Chapter 4 – Queuing Systems

Ioannis Glaropoulos

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1 Exercise 4.1

Packets arrive to a communication node with a single output link according to a Poisson Process. Give the Kendall notation for the following cases:

1. *the packet lengths are exponentially distributed, the buffer capacity at the node is infinite*
2. *the packet length is fixed, the buffer can store n packets*
3. *the packet length is L with probability p_L amd l with probability p_l and there is no buffer in the node*

Solution: Kendall Notation

1. Arrival Process
2. Service Time
3. Number of Servers
4. Number of Total Positions (servers and queues)
5. Population

The Poisson arrivals (M) and the Single server (1) are fixed: M/?/1/?/?

1. M/M/1, as the buffer is infinite
2. M/D/1/n+1, as the service is deterministic and the buffer is n
3. M/G/1/1, as the service is general and there is no buffer

2 Exercise 4.2

Give the Kendall notation for the following systems. Telephone calls arrive to a PBX with C output links. The calls arrive as Poisson process and the call holding times are exponentially distributed.

1. *Calls arriving when all the output links are busy are blocked*
2. *Up to c calls can wait when all the output links are blocked*

Solution:

1. M/M/C/C
2. M/M/C/C+c

3 Exercise 4.3

Why is it not a good idea to have a G/G/10/12/5 System? Solution: 10 servers for 5 users!

4 Exercise 4.4

Which system provides the best performance, an M/M/3/300/100 or an M/M/3/100/100?

Solution: They have the same performance, since the users fit to both queues. Of course, the first system wastes buffer positions!

5 Exercise 4.5

A PBX was installed to handle the voice traffic generated by 300 employees in an office. Each employee on average makes 2 calls per hour with an average call duration of 4.5 minutes The PBX has 90 outgoing links.

1. *What is the offered load to the PBX?*
2. *What is the utilization of the outgoing links? Assume that calls arriving when all the links are busy are queued up.*

Solution: Offered Load $\rho \to \lambda \overline{T} = 300 \cdot \frac{2}{60} \cdot 4.5 = 45$ Erlang. Generally, the actual load is not the offered load.

Link Utilization:

$$\frac{\text{actual load}}{\#\text{ servers}}$$

The existence of an infinite queue here means that no load is dropped, or that the offered load is the actual load. So,

$$\text{utilization} = \frac{\text{offered load}}{90} = \frac{45}{90} = 0.5$$

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