# Classification of Systems

**In general:** $y(n) = H(u(n))$

**Properties:** time invariant — time variable

static — dynamic

linear — non-linear

causal — non-causal

stable — unstable

**Common assumption:** Linear Time Invariant (LTI)

$$y(n) = \sum_{k=-\infty}^{\infty} h(n-k)u(k) = \sum_{k=-\infty}^{\infty} u(n-k)h(k) = h(n) * u(n)$$

$h(n)$ — the impulse response of the system



# Classification of Signals

| | Time-continuous | — | Time-discrete |
|---|---|---|---|
| **Two important classes:** | $\mathcal{L}_1$–signals | | $l_1$–signals |
| | $\mathcal{L}_2$–signals | | $l_2$–signals |

**Common Properties:**

periodic — aperiodic

symmetric — antisymmetric — neither

stochastic — deterministic

## STOCHASTIC SIGNALS

Typical assumption: **(wide sense) stationarity**

**Mean value:** $\mathrm{E}[x(n)] = m_x$, constant, independent on $n$.

**Covariance sequence:** $\mathrm{E}[x(n)x^*(n-k)] = r_{xx}(k)$ independent of $n$.

**Property:** $r_{xx}(-k) = r^*_{xx}(k)$

**Average Power:** $P_x = \mathrm{E}[|x(n)|^2] = r_{xx}(0) \geq 0$



## TWO-SIDED Z-TRANSFORM

$$X(z) = Z\{x(k)\} = \sum_{k=-\infty}^{\infty} x(k)z^{-k}$$

Exists only where the series converges.

This region is called the *Region of Convergence* (ROC).

The signal is uniquely determined by the Z-transform **together with** the ROC.

## Z-TRANSFORMS, PROPERTIES

**Linearity**

$$Z\{ax_1(n) + bx_2(n)\} = aZ\{x_1(n)\} + bZ\{x_2(n)\}$$

**Time shift**

$$Z\{x(n-k)\} = z^{-k}X(z)$$

**Convolution**

$$Z\{x_1(n) * x_2(n)\} = Z\{x_1(n)\}Z\{x_2(n)\}$$

**Scaling**

$$Z\{a^n x(n)\} = X(z/a)$$



## Z-TRANSFORMS, MORE PROPERTIES

**Time reversal**

$$Z\{x(-n)\} = X(z^{-1})$$

**Conjugation** (Complex signals)

$$Z\{x^*(n)\} = X^*(z^*)$$

**Derivation**

$$\frac{dX(z)}{dz} = -z^{-1}Z\{nx(n)\}$$

# Rational Z-transform

Many systems can be described by a difference equation

$$y(n) = -\sum_{k=1}^{N_a} a_k y(n-k) + \sum_{k=0}^{N_b} b_k x(n-k)$$

Corresponds to a rational Z-transform as transfer function ($a_0 = 1$)

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{N_b} b_k z^{-k}}{\sum_{k=0}^{N_a} a_k z^{-k}}$$

Zeros: $z$ such that $H(z) = 0$

Poles: $z$ such that $H(z) = \infty$