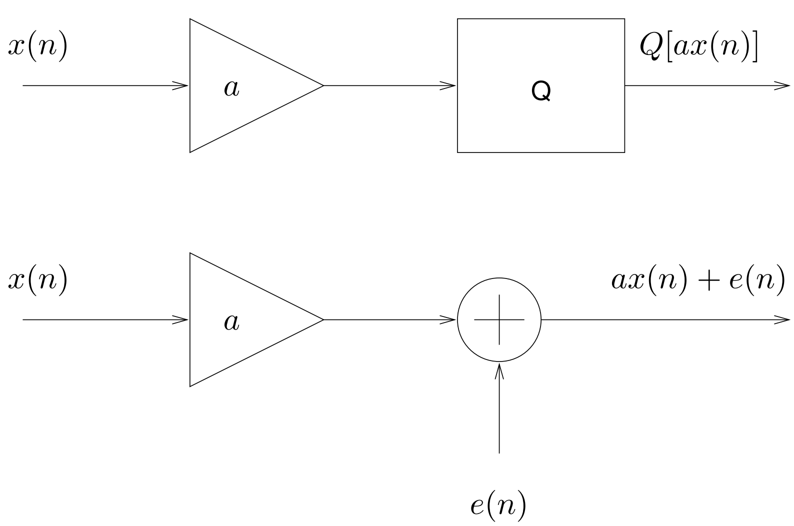
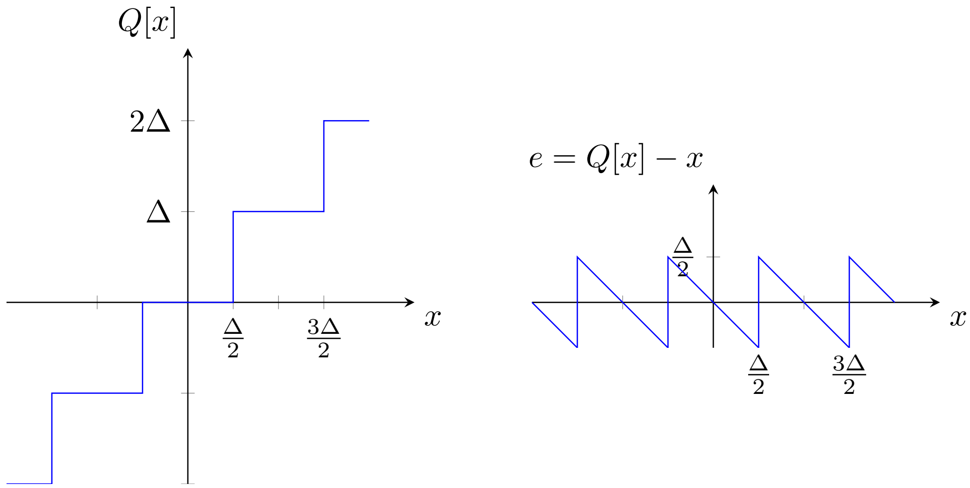
FINITE WORD LENGTH EFFECTS

- quantization noise from A/D conversion
- errors due to quantization of filter coefficients
- uncorrelated round-off noise
- correlated round-off noise (oscillations, limit cycles)

MODEL OF QUANTIZATION AND ROUND-OFF



Uniform Quantization



Number Representations

Fixed point representation $n = B + 1$ bits

$$X = \sum_{i=1}^{B} b_i 2^{-i}$$

$b_0.b_1b_2\cdots b_B$ where b_0 is used in the sign representation (sign-magnitude, one's complement or two's complement).

Yields a number in $[-1, 1-2^{-B}]$ with resolution $\frac{1-(-1)}{2^{B+1}} = 2^{-B}$

Floating point representation

$$X = (-1)^s M 2^E$$

s — sign

M — mantissa (fixed point), $\frac{1}{2} \leq M < 1$

E — exponent (fixed point)

ASSUMPTIONS FOR FIXED-POINT QUANTIZATION NOISE

(i) $e(n)$ is a stationary stochastic process

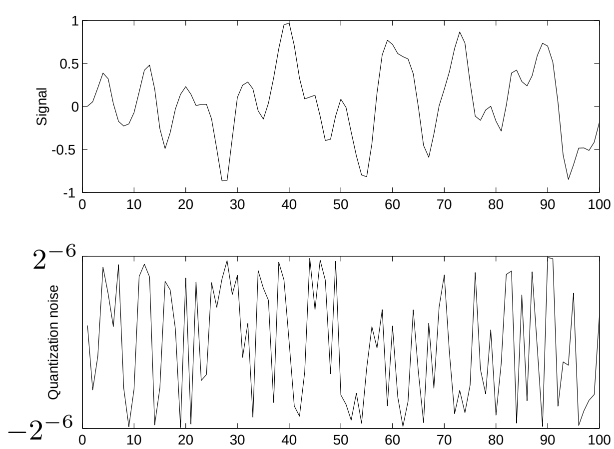
(ii) $e(n)$ is uniformly distributed $[\frac{-2^{-b}}{2}, \frac{2^{-b}}{2}]$

(iii) $e(n)$ is white, $E\{e(n)e(m)\} = \delta(n-m)\sigma_e^2$

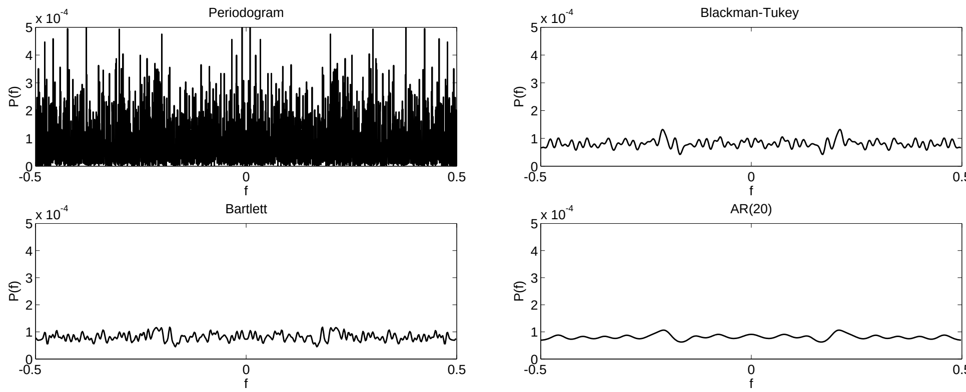
(iv) $e(n)$ is uncorrelated with $x(n)$, $E\{e(n)x(m)\} = 0$

$$(ii) \Longrightarrow \sigma_e^2 = \frac{2^{-2b}}{12}$$

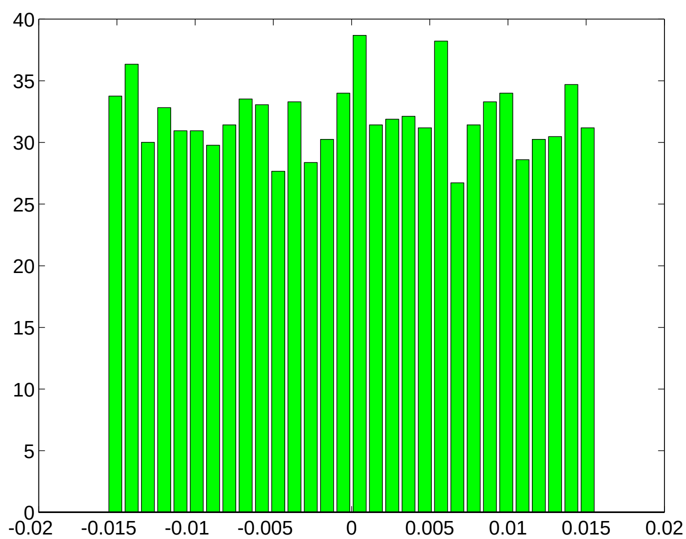
EXAMPLE, QUANTIZATION NOISE



Example, Spectral Estimate of the Quantization Noise



Example, Probability Density of the Quantization Noise



CALCULATION OF QUANTIZATION NOISE — STRATEGY

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- Replace each quantization with an additive noise source $e_k(n)$
- Find transfer function $h_k(n)$ from $e_k(n)$ to the output.
- Noise contribution at the output:

$$\sigma_{q_k}^2 = \sigma_{e_k}^2 \sum_n |h_k(n)|^2$$

- Total quantization noise power at the output:

$$\sigma_{q_{\text{Total}}}^2 = \sum_k \sigma_{q_k}^2$$