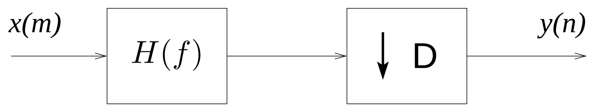
PROPER DECIMATION

Avoid aliasing!



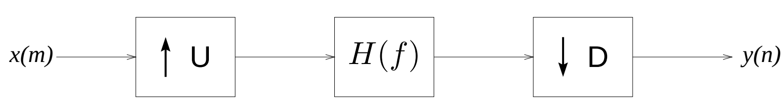
$$H(f) = \begin{cases} 1, & |f| \le \frac{1}{2D} \\ 0, & \frac{1}{2D} < |f| \le \frac{1}{2} \end{cases}$$

$$\Longrightarrow$$

$$Y(f) = \frac{1}{D} X\left(\frac{f}{D}\right) \quad |f| \le \frac{1}{2}$$

RATE CONVERSION BY A FACTOR U/D

U and D relatively prime integers



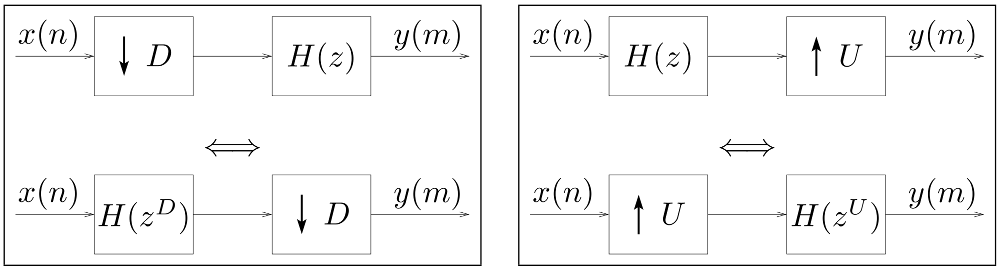
$$H(f) = \begin{cases} U, & |f| \le \min\{\frac{1}{2U}, \frac{1}{2D}\} \\ 0, & \min\{\frac{1}{2U}, \frac{1}{2D}\} < |f| \le \frac{1}{2} \end{cases}$$

$$\Longrightarrow$$

$$Y(f) = \begin{cases} \frac{U}{D} X(f\frac{U}{D}), & |f| \le \min\{\frac{1}{2}, \frac{D}{2U}\} \\ 0, & \text{otherwise} \end{cases}$$

THE *Noble* IDENTITIES

Useful results for multirate systems.



POLYPHASE REPRESENTATION

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n} = \sum_{k=0}^{M-1} z^{-k} P_k(z^M)$$

where the polyphase components are

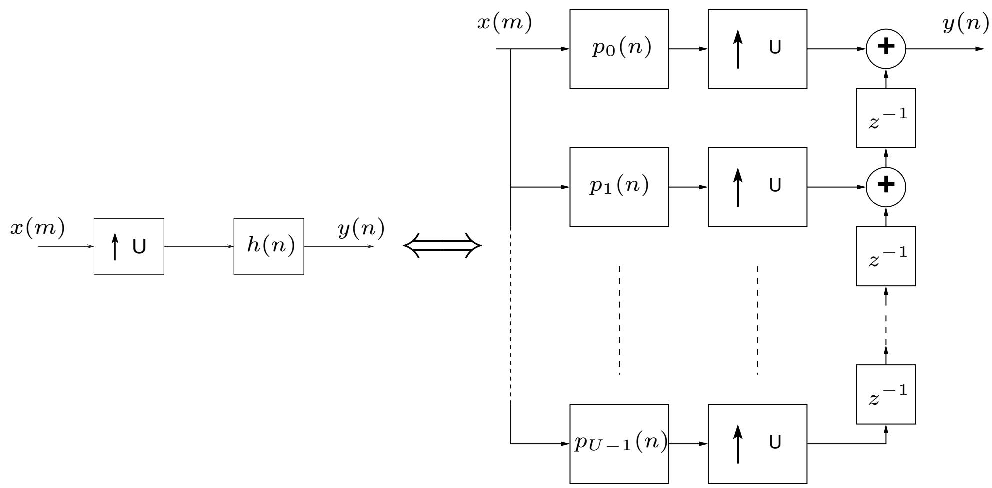
$$P_k(z) = \sum_{n=-\infty}^{\infty} p_k(n) z^{-n}$$

$$p_k(n) = h(nM + k)$$

For interpolation, $M = U$

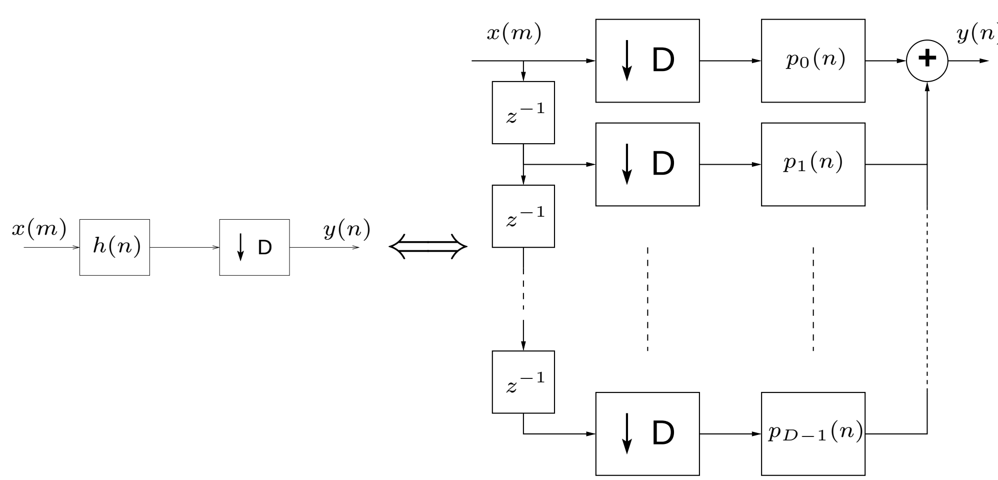
For decimation, $M = D$

POLYPHASE IMPLEMENTATION OF AN INTERPOLATOR



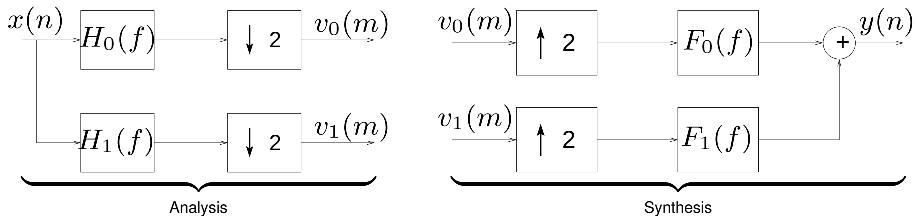
Gain: The number of multiplications is reduced by a factor U.

POLYPHASE IMPLEMENTATION OF A DECIMATOR



Gain: The number of multiplications is reduced by a factor D.

Filter Banks



Possible to get perfect reconstruction with a delay, $y(n) = x(n-L)$, using simple implementable filters, not only ideal low/high pass filters.

General criterion for perfect reconstruction, $y(n) = x(n-L)$:

$$F_0(z)H_0(-z) + F_1(z)H_1(-z) = 0$$
$$F_0(z)H_0(z) + F_1(z)H_1(z) = 2z^{-L}$$

Popular Filter Banks

QMF Quadrature Mirror Filter banks

$$F_0(z) = H_1(-z), \qquad F_1(z) = -H_0(-z)$$
$$F_0(z)H_0(z) - F_0(-z)H_0(-z) = 2z^{-L}$$

QCF Quadrature Conjugate Filter banks (both filter order N and delay L are odd integers)

$$F_0(z) = z^{-L}H_0(z^{-1}), \quad F_1(z) = z^{-L}H_1(z^{-1}), \quad H_1(z) = -z^{-N}H_0(-z^{-1})$$
$$H_0(z)H_0(z^{-1}) + H_0(-z)H_0(-z^{-1}) = 2$$