MULTIRATE SIGNAL PROCESSING

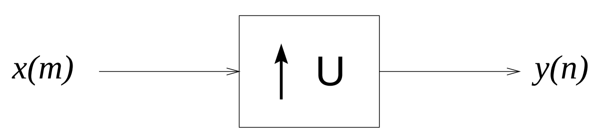
Applications

- Systems with multiple sampling rates
- Dividing a signal into frequency bands
- Filter design
- Time-frequency analysis
- Used in many audio and video compression schemes

Basic operations

- Downsampling/decimation
- Upsampling/interpolation

UPSAMPLING BY A FACTOR U



$$y(n) = \{\ldots, \underset{\uparrow}{x(0)}, \underbrace{0, \ldots, 0}_{U-1 \text{ zeros}}, x(1), \underbrace{0, \ldots, 0}_{U-1 \text{ zeros}}, x(2), 0, \ldots\}$$

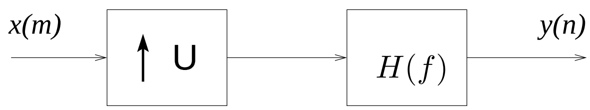
$$= \begin{cases} x(n/U), & n = 0, \pm U, \pm 2U, \ldots \\ 0, & \text{otherwise} \end{cases}$$

$$Y(f) = X(fU)$$

$$Y(z) = X(z^U)$$

Proper Interpolation

Preserve the spectral shape!

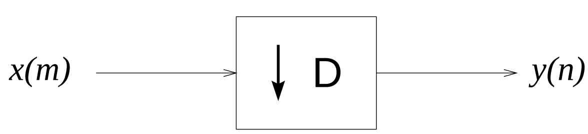


$$H(f) = \begin{cases} U, & |f| \le \frac{1}{2U} \\ 0, & \frac{1}{2U} < |f| \le \frac{1}{2} \end{cases}$$

$$\Longrightarrow$$

$$Y(f) = \begin{cases} UX(fU), & |f| \le \frac{1}{2U} \\ 0, & \frac{1}{2U} < |f| \le \frac{1}{2} \end{cases}$$

Downsampling by a Factor D



$$y(n) = \{\ldots, \underset{\uparrow}{x(0)}, x(D), x(2D), x(3D), \ldots\} = x(nD) \quad n = 0, \pm1, \pm2, \ldots$$

$$Y(f) = \frac{1}{D} \sum_{k=0}^{D-1} X\left(\frac{f-k}{D}\right)$$

$$Y(z) = \frac{1}{D} \sum_{k=0}^{D-1} X\left(z^{1/D} e^{-j2\pi k/D}\right)$$