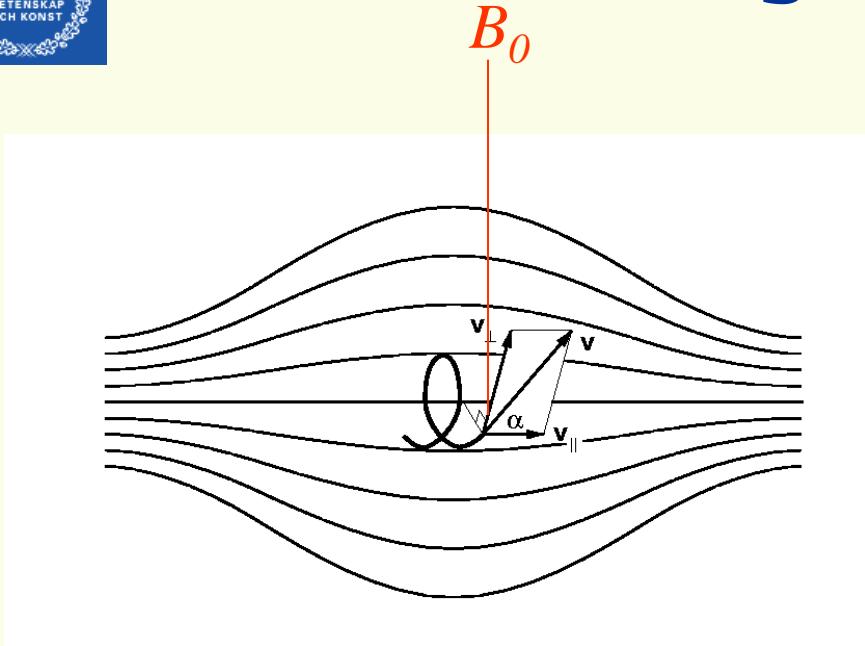
What happens with  $\alpha$  as the particle moves into the stronger magnetic field?

$$\sin \alpha = \sqrt{B \cdot \text{konst}}$$

Red

$\alpha$  increases

# Magnetic mirror



The magnetic moment  $\mu$  is an *adiabatic invariant*.

$$\mu = \frac{mv_{\perp}^2}{2B} = \frac{mv^2 \sin^2 \alpha}{2B}$$

$mv^2/2$  constant (energy conservation) →

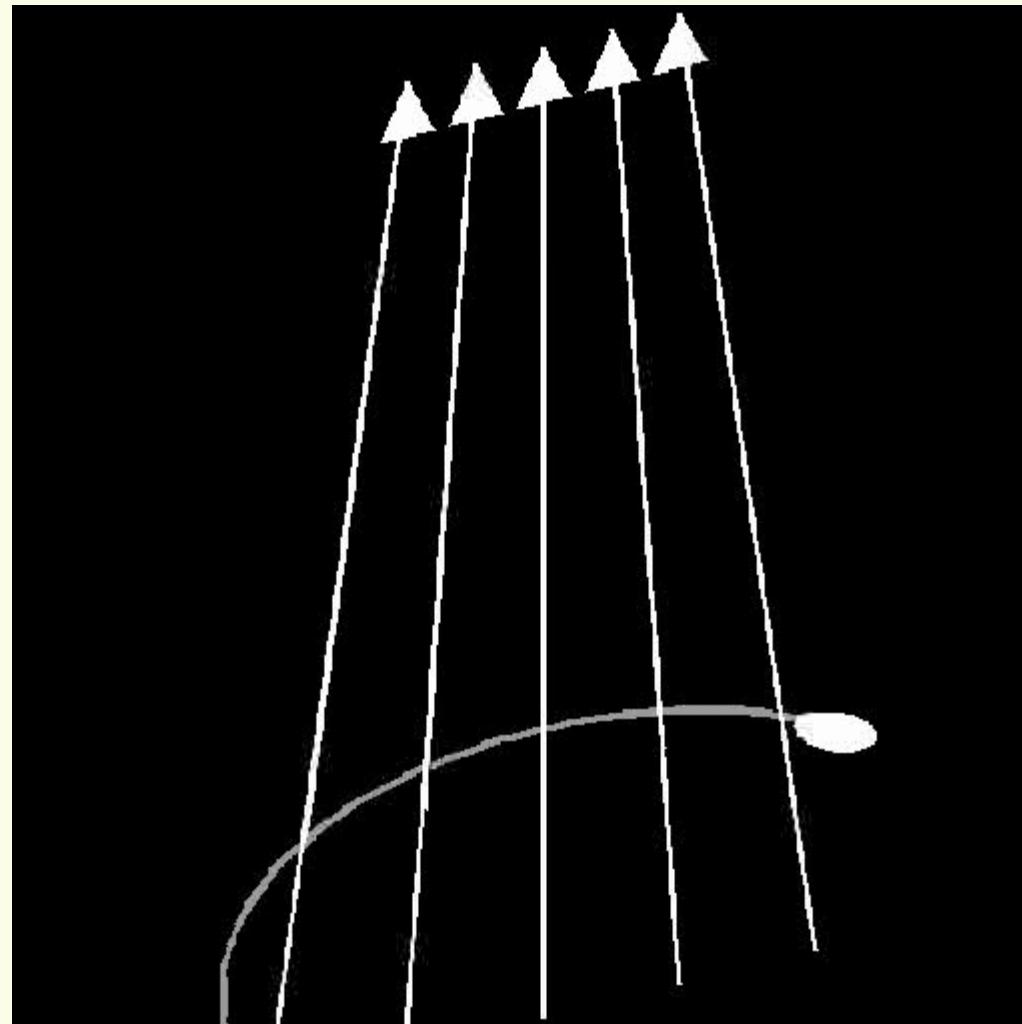
$$\frac{\sin^2 \alpha}{B} = \text{konst}$$

particle turns when  $\alpha = 90^\circ$  →

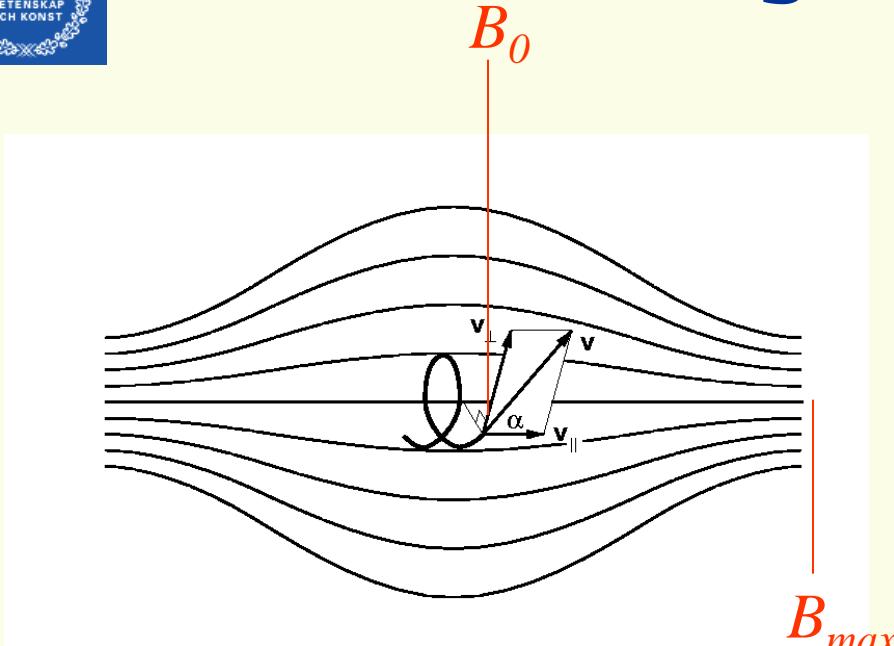
$$\frac{B_{\text{turn}}}{\sin^2 90^\circ} = \frac{B_0}{\sin^2 \alpha}$$

$$B_{\text{turn}} = \frac{B_0}{\sin^2 \alpha}$$

# Magnetic mirror



# Magnetic mirror



The magnetic moment  $\mu$  is an *adiabatic invariant*.

$$\mu = \frac{mv_{\perp}^2}{2B} = \frac{mv^2 \sin^2 \alpha}{2B}$$

$mv^2/2$  constant (energy conservation) →

$$\frac{\sin^2 \alpha}{B} = \text{konst}$$

particle turns when  $\alpha = 90^\circ$  →

$$B_{turn} = B_0 / \sin^2 \alpha$$

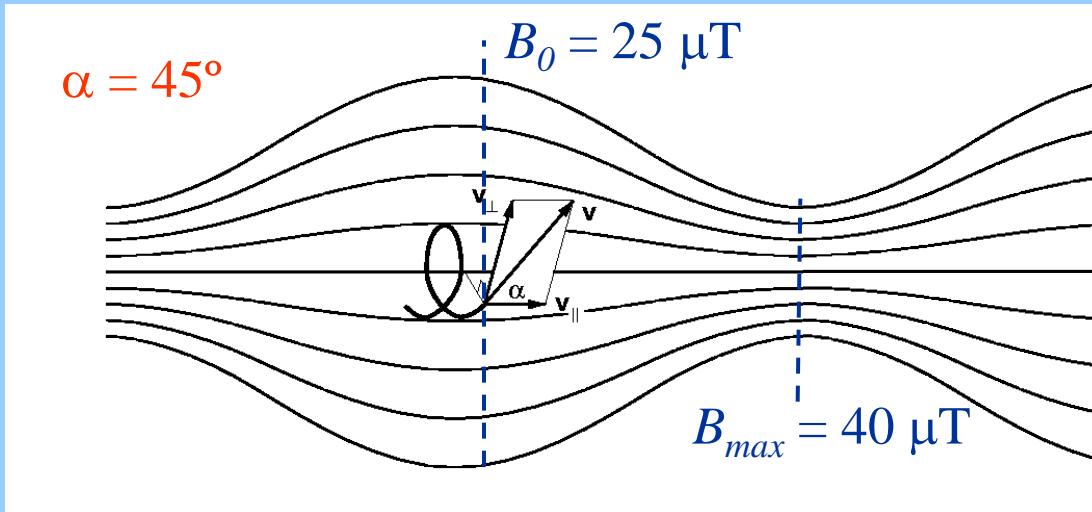
If maximal  $B$ -field is  $B_{max}$  a particle with pitch angle  $\alpha$  can only be turned around if

$$B_{turn} = B_0 / \sin^2 \alpha \leq B_{max} \rightarrow$$

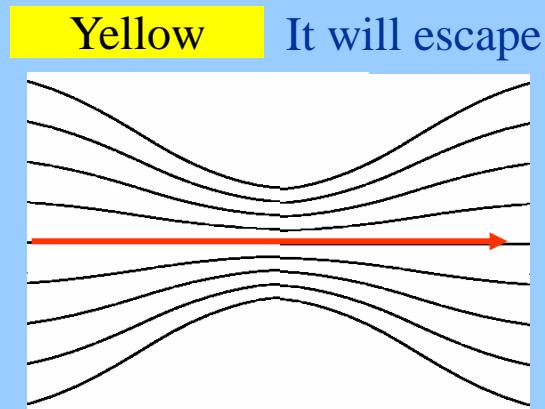
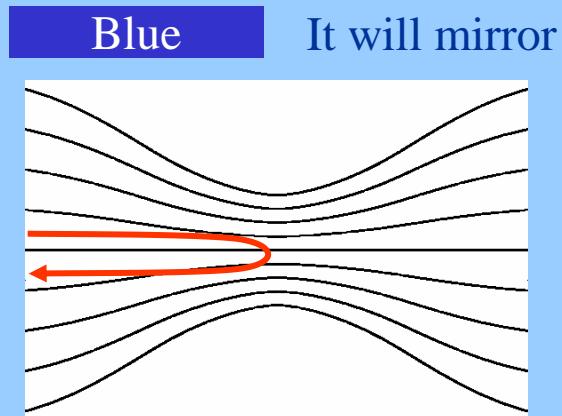
$$\alpha > \alpha_{lc} = \arcsin \sqrt{B_0 / B_{max}}$$

Particles in *loss cone* :  $\alpha < \alpha_{lc}$

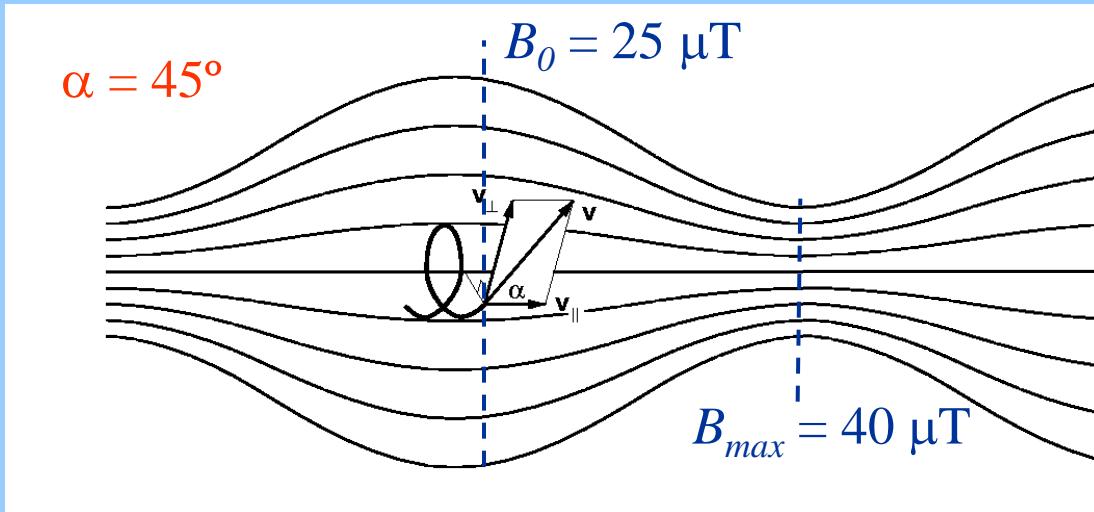
# What will happen to the particle?



$$\alpha_{lc} = \arcsin \sqrt{B_0 / B_{max}}$$

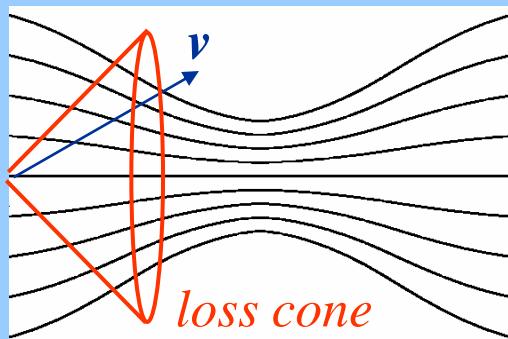


# What will happen to the particle?



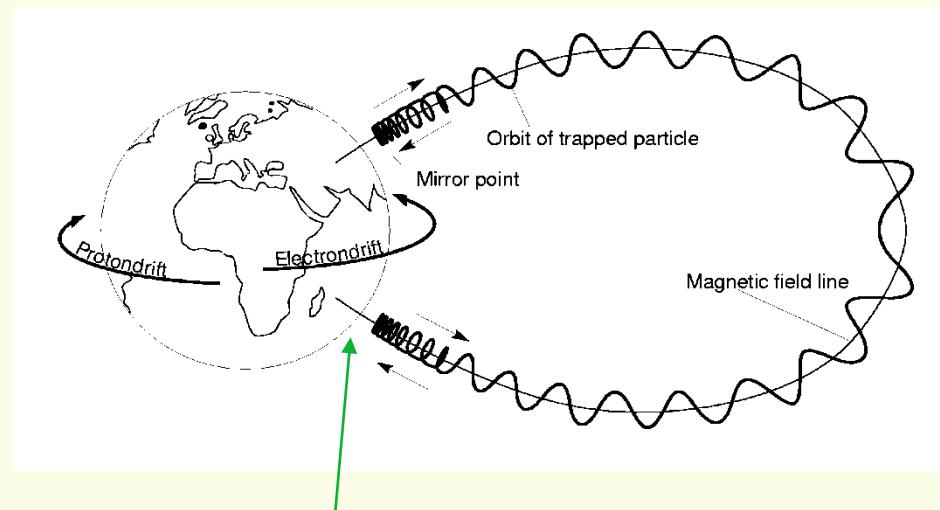
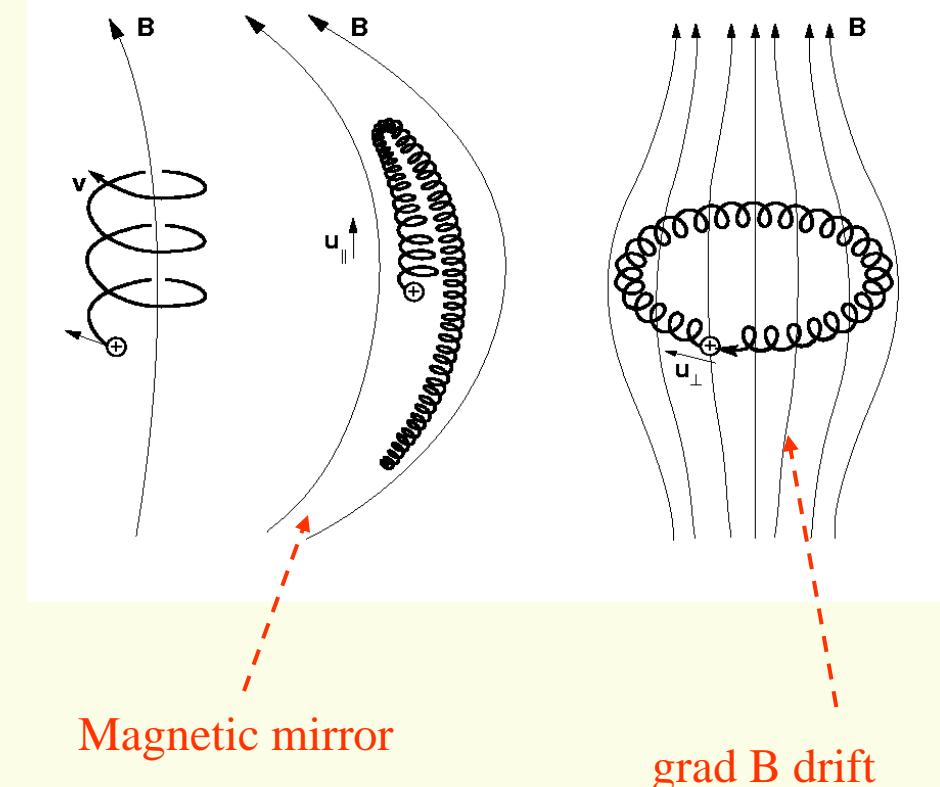
$$\alpha_{lc} = \arcsin \sqrt{B_0 / B_{max}} = \arcsin \sqrt{25 / 40} = 52^\circ$$

Yellow It will escape



# Particle motion in geomagnetic field

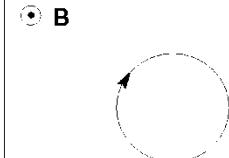
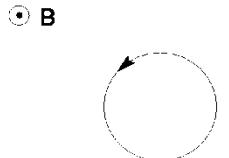
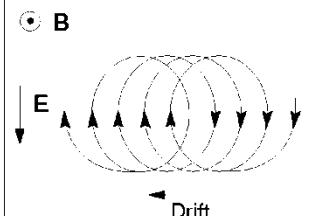
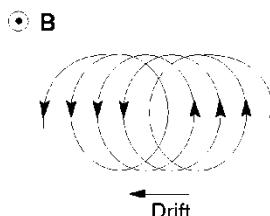
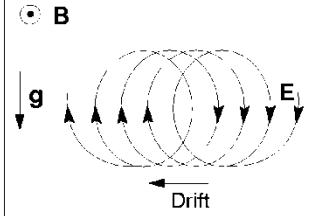
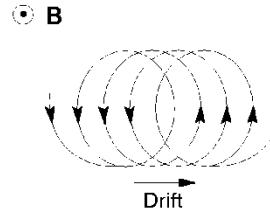
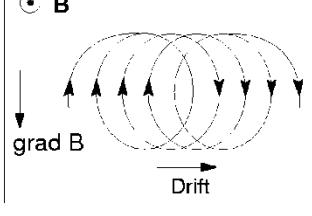
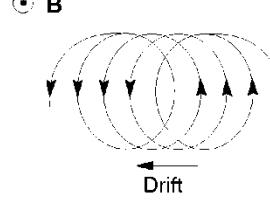
gyration      longitudinal oscillation      azimuthal drift



Particles in the loss cone create the aurora!

# Drift motion

$$\mathbf{u} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

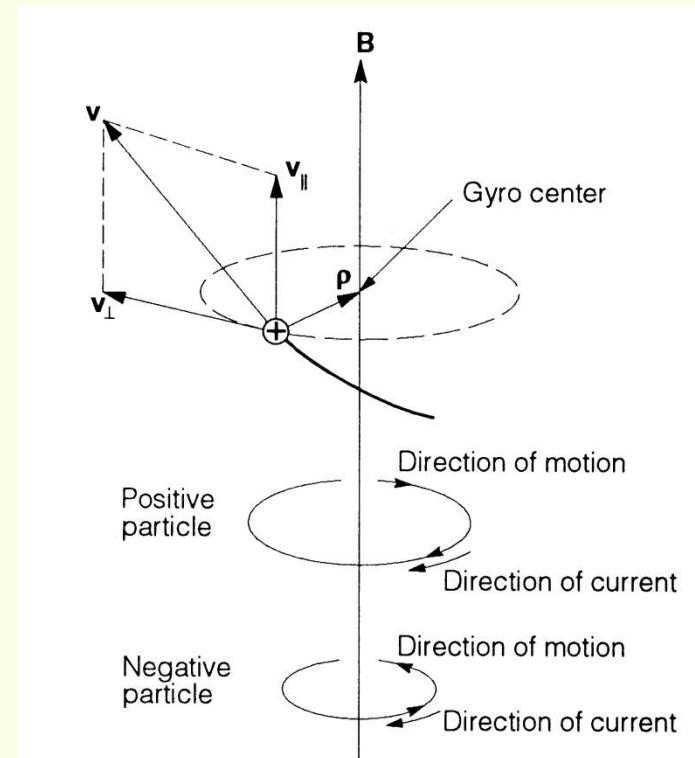
	Positive particles	Negative particles
Homogeneous magnetic field No disturbing force $\mathbf{F} = 0$		
Homogeneous magnetic field Homogeneous electric field $\mathbf{F} = q\mathbf{E}$		
Homogeneous magnetic field Gravitation $\mathbf{F} = mg$		
Inhomogeneous magnetic field $\mathbf{F} = -\mu \text{ grad } \mathbf{B}$		

# Force on magnetic dipole

$$\mu \sim -\mathbf{B} \Rightarrow \mu = -\mu \frac{\mathbf{B}}{B}$$

$$\mathbf{F} = \nabla(\mu \cdot \mathbf{B}) = -\mu \nabla \left( \frac{\mathbf{B}}{B} \cdot \mathbf{B} \right) =$$

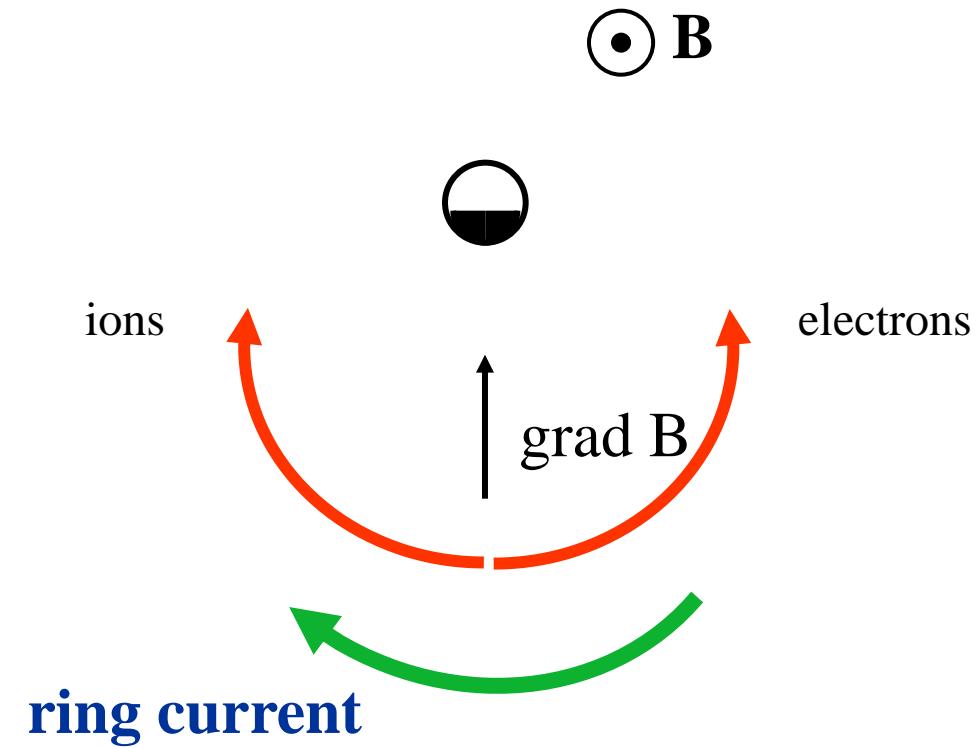
$$= -\mu \nabla \left( \frac{B^2}{B} \right) = -\mu \nabla B$$



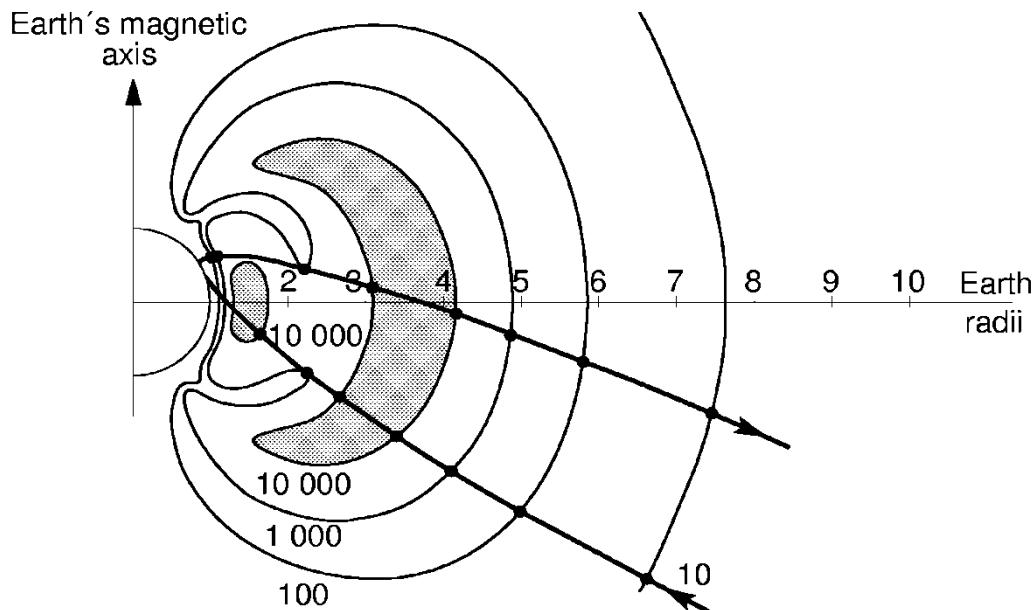
# Ring current and particle motion

$$\mathbf{u} = -\frac{\mu \nabla B \times \mathbf{B}}{qB^2}$$

$$\mu = \frac{mv_{\perp}^2}{2B}$$



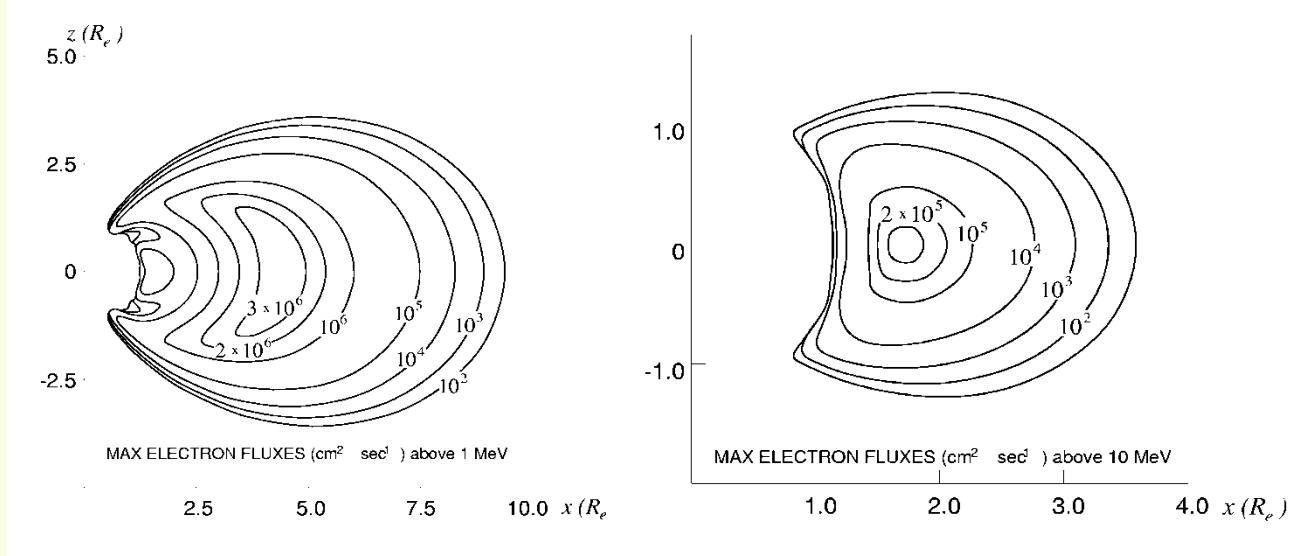
# Radiation belts



## I. Van Allen belts

- Discovered in the 50s , Explorer 1
- Inner belt contains protons with energies of ~30 MeV
- Outer belt (Explorer IV, Pioneer III): electrons,  $W > 1.5 \text{ MeV}$

# Radiation belts



- At lower energies there is a more or less continuous population of energetic particles in the inner magnetosphere. (Inner part of ***plasma sheet***)
- source: **CRAND** (Cosmic Ray Albedo Neutron Decay).
- a danger for satellites and astronauts.
- associated with a current (***ring current***) which distorts the inner part of the geomagnetic field.

# CRAND (Cosmic Ray Albedo Neutron Decay)

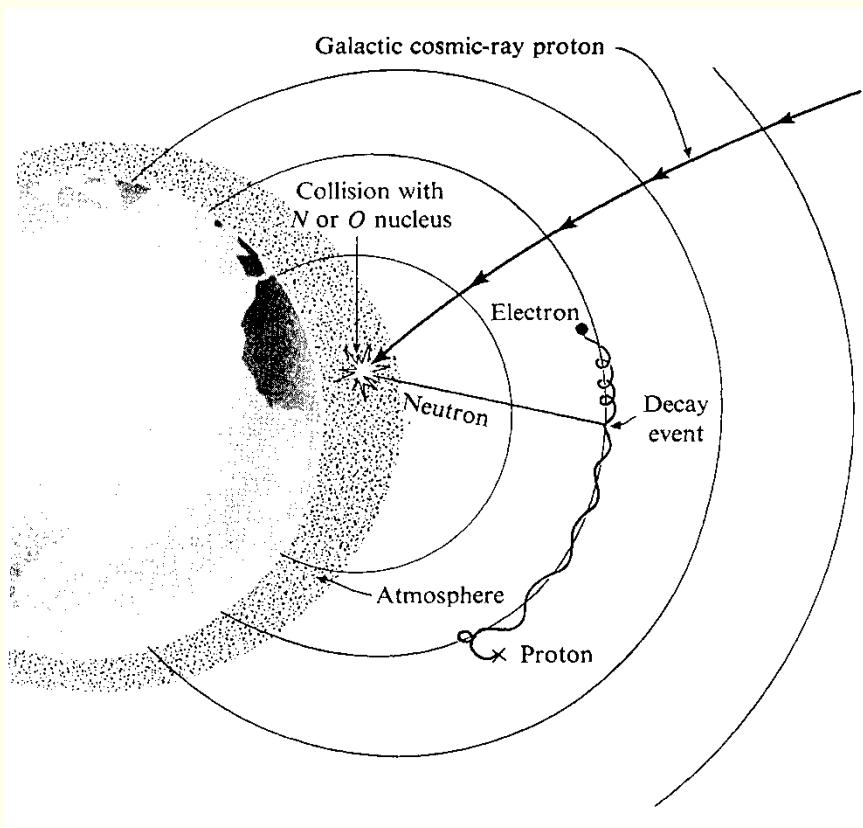
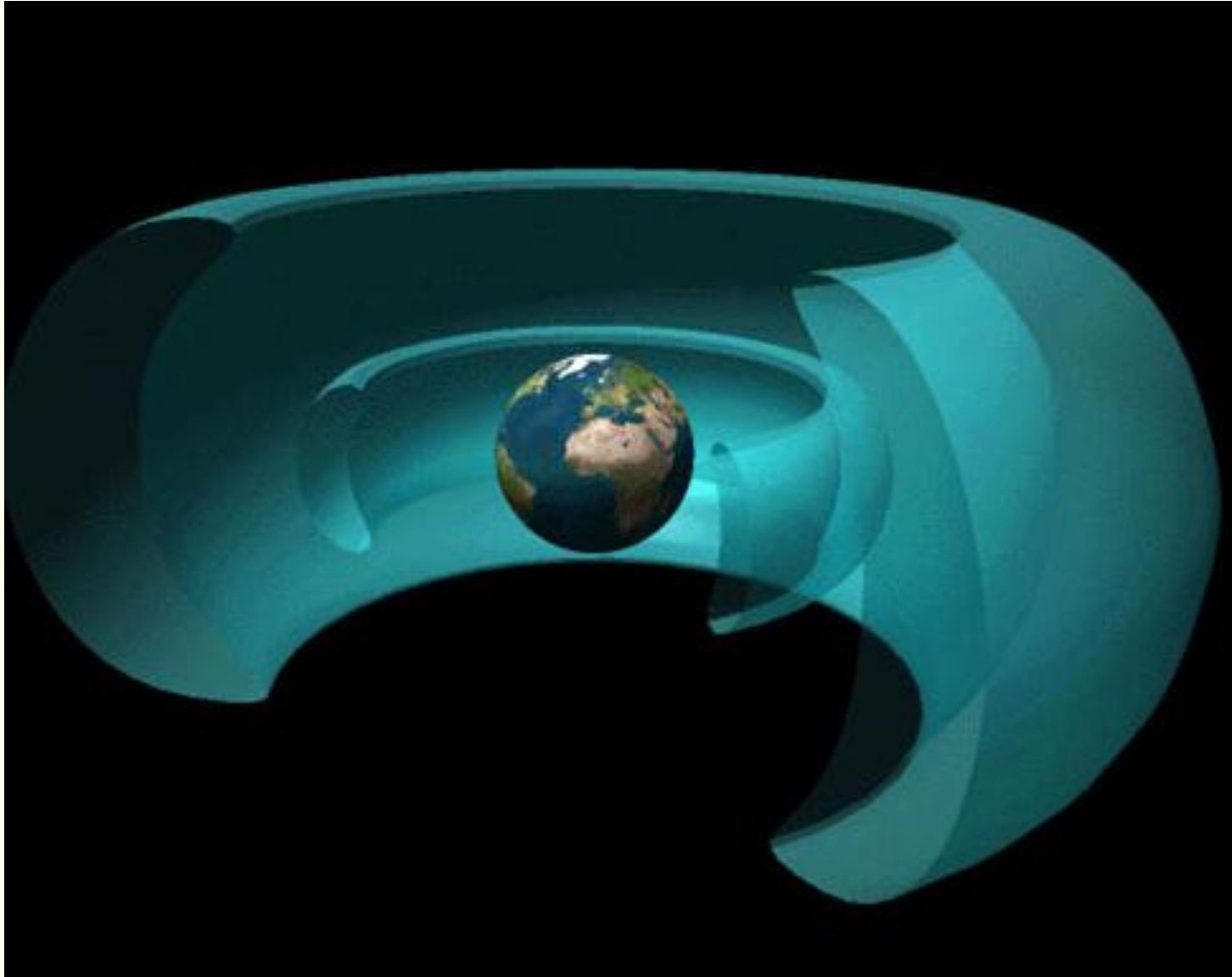


Figure 8. An illustration of the CRAND process for populating the inner radiation belts [Hess, 1968].

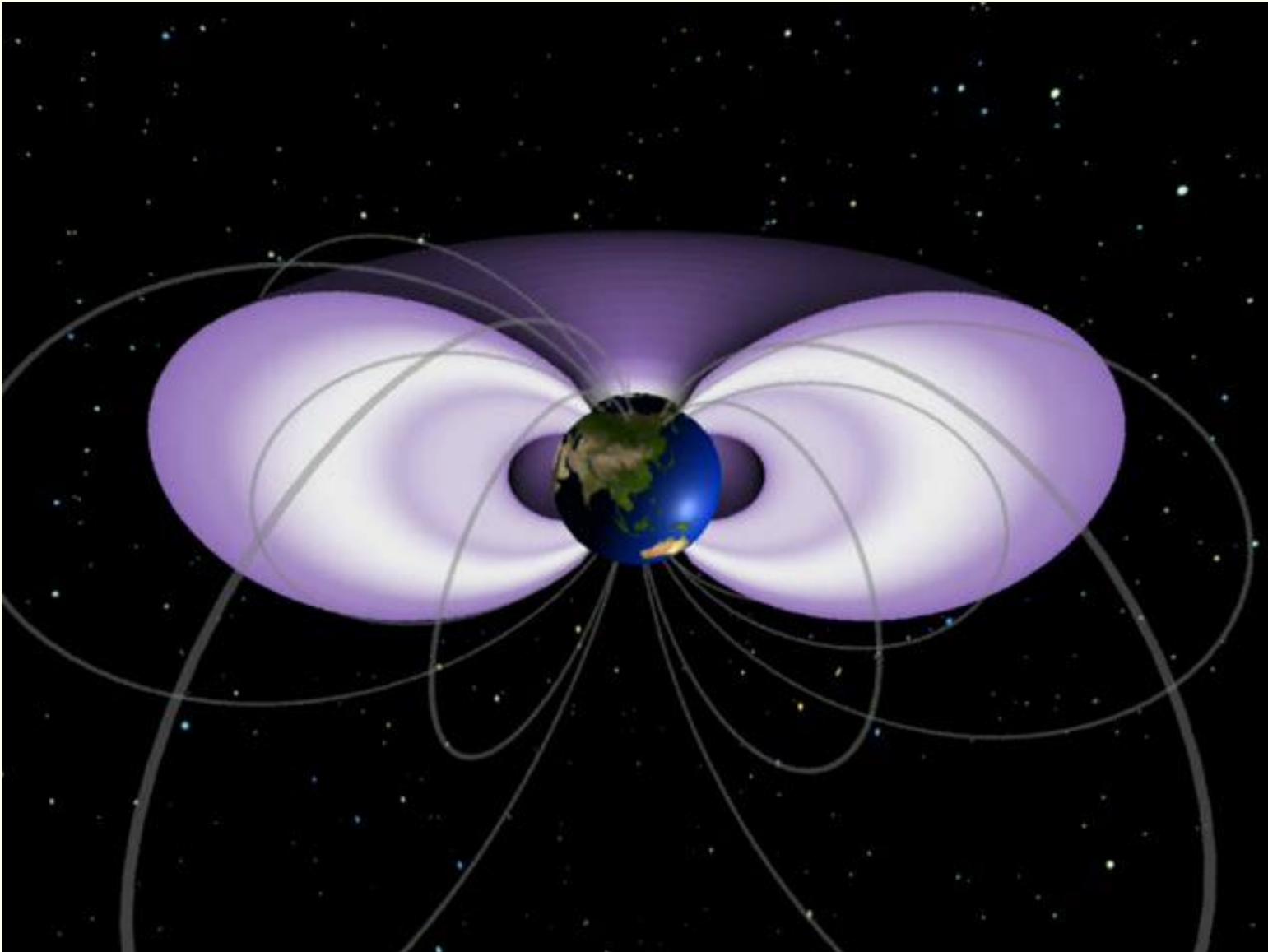
Collisions between cosmic ray particles and the Earth create new particles. Among these are neutrons, that are not affected by the magnetic field. They decay, soon eof them when they happen to be in the radiation belts. The resulting protons and electrons are trapped in the radiation belts.

This contribution to the radiation belts are called the ***neutron albedo***.

# Radiation belts



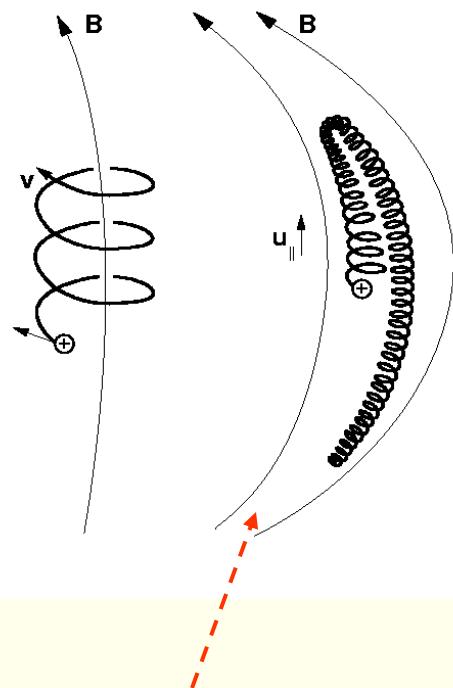
# Radiation belts



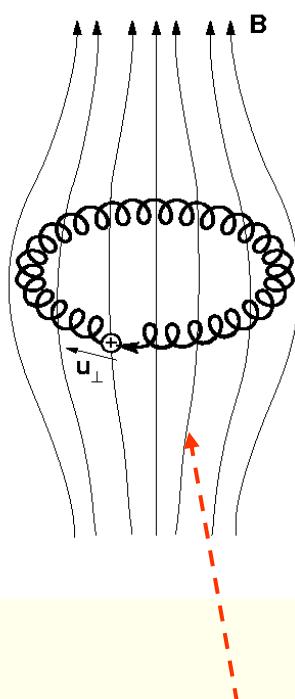
# Particle motion in geomagnetic field

longitudinal oscillation

gyration

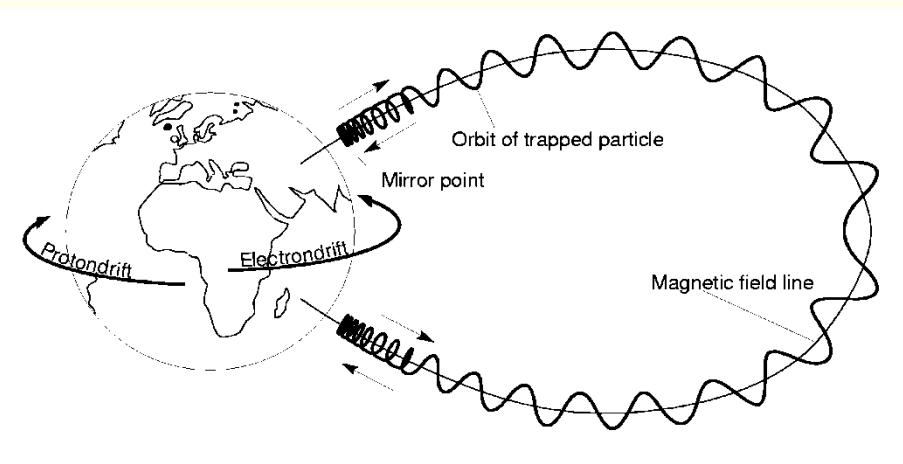


azimuthal drift

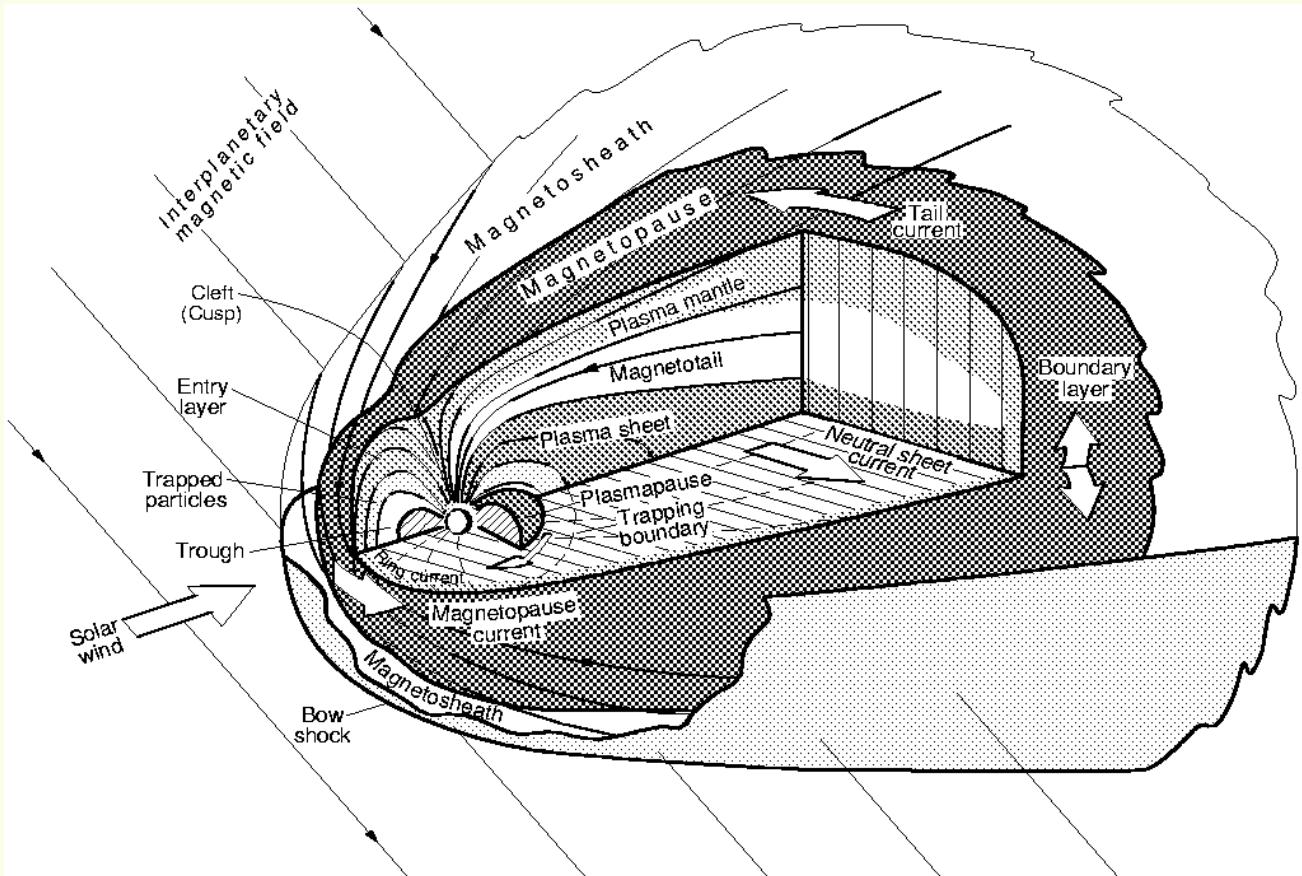


Magnetic mirror

grad B drift



# Structure of magnetosphere

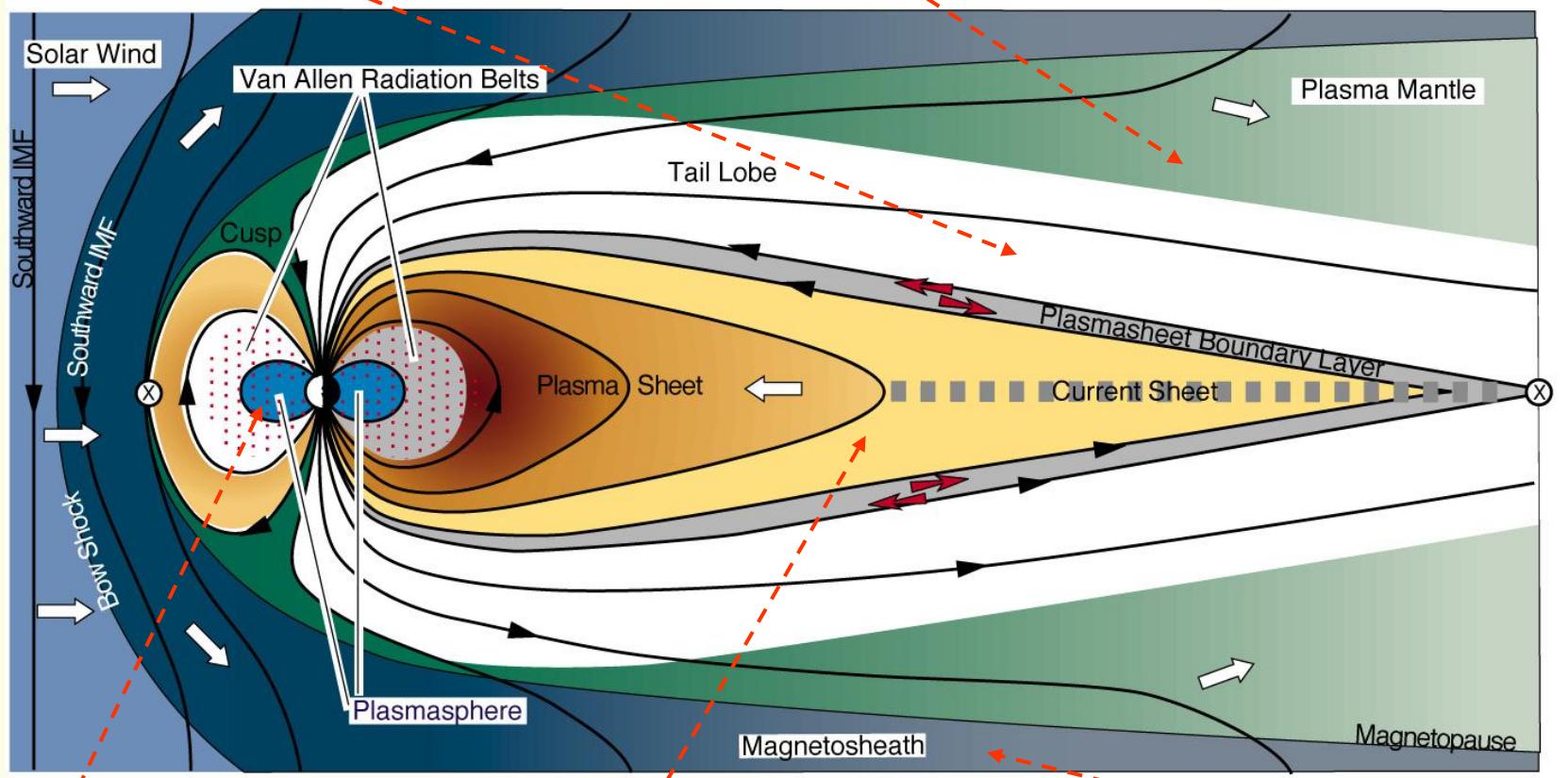


- The plasma in the magnetosphere is made up of approximately equal parts of  $H^+$  and  $O^+$ .
- Plasma populations are organized by the geomagnetic field.
- Particles will mirror between northern and southern hemispheres on closed field lines.

# Magnetospheric structure

polar plumes = tail lobe  
 $n_e \sim 0,01 \text{ cm}^{-3}$ ,  $T_e \sim 10^6 \text{ K}$

plasma mantle  
 $n_e \sim 0,1-1 \text{ cm}^{-3}$ ,  $T_e \sim 10^6 \text{ K}$

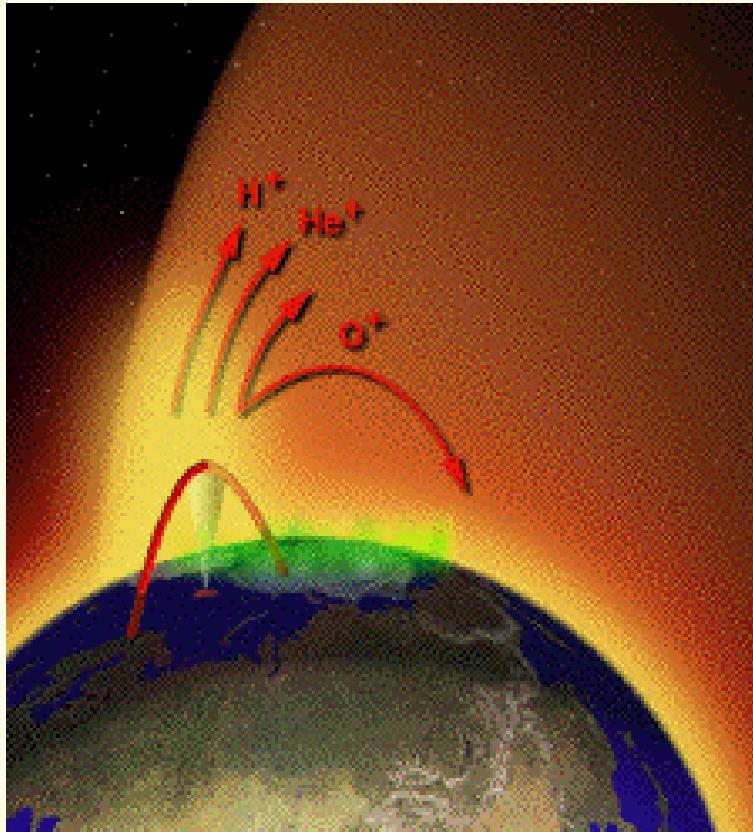


plasmasphere:  
 $n_e \sim 10-100 \text{ cm}^{-3}$ ,  $T_e \sim 1000 \text{ K}$

plasma sheet:  
 $n_e \sim 1 \text{ cm}^{-3}$ ,  $T_e \sim 10^7 \text{ K}$

magnetosheath:  
 $n_e \sim 5 \text{ cm}^{-3}$ ,  $T_e \sim 10^6 \text{ K}$

# Outflow from the ionosphere

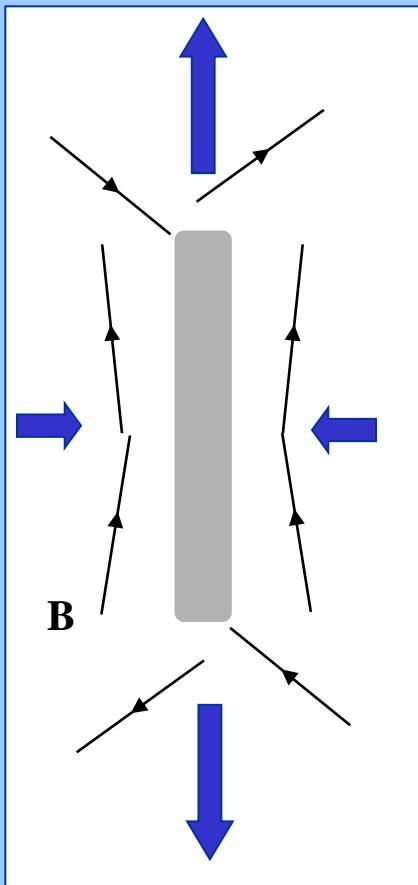


An important source for the magnetospheric plasma.  
Research is ongoing.

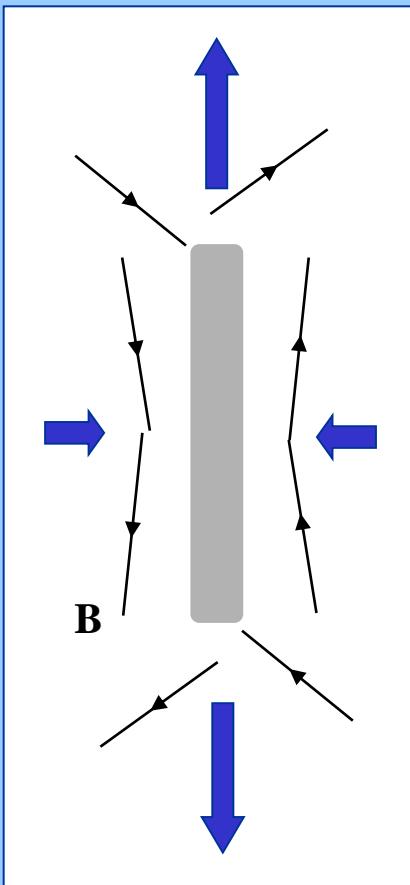
# Magnetic reconnection

Which figure is correct?

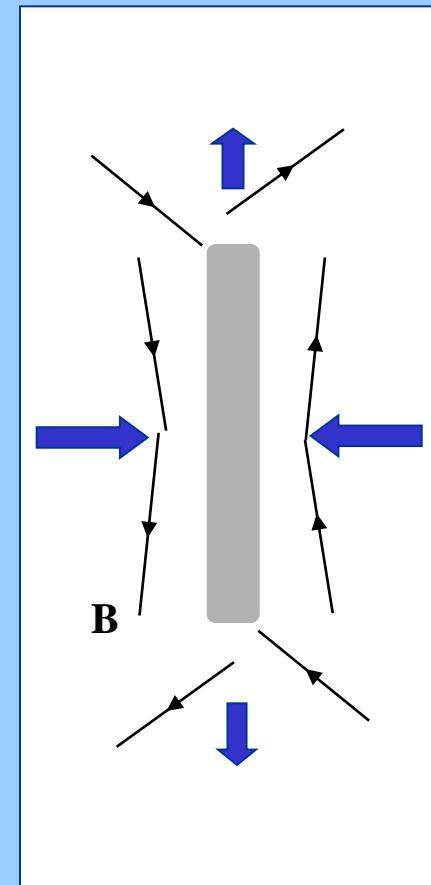
Size of arrows represents plasma flow velocity



Green



Yellow

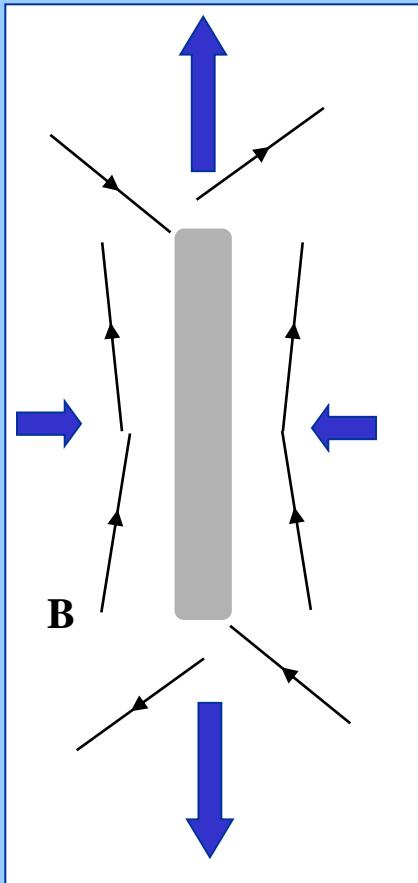


Red

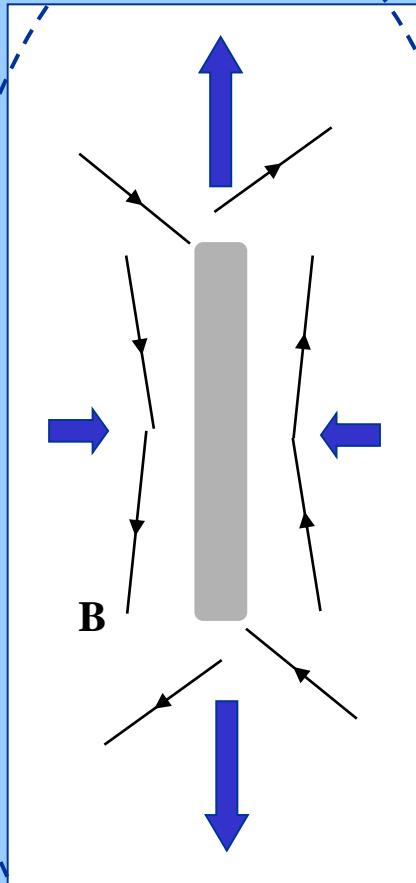
# Magnetic reconnection

*Which figure is correct?*

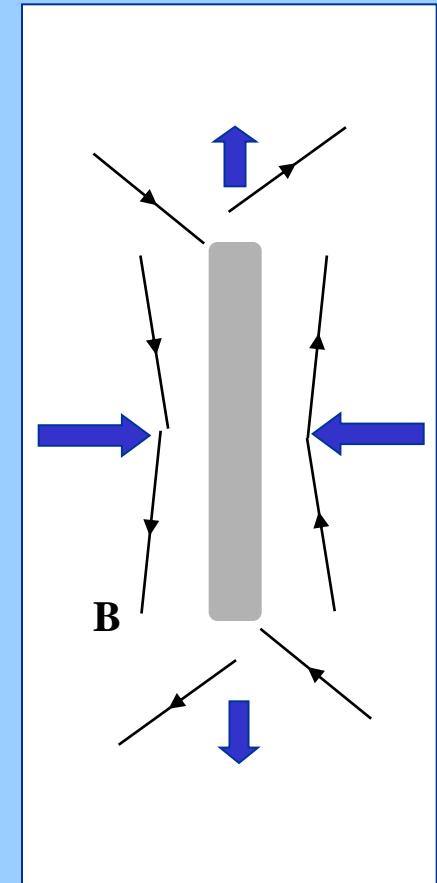
*Size of arrows represents plasma flow velocity*



Green

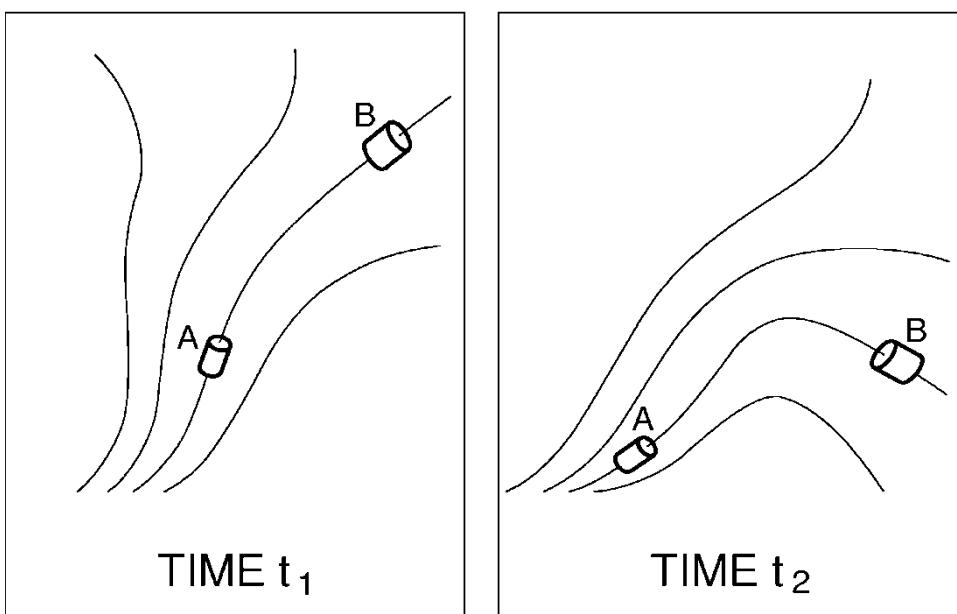


Yellow



Red

# Frozen in magnetic field lines

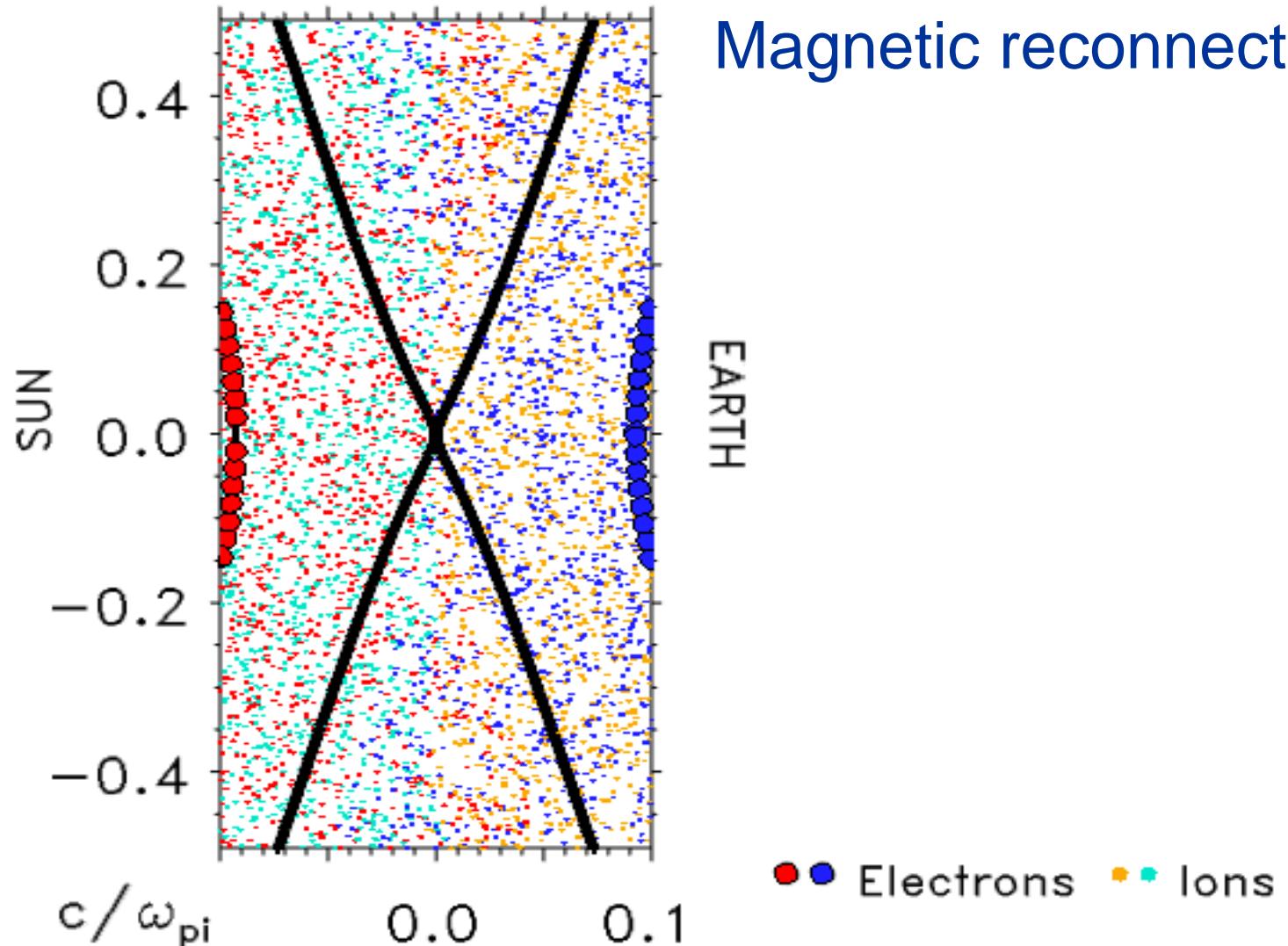


In fluid description of plasma two plasma elements that are connected by a common magnetic field line at time  $t_1$ , will be so at any other time  $t_2$ .

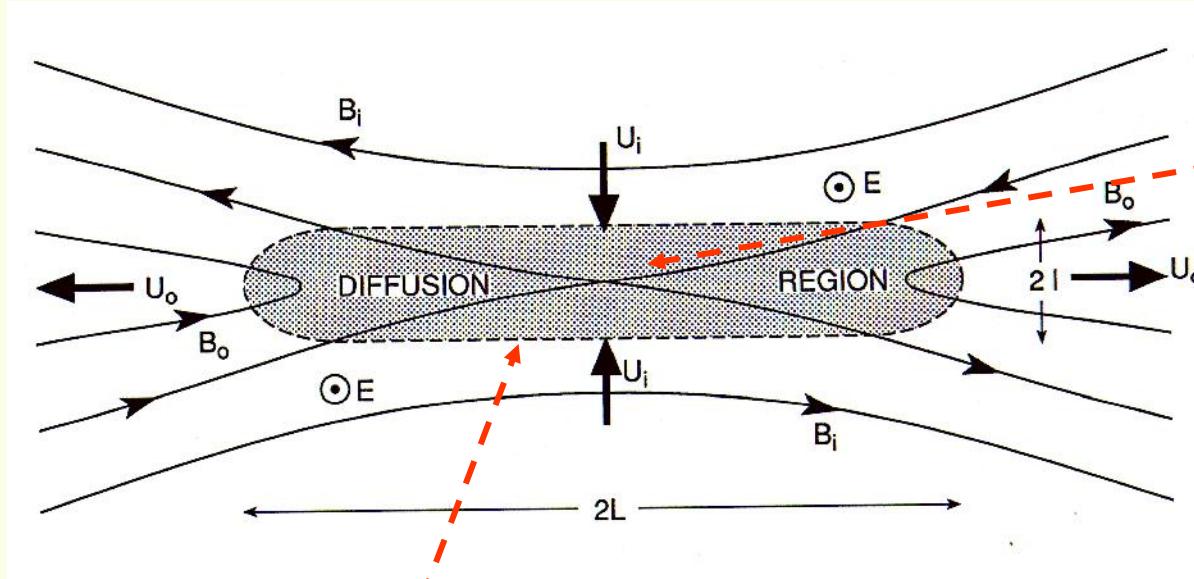
This applies if the magnetic Reynolds number is large:

$$R_m = \mu_0 \sigma l_c v_c \gg 1$$

*An example of the collective behaviour of plasmas.*



# Reconnection



- Field lines are “cut” and can be re-connected to other field lines
- **Magnetic energy is transformed into kinetic energy ( $U_o \gg U_i$ )**
- **Plasma from different field lines can mix**

In ‘diffusion region’:

$$R_m = \mu_0 \sigma l v \sim 1$$

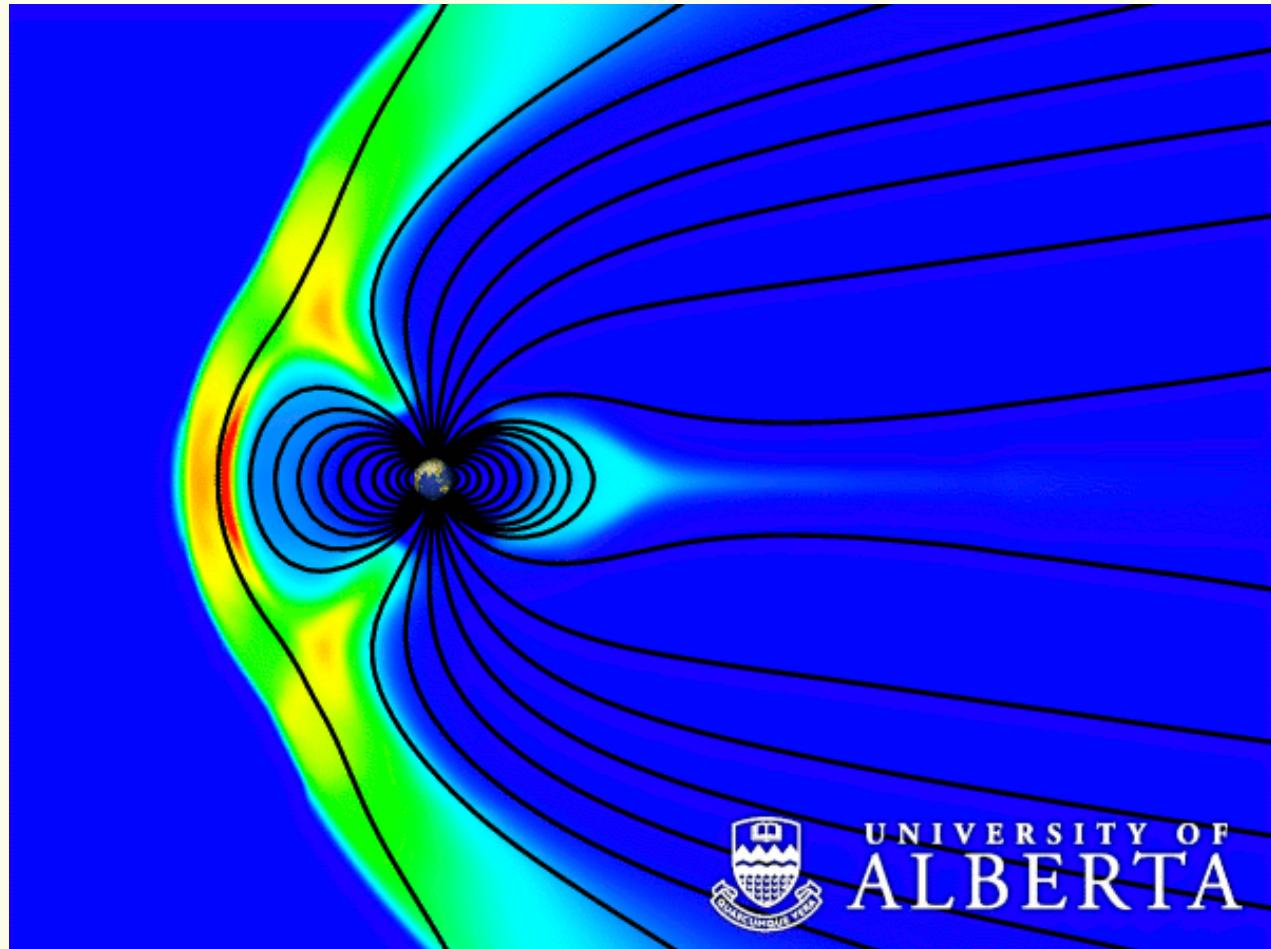
Thus: **condition for frozen-in magnetic field breaks down.**

A second **condition** is that there are two regions of magnetic field pointing in **opposite direction**:



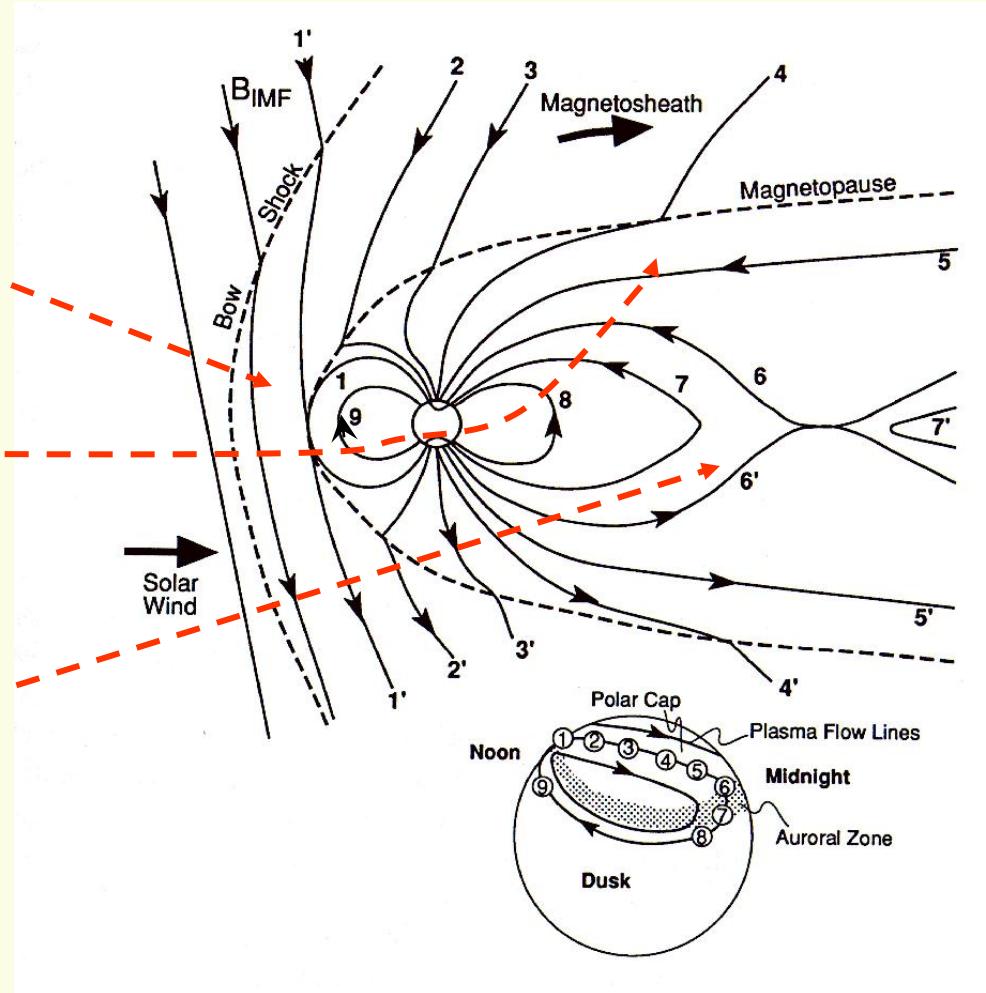
# Reconnection and plasma convection

Solar wind

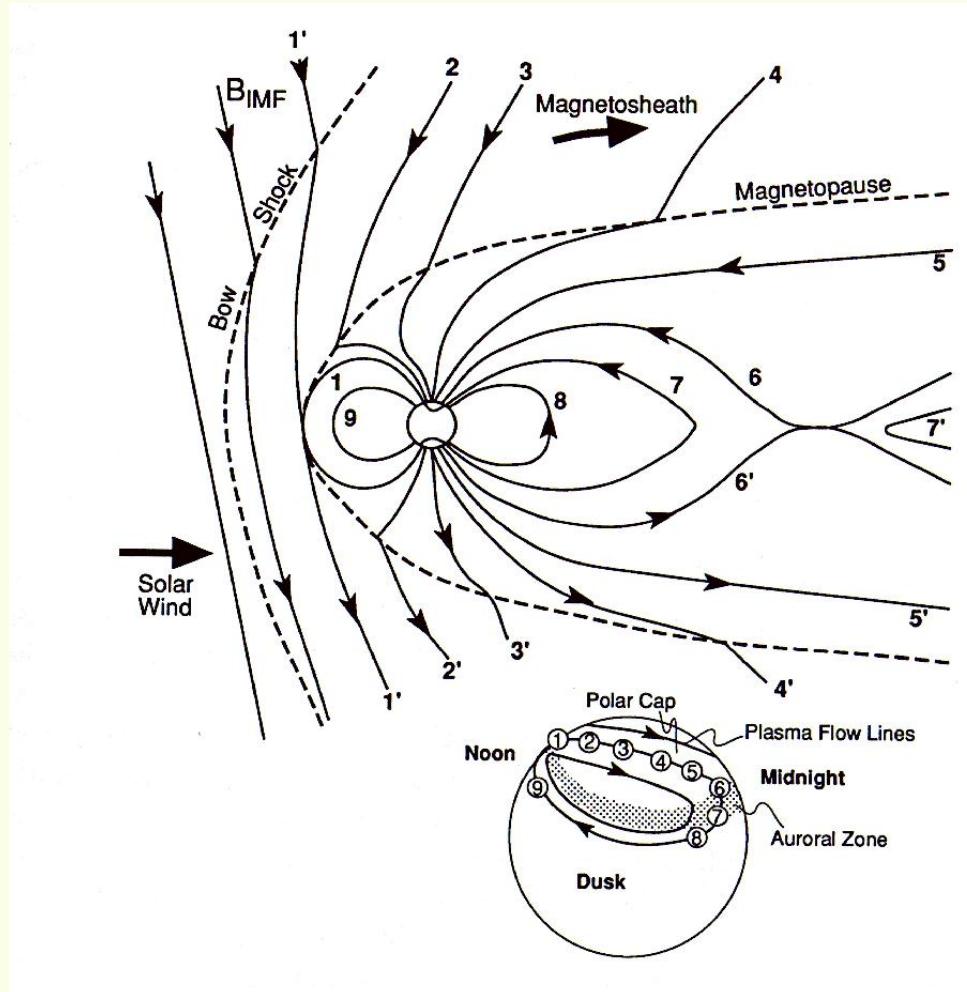


# Reconnection och plasma convection

- Reconnection on the dayside “re-connects” the solar wind magnetic field and the geomagnetic field
- In this way the plasma convection in the outer magnetosphere is driven
- In the night side a second reconnection region drives the convection in the inner magnetosphere.  
The reconnection also heats the plasmashell plasma.

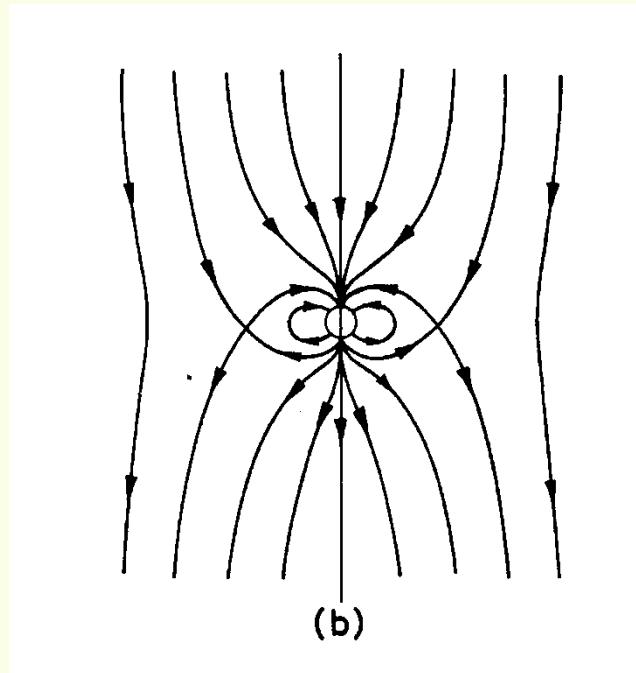


# What happens if IMF is northward instead?

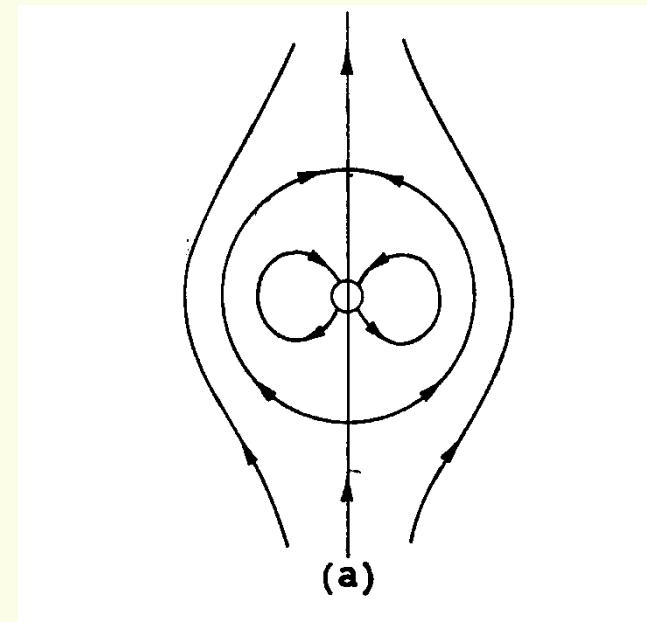


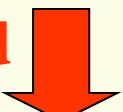
# Magnetospheric dynamics

*open magnetosphere*

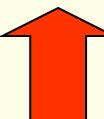


*closed magnetosphere*



southward 

Interplanetary  
magnetic field (IMF)

 northward



# What do the magnetospheres of the other planets look like?