Aperiodic Time-continuous Signal

Fourier transform $X(F) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi Ft}dt$

(often $\Omega = 2\pi F$)

Inverse Fourier transform $x(t) = \int_{-\infty}^{\infty} X(F)e^{j2\pi Ft}dF$

Parseval's Relation Total energy $= \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(F)|^2 dF$

Energy spectrum (energy spectral density): $|X(F)|^2$

Connection to the Laplace-transform $X(F) = X(s)$ where $s = j2\pi F = j\Omega$

Periodic Time-continuous Signal

Fundamental period, T_p $x(t) = x(t + T_p)$, $F_0 = 1/T_p$.

Fourier series $x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi kt}{T_p}} = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi ktF_0}$

Fourier coefficients $c_k = \frac{1}{T_p}\int_{t_0}^{t_0+T_p} x(t)e^{-j2\pi ktF_0}dt$

Parseval's Relation Total power $= \frac{1}{T_p}\int_{t_0}^{t_0+T_p} |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |c_k|^2$

Power spectrum (power spectral density): $|c_k|^2$

Frequency Analysis

Aperiodic Time-discrete Signal

Discrete Time Fourier Transform, DTFT $X(f) = \sum_{n=-\infty}^{\infty} x(n)e^{-j2\pi fn}$

(often $\omega = 2\pi f$)

Inverse Discrete Time Fourier Transform, IDTFT $x(n) = \int_0^1 X(f)e^{j2\pi fn}df$

Parseval's Relation Total energy $= \sum_{n=-\infty}^{\infty} |x(n)|^2 = \int_0^1 |X(f)|^2 df$

Energy spectrum (energy spectral density): $|X(f)|^2$

Connection to the Z-transform $X(f) = X(z)$ where $z = e^{j2\pi f} = e^{j\omega}$.

Frequency Analysis

Periodic Time-discrete Signal

Fundamental period: $N, \quad x(n) = x(n+N)$

Discrete Fourier Transform, DFT $X(k) = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi nk}{N}}$

Inverse Discrete Fourier Transform $x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi nk}{N}}$

Parseval's Relation Total Energy $= \sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |X(k)|^2$

Energy spectrum (energy spectral density): $\frac{1}{N}|X(k)|^2$

Note: Scaling can differ from book to book!

IMPORTANT RELATIONSHIPS

Time-discrete signal $\Longleftrightarrow$ Periodic Spectrum

Periodic signal $\Longleftrightarrow$ "Frequency-discrete" Spectrum

DIFFERENT NOTATIONS, FOURIER TRANSFORMS

Continuous Time Fourier

Lectures: $X(F)$

Diniz, da Silva, Netto: $X(j\Omega)$

Other books: $X(\Omega)$

Frequency: F [Hz]

Angular frequency: Ω [rad/s]

Discrete Time Fourier (DTFT)

Lectures: $X(f)$

Diniz, da Silva, Netto: $X(e^{j\omega})$

Other books: $X(\omega)$

Normalized frequency: f [per sample]

Normalized angular frequency:

ω [rad/sample]

For sampled signal:

$$f = \frac{F}{F_s}, \quad \omega = \frac{\Omega}{F_s}$$