OPTIMAL FILTERING

LECTURE 8

- The Continuous-Time Kalman Filter
  - Derivation using discrete-time approximations
  - Derivation using the innovations process

Reading instructions: Kailath, Sect. 16.1-16.5

Continuous Time Kalman, Problem Formulation

Assumptions:

$$\dot{x}(t) = F(t)x(t) + G(t)w(t)$$
$$y(t) = H(t)x(t) + v(t)$$

$$E\begin{bmatrix} w(t_1) \\ v(t_1) \end{bmatrix} \begin{bmatrix} w(t_2)^* & v(t_2)^* \end{bmatrix} = \begin{bmatrix} Q(t_1) & S(t_1) \\ S(t_1)^* & R(t_1) \end{bmatrix} \delta(t_1 - t_2)$$

$$Ex_0 = 0, \qquad Ew(t) = 0, \qquad Ev(t) = 0$$

$$Ex_0x_0^* = \Pi_0, \qquad Ex_0v(t)^* = 0, \qquad Ex_0w(t)^* = 0$$

Problem: Determine $\hat{x}(t) = \hat{x}(t|t^-)$, the l.l.s.e. of $x(t)$ given $\{y(s); 0 \leq s < t\}$, $x(0) = x_0$.

Discrete Time Approximation of Stochastic Continuous Time Signals

Note: Not the same scaling as in the book!

Approximation:

$$m(t) \approx \sum_{i=-\infty}^{\infty} m_i p(t-iT), \text{ with } p(t) = \begin{cases} 1 & 0 \le t < T \\ 0 & \text{otherwise} \end{cases}$$

where $m_i = \frac{1}{T}\int_{iT}^{(i+1)T} m(t)dt$

If $m(t)$ **is continuous:** $m_i \approx m(iT)$ with autocorrelation function $R_{m_i}(k) \approx \lfloor R_m(\tau) \rfloor_{\tau=kT}$.

White noise: Note, not continuous!

If $E[n(t_1)n^*(t_2)] = R_n(t_1)\delta(t_1-t_2) \implies$

n_i white noise with $E[n_i n_k^*] \approx \frac{1}{T} R_n(iT)\delta_{i-k}$

Resulting Kalman Filter

$$\dot{\hat{x}}(t) = F(t)\hat{x}(t) + K(t)\big(y(t) - H(t)\hat{x}(t)\big)$$

$$K(t) = \big(P(t)H^*(t) + G(t)S(t)\big)R^{-1}(t)$$

$$\dot{P}(t) = F(t)P(t) + P(t)F^*(t) + G(t)Q(t)G^*(t) - K(t)R(t)K^*(t)$$

$$P(0) = \Pi_0$$

Innovations

$$e(t) = y(t) - H(t)\hat{x}(t) = y(t) - \hat{y}(t|t^-)$$

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$e(t)$ is the continuous-time limit of the discrete-time innovations e_k.

Interpretation: The additional information at time t after all information $\hat{y}(t|t^-)$ in past observations has been removed.

Can be used for alternative derivation of the Kalman equations, see the board.