Group Delay and Linear Phase

Group Delay $\tau(f) = -\frac{1}{2\pi}\frac{\partial}{\partial f}\angle\{H(f)\}$ "=" $-\frac{\partial}{\partial \omega}\angle\{H(\omega)\}$
"time delay" for frequency components with frequency f.

Linear phase $\tau(f) = \tau_0$ constant $\Longleftrightarrow$

$$H(f) = B(f)e^{-j(2\pi f \tau_0 + \phi)}$$

where $B(f)$ is real valued $\Longrightarrow$ all frequencies delayed equally much

FIR Linear Phase Filters

Assume FIR: $h(n) = \{h(0), h(1), \ldots, h(M)\}$

General form for linear phase: $h(n) = (-1)^k h(M - n)$

Delay: $\tau_0 = M/2$

Phase: $\phi = k\pi/2$

Four options:

Type 1: $k = 0$, M even

Type 2: $k = 0$, M odd

Type 3: $k = 1$, M even

Type 4: $k = 1$, M odd

FIR APPROXIMATIONS OF DESIRED FILTERS

Alternative methods:

Frequency sampling: Match values, $H_{\text{approx}}(f) = H_{\text{desired}}(f)$ at $f = k/M$, take IDFT

Truncate impulse response: $h_{\text{approx}}(n) = \begin{cases} h_{\text{desired}}(n) & |n| \leq M/2 \\ 0 & \text{otherwise} \end{cases}$

delay to make it causal: $h_{\text{approx}}(n) = \begin{cases} h_{\text{desired}}(n - M/2) & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$

Problems: Both methods give large ripple in the frequency response.

FIR APPROXIMATIONS USING WINDOWING

$$h_{\text{approx}}(n) = h_{\text{desired}}(n)w(n)$$

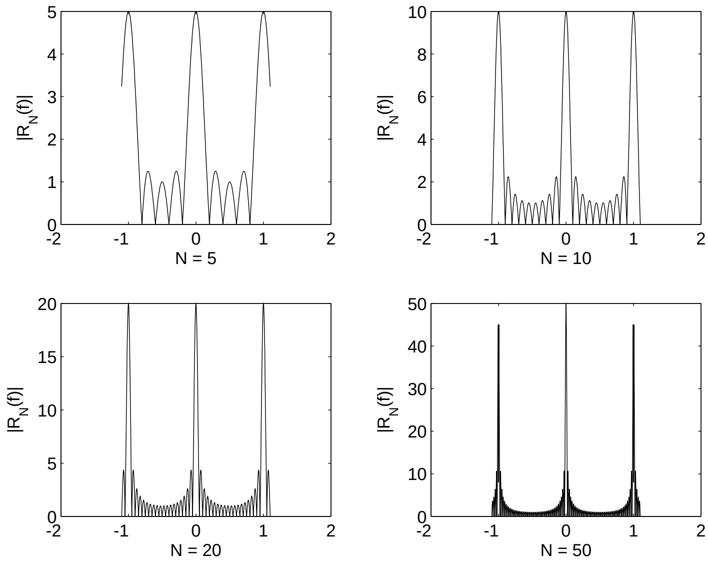
$$\Longleftrightarrow$$

$$H_{\text{approx}}(f) = \int_{-1/2}^{1/2} H_{\text{desired}}(\nu)W(f - \nu)\, d\nu$$

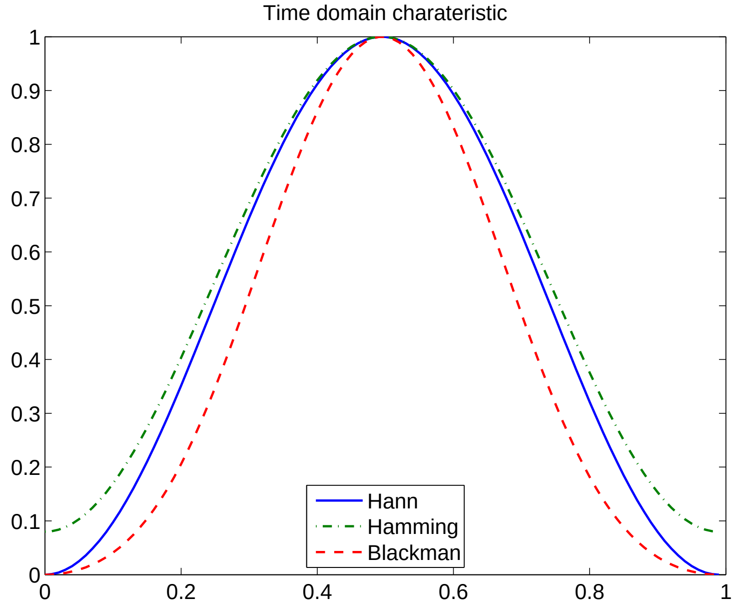
- Truncation corresponds to rectangular window
- Smoother window gives less ripple in frequency domain
- Popular choices: triangular (Bartlett), Hamming, Hann, Blackman, ...

Rectangular Window

$$r_N(n) = 1, n = 0, \ldots, N-1 \Longleftrightarrow R_N(f) = e^{-j\pi f(N-1)} \frac{\sin(N\pi f)}{\sin(\pi f)}$$



Different windows, time domain



DIFFERENT WINDOWS, FREQUENCY DOMAIN