KTH Electrical Engineering

Pattern Recognition — Corrections

Fundamental Theory and Exercise Problems

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Lecture Notes www.kth.se/social/course/EN2202/

Chapter 5

Hidden Markov Models

5.4 Probability of Observed Sequence

Equation (5.26):

$$P\left[\underline{\boldsymbol{X}} = \underline{\boldsymbol{x}} \mid \lambda\right) = \sum_{i_1=1}^{N} q_{i_1} b_{i_1}(\boldsymbol{x}_1\right] \sum_{i_2=1}^{N} a_{i_1 i_2} b_{i_2}(\boldsymbol{x}_2) \cdots \sum_{i_T=1}^{N} a_{i_{T-1} i_T} b_{i_T}(\boldsymbol{x}_T)$$

should be

$$P\left[\underline{\boldsymbol{X}} = \underline{\boldsymbol{x}} \mid \lambda\right] = \sum_{i_1=1}^{N} q_{i_1} b_{i_1}(\boldsymbol{x}_1) \sum_{i_2=1}^{N} a_{i_1 i_2} b_{i_2}(\boldsymbol{x}_2) \cdots \sum_{i_T=1}^{N} a_{i_{T-1} i_T} b_{i_T}(\boldsymbol{x}_T)$$

Equation (5.46):

$$P\left[\boldsymbol{X}_2 = \boldsymbol{x}_2 \mid S_2 = j, \boldsymbol{x}_1, \lambda\right] = P\left[\boldsymbol{X}_2 = \boldsymbol{x}_2 \mid S_2 = j, \lambda\right) = b_j(\boldsymbol{x}_2\right]$$

should be

$$P\left[\boldsymbol{X}_2 = \boldsymbol{x}_2 \mid S_2 = j, \boldsymbol{x}_1, \lambda\right] = P\left[\boldsymbol{X}_2 = \boldsymbol{x}_2 \mid S_2 = j, \lambda\right] = b_j(\boldsymbol{x}_2)$$

5.6 Most Probable State Sequence — Viterbi

Equation (5.78) should be (with corrected parentheses):

$$\chi_{j,1} = P\left[S_1 = j, \boldsymbol{x}_1\right] = q_j b_j(x_1)$$

Chapter 7

Expectation Maximization

7.4 EM Examples with Solutions

Some equations should have an index i_t on the μ an σ parameters. Equation (7.17) should be:

$$\begin{aligned}\ln P[\underline{S} = (i_1 \dots i_T) \cap (x_1 \dots x_T) \mid \theta'] &= \ln \prod_{t=1}^{T} 0.5 g(x_t, \mu'_{i_t}, \sigma'_{i_t}) \\ &= \sum_{t=1}^{T} (\ln 0.5 + \ln g(x_t, \mu'_{i_t}, \sigma'_{i_t}))\end{aligned}$$

Similarly, Eq. (7.20) and following should be:

$$\begin{aligned}&Q(\theta', \theta) = \\ &= \sum_{i_1=1}^{2} \sum_{i_2=1}^{2} \dots \sum_{i_T=1}^{2} P[\underline{S} = (i_1 \dots i_T) \mid \underline{x}, \theta] \sum_{t=1}^{T} (\ln 0.5 + \ln g(x_t, \mu'_{i_t}, \sigma'_{i_t}))\end{aligned} \tag{7.20}$$

$$\begin{aligned}&Q(\theta', \theta) = \\ &= \sum_{t=1}^{T} \sum_{i_1=1}^{2} \sum_{i_2=1}^{2} \dots \sum_{i_T=1}^{2} P[\underline{S} = (i_1 \dots i_T) \mid \underline{x}, \theta] (\ln 0.5 + \ln g(x_t, \mu'_{i_t}, \sigma'_{i_t}))\end{aligned} \tag{7.21}$$

Finally, in Eq. (7.23) we first use the summation index i_t as before, and then change it to just i, when only one state sum remains.

$$
\begin{aligned}
Q(\theta', \theta) = \sum_{t=1}^{T} \sum_{i_t=1}^{2} & \underbrace{P[S_t = i_t \mid x_t, \theta]}_{\gamma_{i_t,t} \text{ from Eq. 7.19}} \left(\ln 0.5 + \ln g(x_t, \mu'_{i_t}, \sigma'_{i_t}) \right) \cdot \\
& \cdot \underbrace{\sum_{i_1=1}^{2} \cdots \sum_{i_{t-1}=1}^{2} \sum_{i_{t+1}=1}^{2} \cdots \sum_{i_T=1}^{2} P[(i_1 \ldots i_{t-1} i_{t+1} \ldots i_T) \mid S_t = i_t, \underline{x}, \theta]}_{=1} \\
= & \sum_{t=1}^{T} \sum_{i=1}^{2} \gamma_{i,t} \left(\ln 0.5 + \ln g(x_t, \mu'_i, \sigma'_i) \right) = /\text{Eq. } 7.15/ \\
= & \sum_{t=1}^{T} \sum_{i=1}^{2} \gamma_{i,t} \left(\ln 0.5 - \frac{1}{2} \ln 2\pi - \ln \sigma'_i - \frac{(x_t - \mu'_i)^2}{2{\sigma'_i}^2} \right) = \\
= & \sum_{t=1}^{T} \gamma_{1,t} \left(\ln 0.5 - \frac{1}{2} \ln 2\pi - \ln \sigma'_1 - \frac{(x_t - \mu'_1)^2}{2{\sigma'_1}^2} \right) + \\
& \qquad + \gamma_{2,t} \left(\ln 0.5 - \frac{1}{2} \ln 2\pi - \ln \sigma'_2 - \frac{(x_t - \mu'_2)^2}{2{\sigma'_2}^2} \right)
\end{aligned}
\tag{7.23}
$$