EG2080 Monte Carlo Methods
in Engineering

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Theme 7

SIMULATION DESIGN

PROJECT ASSIGNMENT - Problems

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- Formulate a simulation problem.
  Notice that it is more important that you have an interesting simulation problem than that your problem is realistic.
- Analyse possible simulation methods.
- Test suggested simulation methods.
- Write report.

PLANNING A MONTE CARLO SIMULATION

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- Overview of the simulation problem
- Objective
- Define symbols
- Formulate model
- Choose simulation method
- Testing and verification

OVERVIEW OF THE SIMULATION PROBLEM

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- Try to write a short description of the system to be studied.
- What is the general background of the problem, i.e., why is it interesting to study this problem?
- Describe related work, i.e., if the problem has been studied by other authors.
- Summarise how the system is modelled and provide references to the theory necessary to understand the model.

OBJECTIVE

- Which outputs are of interest?
- How detailed results are required?
  - Expectation value
  - Variance
  - Probability distribution
- Which accuracy is necessary?

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DEFINE SYMBOLS

- Try to list all the variables and parameters that you are going to need in to describe your system.
- Use symbols that are as clear as possible.
- Try to follow the convention that random variables are denoted by capital letters and use lower-case letters for parameters.
- Sort the symbols under different headings as for example Inputs, Model parameters and Outputs.

  Cf. example 1.

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FORMULATE MODEL

- List and motivate all assumptions that you have made, for example if you have neglected some properties of your system.
- Describe the equations used to calculate the output values given a specific set of input values.

  It is not necessary to include detailed descriptions of well-known algorithms; you can refer to relevant literature or include the background theory in an appendix.

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FORMULATE MODEL

- Test the model thoroughly, i.e., make sure that you get correct output values for some typical scenarios.
- Pay specific attention to possible exceptions (such as for example division by zero)!

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SIMULATION METHOD

The choice of simulation method is not only depending on the characteristics of the system you are simulating, but also on the reason why you are studying it.

- A specific simulation problem is a simulation problem that only has to be solved once.
- A general simulation problem refers to a class of systems with the same basic structure, but which can vary in size, parameter values, etc., and which therefore will be solved repeatedly.

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SIMULATION METHOD
- Specific simulation problem

- Preparing a smart simulation method may take longer time than the reduction in simulation time compared to simple sampling.
- Hence, it is better to use a straight-forward, reliable simulation method (such as simple sampling or correlated sampling) and to collect a very large number of samples.
- Complementary random numbers and dagger sampling can be useful.

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SIMULATION METHOD
- General simulation problem

- Since the same simulation method can be applied more than once, it is probably worth-while to spend some time on developing an efficient simulation method.
- Investigate if there are any predictable corre-lations between the input values and the output values.
- Try to formulate a simplified model of the system; the simplified model can then be used as a control variate.

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SIMULATION METHOD

General

- Which stopping rule should be used?
- Which batch size is reasonable?
- Which variance reduction techniques are applicable?
- Is it possible to combine different variance reduction techniques?

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SIMULATION METHOD

Complementary random numbers

- Have you identified any inputs which if negatively correlated will result in negative correlations for one or more outputs?
- If no such inputs exist in the original problem, can you add extra variables, which is a direct function of the original inputs, but which is also are correlated somehow to the output values?

  Cf. examples 17 and 18.

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SIMULATION METHOD

Dagger sampling

- Dagger sampling is an obvious possibility whenever you have two-state input variables.
- However, consider alternative methods, such as combining several two-state variables in a multi-variate discrete distribution, or to apply importance sampling.
- Which dagger cycle length is optimal?

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SIMULATION METHOD

Control variate

- If a control variate is available—use it!
- However, the control variate method is changing the population which we are sampling; other scenarios are now more important compared to if only the original system is sampled $\Rightarrow$ Modify importance sampling function or stratification?

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SIMULATION METHOD

Correlated sampling

- If you need to compare several systems—use correlated sampling!
- However, if the inputs have different probability distributions, it might be impossible to use correlated sampling in combination with importance sampling or stratified sampling.
- If the systems have different values of critical parameters, it might be necessary to consider both systems when designing importance sampling functions and stratification.

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SIMULATION METHOD

Importance sampling

- Investigate if there is a control variate which can be used to design an importance sampling function.
- Consider the possibility to focus the simulation of scenarios which have low probability but large impact on $E[X]$.

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SIMULATION METHOD

Stratified sampling

- Investigate if there is a control variate which can be used to design strata.
- Investigate if the scenarios with similar output values can be identified using a strata tree.
- Which sample allocation method is optimal?

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TESTING AND VERIFICATION

- Test your simulation methods on a small-scale problem such that it is possible to verify that the simulation provides correct results.
  - It is convenient if it possible to calculate $E[g(Y)]$ using analytical calculations.
  - Try to keep the main features of the system.
- Test your simulation methods on large-scale problems.
  - Are the results reasonable?
  - Estimate the variance of the expectation value for different methods.

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PROJECT ASSIGNMENT - Report

Sections marked * are optional.

- Introduction
  - Background
  - Problem definition
  - *Related research
  - Overview of the report
- Modelling
  - Assumptions and limitations
  - Symbols
  - Mathematical model

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PROJECT ASSIGNMENT - Report

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- Variance reduction techniques
  Analyse the possibility to use the variance reduction techniques described in this course.
- Case study
  - System description
  - Simulation method
  - Results

PROJECT ASSIGNMENT - Report

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- Conclusions
  - General conclusions
  - *Conclusions from the case study
  - *Future work
- References