EG2080 Monte Carlo Methods in Engineering

EXERCISES

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MOTIVATION

It has been claimed that people learn...

- 10% of what they read
- 20% of what they hear
- 30% of what they see
- 50% of what they see and hear
- 70% of what they talk over with others
- 80% of what they use and do in real life
- 95% of what they teach someone else

⇒ Hence, you are welcome to discuss these exercises together with other students, and explain your solutions to each other!

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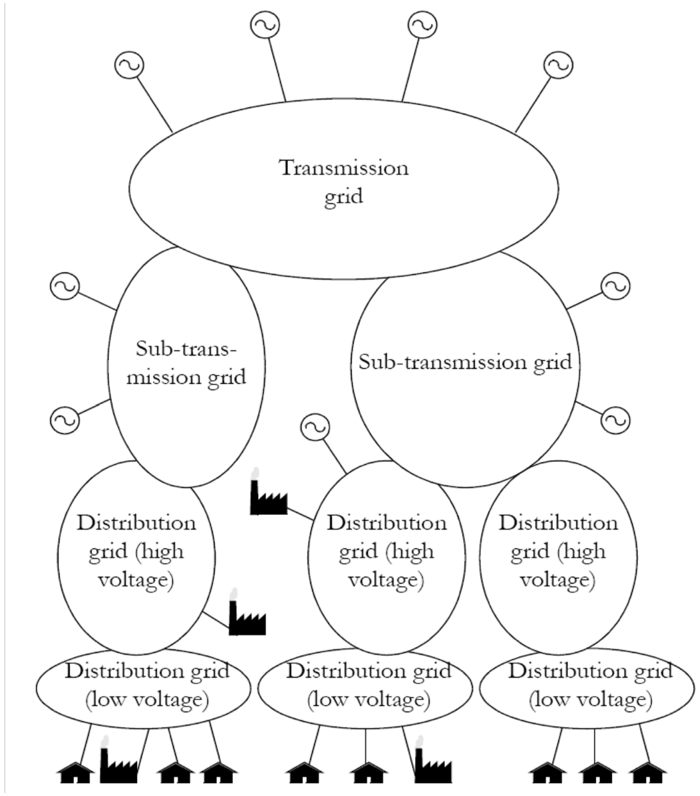
PROBLEM 1 - Monte Carlo model of a power system

- Assume that there is a power system with a certain load and a certain generation, which covers the load. In other words, every-thing in the garden is lovely.
- Then a disturbance occurs somewhere in the grid.



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PROBLEM 1 - Monte Carlo model of a power system

- If there at the time of the disturbance is an alert operator in the control room and this operator immediately comes to decision about the appropriate countermeasures, the power system can cope with the disturbance.
- If the operator is on a coffee break or for some reason does not manage to take the right actions in time, then extensive disturbances will come up.

PROBLEM 1 - Questions

Classify the following items as either inputs, outputs or model properties:

a) Action taken by the operator in the control room.

b) Available generation capacity in different power plants.

c) Electricity demand in different points of the system.

d) Impedence of the different lines in the grid.

e) System state (normal operation/extensive disturbances).

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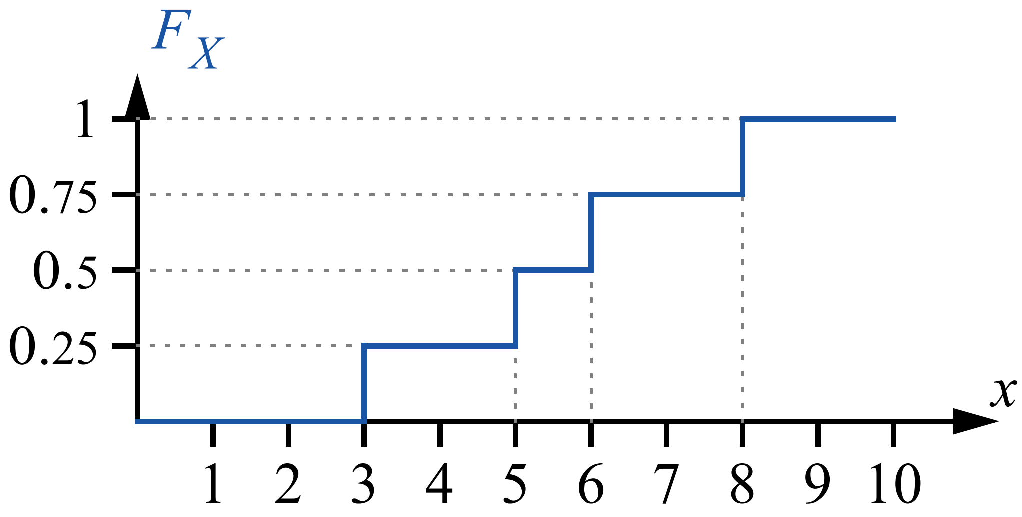
PROBLEM 2 - Probability distribution 1

Consider the probability distribution of X as shown below. Are the following statements true or false?

a) X is a discrete random variable.

b) $E[X] = 6$.

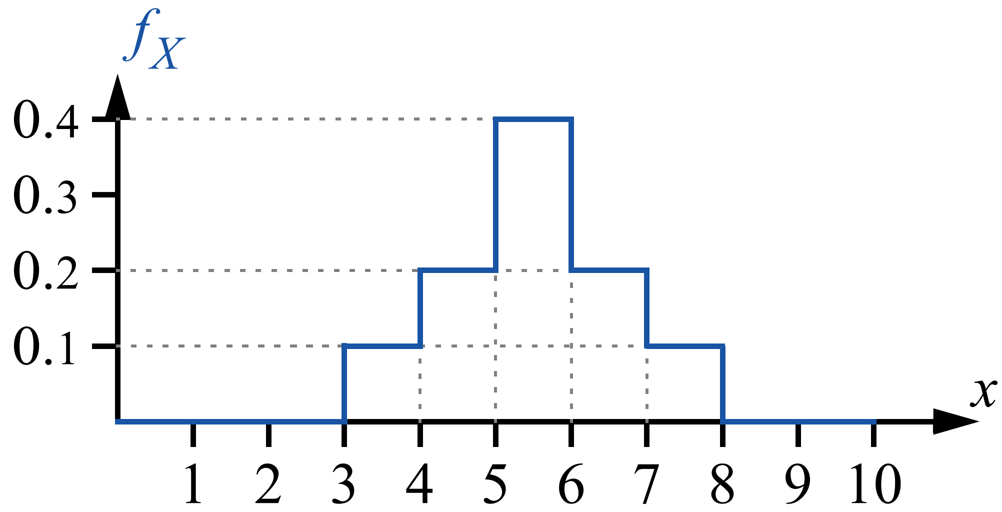


PROBLEM 3 - Probability distribution 2

Consider the probability distribution of X as shown below. Are the following statements true or false?

a) X is a discrete random variable.

b) $P(X < 4) = 0.1$.



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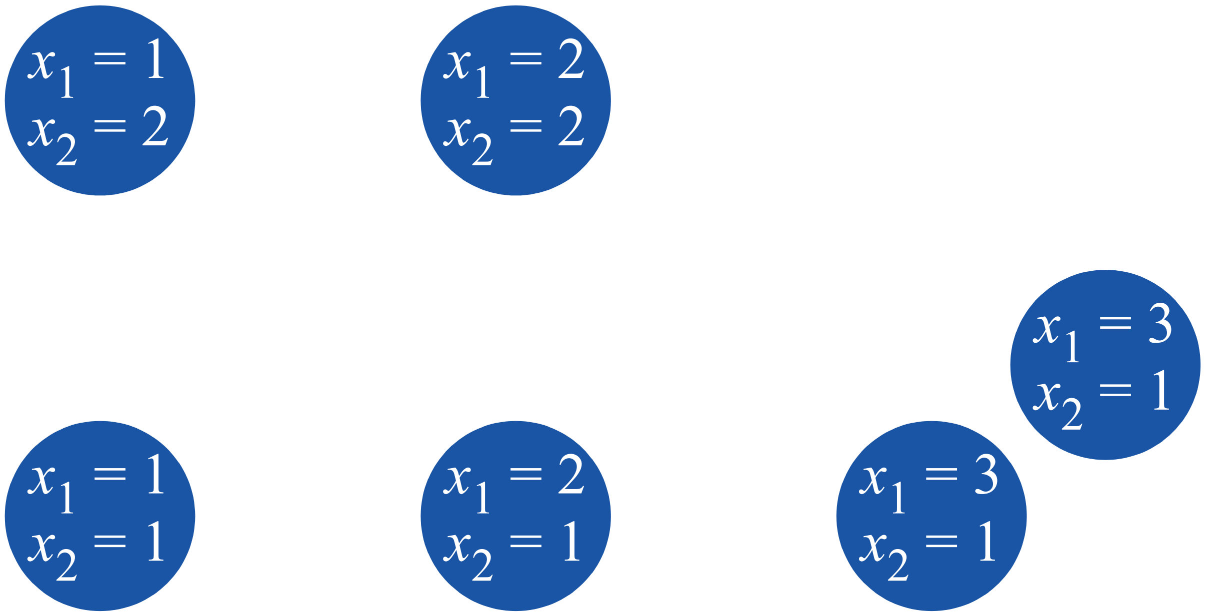
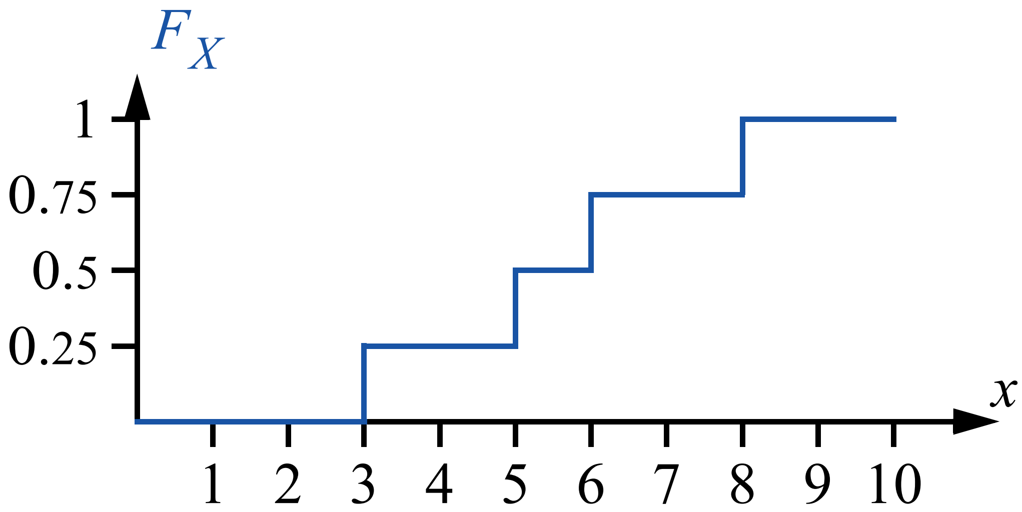
PROBLEM 4 - Correlated or not?

Consider the multivariate probability distribution represented by the population shown below. Are the random variables X_1 and X_2 positively correlated, uncorrelated or negatively correlated?



PROBLEM 5 - Inverse Transform Method

Consider the probability distribution of X as shown below. Generate a random value of X based on the value 0.75 from a $U(0, 1)$-distribution.



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PROBLEM 6 - Another sample

$1\,000$ samples have been collected of the random variable X (representing the outcome of rolling a dice), and the results are

$$\sum_{i=1}^{1\,000} x_i = 3\,480, \quad \sum_{i=1}^{1\,000} x_i^2 = 15\,028.$$

Let $m_{X1\,000}$ and $s^2_{X1\,000}$ be the estimates of $E[X]$ and $Var[X]$ respectively.

Assume that another batch of $1\,000$ samples is generated, providing the estimates $m_{X2\,000}$ and $s^2_{X2\,000}$.

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PROBLEM 6 - Questions

Are the following statements true or false?

a) $m_{X2\ 000} > m_{X1\ 000}$

b) $s^2_{X1\ 000} > s^2_{X2\ 000}$

c) $E[S^2_{X2\ 000}] < s^2_{X1\ 000}$

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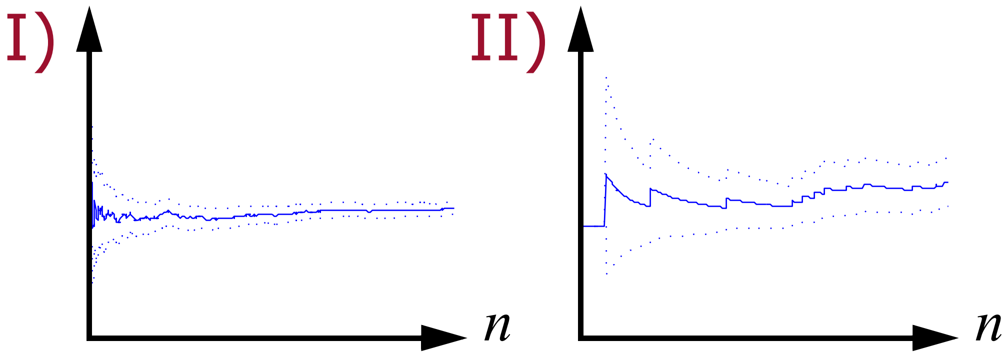
PROBLEM 7 - Convergence of simple sampling



The figures above show the estimated expectation value (solid line) and the confidence interval of the estimate (dashed lines) as a function of the number of collected samples.

One of the variables is duogeneous and the other is heterogeneous. Which one is which?

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PROBLEM 8 - Duogeneous populations

State if the following random variables are duogeneous or heterogeneuos.

a) The available generation capacity in a nuclear power plant.

b) The available generation capacity in a wind power plant.

c) The result of rolling a dice, where one side is labelled “1” and the other sides are labelled “2”.

d) The result of tossing a coin.

e) The temperature in Stockholm at noon.

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PROBLEM 9 - Complementary random number

Consider the probability distribution

$$f_Y(x) = \begin{cases} 0.23 & x = 0, \\ 0.77 & x = 100, \\ 0 & \text{all other } x. \end{cases}$$

a) Use the pseudorandom number 0.34 to generate a value of Y.

b) Calculate the corresponding complementary random number.

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PROBLEM 10 - Dagger sampling

Consider the probability distribution

$$f_Y(x) = \begin{cases} 0.23 & x = 0, \\ 0.77 & x = 100, \\ 0 & \text{all other } x. \end{cases}$$

a) How many random values of Y can be generated from the pseudorandom number 0.96?

b) Which of these random values is equal to 0?

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PROBLEM 11 - Randomisation method for a power system model

Which randomisation method (simple sampling, complementary random numbers or dagger sampling) do you think is most appropriate for the following inputs to a power system model? The objective of the simulation is to investigate the total operation cost.

a) The available generation capacity in different power plants.

b) The available transmission capacity in different high voltage lines.

c) The total load in the system.

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PROBLEM 12 - Control variate

Which simulation method, simple sampling or using a control variate, is most efficient in the following cases?

a) $E[X] = 100$, $E[Z] = 90$, $Var[X] = 9$, $Var[Z] = 16$, $Cov[X, Z] = -9$.

b) $E[X] = 100$, $E[Z] = 90$, $Var[X] = 9$, $Var[Z] = 16$, $Cov[X, Z] = 0.9$.

c) $E[X] = 100$, $E[Z] = 90$, $Var[X] = 9$, $Var[Z] = 16$, $Cov[X, Z] = 9$.

PROBLEM 13 - Correlated sampling

Which simulation method, simple sampling or correlated sampling, is most efficient in the following cases?

a) $E[M_{X1}] = 100$, $E[M_{X2}] = 90$, $Var[M_{X1}] = 9$, $Var[M_{X2}] = 16$, $Cov[M_{X1}, M_{X2}] = -9$.

b) $E[M_{X1}] = 100$, $E[M_{X2}] = 90$, $Var[M_{X1}] = 9$, $Var[M_{X2}] = 16$, $Cov[M_{X1}, M_{X2}] = 0.9$.

c) $E[M_{X1}] = 100$, $E[M_{X2}] = 90$, $Var[M_{X1}] = 9$, $Var[M_{X2}] = 16$, $Cov[M_{X1}, M_{X2}] = 9$.

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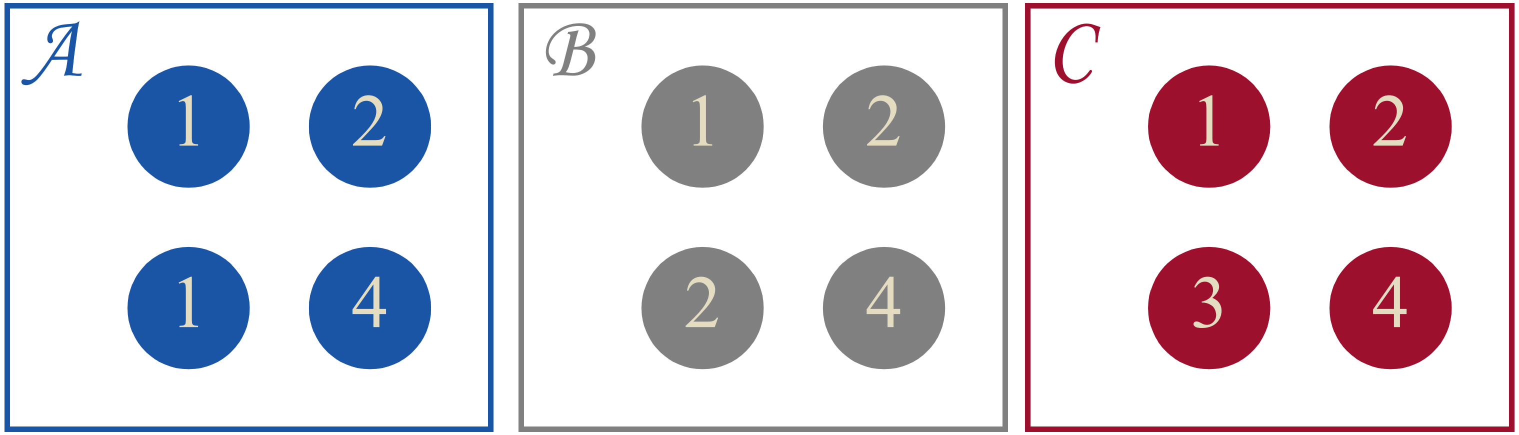
PROBLEM 14 - Importance sampling function

Consider the three random variables A, B and C below.



a) Is $f_B(x)$ an optimal importance function when sampling A?

b) Is $f_C(x)$ an optimal importance function when sampling A?

PROBLEM 15 - Neyman allocation

The pilot study of a Monte Carlo simulation included 12 samples (four per stratum) and the following results were obtained:

Stratum, h	1	2	3
Stratum weight, ω_h	0.25	0.25	0.5
Estimated standard deviation, s_h	0	0.200	0.025

Assume that the next batch will comprise of another 4 samples.

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PROBLEM 15 - Neyman allocation

a) How many samples should be allocated to stratum 1?

b) How many samples should be allocated to stratum 2?

c) How many samples should be allocated to stratum 3?

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PROBLEM 16 - Strata tree

Consider the input population to the system

$$g(Y) = \begin{cases} 0 & \text{if } Y_1 \geq Y_2, \\ 1 & \text{if } Y_1 < Y_2, \end{cases}$$

as shown below.

$x_1 = 1$, $x_2 = 2$	$x_1 = 2$, $x_2 = 2$	$x_1 = 2$, $x_2 = 3$	$x_1 = 3$, $x_2 = 3$
$x_1 = 1$, $x_2 = 1$	$x_1 = 2$, $x_2 = 1$	$x_1 = 2$, $x_2 = 2$	$x_1 = 3$, $x_2 = 2$

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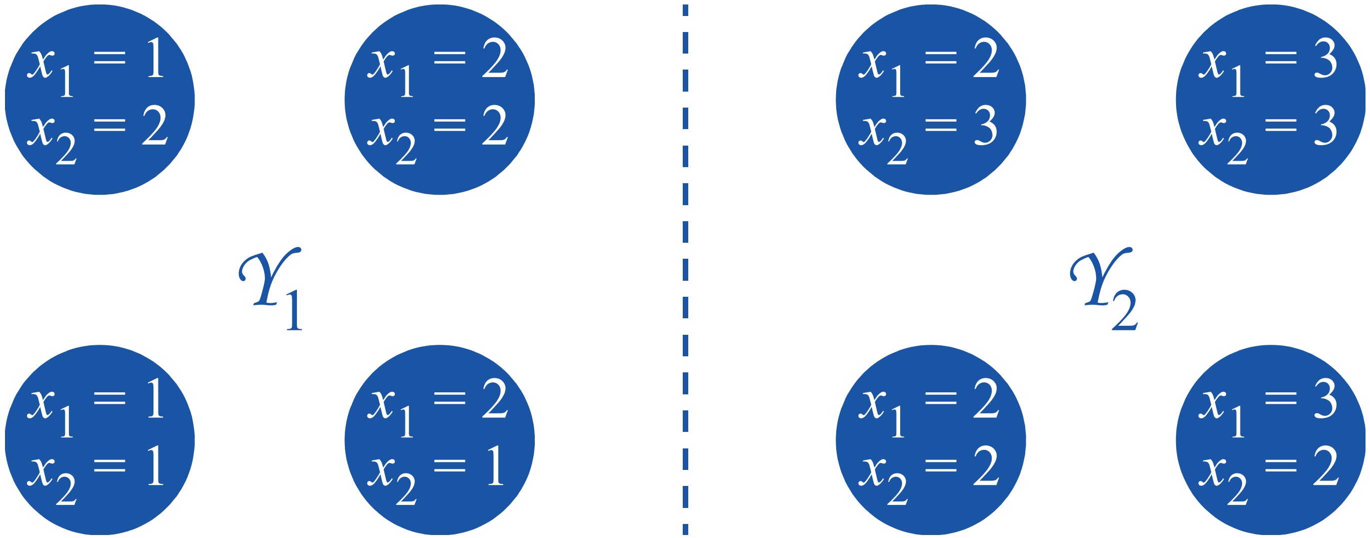

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PROBLEM 16 - Strata tree

Is the strata tree below suitable for simulation of this system?