# EP2200 Course Project – 2012
# Project I - Error Control in Relay Networks

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## 1   Introduction

Queuing theory provides us a very useful mathematic tool that can be used to analytically evaluate the performance of different applications in the area of telecommunications, computing, traffic engineering, etc.

In this project we consider the multi-hop relay network architecture that has been introduced in the next generation wireless cellular networks such as LTE advanced and WiMax systems as a promising technique to improve the coverage and the throughput of the network. We aim to compare the performance of different error control schemes for relay networks by building analytic models based on queuing theory.

## 2   System Description - Relaying and Error Control

Fig. 1 shows an example of a multi-hop relay system where a Base Station (BS) can transmit/receive data to/from a Mobile Station (MS, a wireless communication terminal) via multiple Relay Stations (RSs). Without loss of generality, we consider the downlink data transmission, that is transmission from the BS to the MS via multiple RSs.
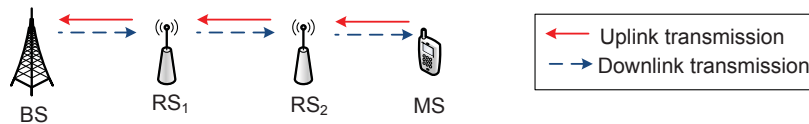


Figure 1: An example multi-hop relay system, where there are two RSs.

We consider two basic types of relaying operation: Amplify-and-Forward (AF) and Decode-and-Forward (DF) relaying. With AF relaying, a RS just amplifies the received signals without decoding. Due to fading and interference of radio channels, data packets cannot always be transmitted without error. So when data packets are forwarded across multiple AF-RSs, these errors are propagated. This is avoided with DF-relaying, where a DF-RS first decodes the packet and then forwards the re-encoded packet to the next-hop receiver. If a

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packet is received with error, it cannot be decoded successfully at a DF-RS, and is dropped. Consequently, only error-free packets are forwarded.

Packets that arrive with error to the MS or are dropped on the multihop path need to be retransmitted. Automatic Repeat reQuest (ARQ) is a widely used concept to control retransmissions. ARQ has several versions, typically building on packet numbering, timeouts and positive and negative acknowledgements (ACKs). If a packet is received with error, retransmission is requested with a negative ACK (NACK). Packet drops are detected based on the sequence numbers of the received ones, or based on timeouts. The retransmission of these packets are requested with a NACK as well. We consider two simple ARQ schemes, end-to-end ARQ (e2e-ARQ) and hop-by-hop ARQ (hyh-ARQ).

With e2e-ARQ retransmissions happen end to end, that is, the destination (in our case the MS) requests retransmissions by generating a NACK message, and then sending the NACK via the reverse multi-hop path to the original source (in our case the BS). The original source (in our case the BS) retransmits the packet. With hyh-ARQ error control is performed in each transmission hop, and each node (an RS or the MS) requests the retransmission of the packet directly from the previous node.

## 2.1 AF relaying with end-to-end ARQ

The AF-RSs forward packets from the BS to the MS simply by amplifying the received signals, and do not decode and check the correctness of the packets. A packet is decoded and checked only at the last hop when the MS receives it. So only end-to-end ARQ (e2e-ARQ) can be used in this case.

## 2.2 DF relaying with end-to-end or hop-by-hop ARQ

The DF-RSs can only decode the packets that are received without errors, so they will drop the ones that cannot be decoded successfully, and only forward the correct ones to the next hop.

If end-to-end ARQ (e2e-ARQ) is used in this case, the MS sends the BS a NACK if a packet is not received or is received with error, and the BS is responsible for retransmitting the packets.

If hop-by-hop ARQ (hyh-ARQ) is used, retransmission is performed in each hop. If an error occurs during the transmission in a hop, it is detected immediately by the receiver of that hop (a RS or the MS), and the previous node retransmits the packet until it is received correctly.

# 3 Performance Analysis

## 3.1 Configuration and Parameters

We build a simplify model of the relay network to make analytic performance evaluation possible. To simplify the problem, we assume that each of the $N$ hops use a separate data channel, so we do not need to model interference. We assume that the packet transmission and prorogation time is negligible compared to the packet processing time for AF/DF relaying. Also we assume that NACK messages are transmitted in a separate and reliable control channel, they are never lost and their transmission and prorogation time can be ignored.

Also, first we consider the case when packets are always retransmitted until they are received and decoded successfully, i.e. there is no limit on the maximum number of retransmission.

Fig. 2, shows the resulting queuing networks for (a) AF relaying with end-to-end ARQ, (b) DF relaying with end-to-end ARQ, and (c) DF relaying with hop-by-hop ARQ.

We consider an MS that receives packets from a BS via $r$ RSs, so there are totally $r + 1$ hops. Packets are generated at the BS according to a Poisson process with rate $\lambda$, and are transmitted immediately. The packets received at an RS or the MS need to wait in a queue, and are processed one by one. An AF-RS just amplifies the signal of a received packet, whereas a DF-RS has to decode and then re-encode the packet, and the MS only has to decode the packet. We assume the processing time of every packet is exponentially distributed with mean $1/\mu_{AF}$ at a AF-RS, $1/\mu_{DF}$ at a DF-RS, and $1/\mu_{MS}$ at the MS. Based on the complexity of the processing procedure, we assume $\mu_{AF} > \mu_{DF}$ and $\mu_{MS} > \mu_{DF}$.

Let $p_{e,j}$ with $j \in \{1, 2, ..., r + 1\}$ denote the error probability of the packet transmission at the $j$th hop. $p_{e,j} \in [0, 1]$. Packets that need to be retransmitted do not need to be processed again at the transmitter side.
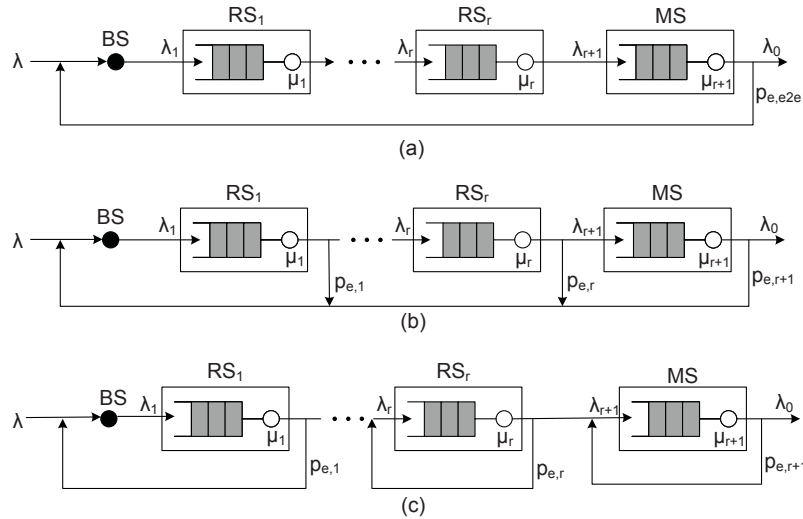


Figure 2: Queuing networks for (a)AF relaying with end-to-end ARQ, (b)DF relaying with end-to-end ARQ, and (c)DF relaying with hop-by-hop ARQ.

## 3.2 Theoretic Analysis

First you will need to derive analytic expressions that describe the performance of the different multi-hop relay systems. We summarize the system parameters in Table 1.

| | |
|---|---|
| Number of relays | $r$ |
| Packet arrival rate | $\lambda$ |
| Packet processing time | $\sim exp(\mu)$, |
| | $\mu = \mu_{AF}$ for AF-RS, |
| | $\mu = \mu_{DF}$ for DF-RS, |
| | $\mu = \mu_{MS}$ for MS |
| Packet error probability of $j$th hop | $p_{e,j} \in [0,1]$ with $j = \{1, 2, ..., r+1\}$ |

Table 1: List of parameters

### 3.2.1 AF relaying with end-to-end ARQ

Consider the queuing network for AF relaying with end-to-end ARQ shown in Fig. 2(a). Give the queuing model for the RS and MS nodes.
Please derive:

- the end-to-end error probability of packet transmission ($p_{e,e2e}$), and the packet arrival rate at the first relay ($\lambda_1$)

- the average queuing delay and the average number of packets (waiting or under processing) at every RS and MS ($\bar{W}_j$ and $\bar{N}_j$),

- the average end-to-end delay, that is the average time from the generation of a packet until it is received successfully at the MS ($\bar{T}$).

### 3.2.2 DF relaying with end-to-end or hop-by-hop ARQ

Consider the queuing network for DF relaying with end-to-end ARQ and with hop-by-hop ARQ shown in Fig. 2 (b) and (c), respectively. Please derive for both cases:

- the packet arrival rate at each RS and MS ($\lambda_j$),

- the average queuing delay and the average number of packets (waiting or under processing) at every RS and MS ($\bar{W}_j$ and $\bar{N}_j$),

- the average end-to-end delay, that is, the average time from the generation of a packet until it is received successfully at the MS ($\bar{T}$).

## 3.3 Numerical Evaluation

Now you need to compare the performance of the three relaying systems shown in Fig. 2, based on the analytic results derived in Section 3.2, and for given system parameter values. You are strongly encouraged to use Matlab to draw the performance figures, but you are also allowed to use other programming languages and tools.

### 3.3.1 Stability region

Consider $p_{e,j} = 0.1$ with $j = \{1, 2, ..., r+1\}$, $\mu_{DF} = 1$ and $\mu_{AF} = \mu_{MS} = 2\mu_{DF}$. Please draw the maximal allowed arrival rate of packets $\lambda_{max}(r)$ for the three

relaying systems respectively as $r$ increases from 1 to 8. The maximal allowed arrival rate $\lambda_{max}(r)$ is the rate where for all $\lambda < \lambda_{max}(r)$ the queuing network remains stable. (A queuing network is stable if all the queues are stable.) How does $\lambda_{max}(r)$ change as $r$ goes to infinity? Give an intuitive explanation.

### 3.3.2 Arrival rate-delay characteristics

With the same set of parameters considered in Section 3.3.1 and $r = 4$, plot the delay as a function of the arrival rate , for of the three relaying systems under their stability regions. The x axis is the arrival rate $\lambda$, whereas the y axis is the average delay from packet generation to correct reception. What happens with the arrival rate-delay curves when $p_{e,j}$ increases?

### 3.3.3 AF or DF?

- Consider $r = 4$, $\mu_{DF} = 1$ and $\mu_{AF} = \mu_{MS} = 2\mu_{DF}$, $\lambda = 0.5\lambda_{max}(r)$. Discuss which relaying+ARQ scheme would you select? What do you need to consider to make a choice? Select relaying+ARQ solution for $p_{e,j}$, $j = \{1, 2, ..., r + 1\}$ values in the interval from 0.05 to 0.95.

- Consider $r = 4$, $\lambda = 0.5\lambda_{max}(r)$, $p_{e,j} = 0.1$ for $j = \{1, 2, ..., r + 1\}$, $\mu_{DF} = 1$, $\mu_{MS} = 2\mu_{DF}$, and $\mu_{AF} = k\mu_{DF}$. Which relaying+ARQ scheme would you select to use when $k$ increases from 1 to 4?

Please motive your answers.

# 4 Finite Number of Retransmissions - Extra Exercise for the Best

Next, we consider more practical systems, that is, there is a limit on the maximum number of retransmission. Let $N$ denote the maximum number of transmission, so $N - 1$ denotes the maximum number of retransmission. A packet is dropped if it cannot be retransmitted successfully in $N - 1$ times. Fig. 3(b) shows the queuing networks for AF relaying with end-to-end ARQ in this case.
Consider the system shown in Fig. 3(b).

- Please derive the packet drop probability given that the packet needs to be retransmitted ($p_{d,e2e}$ in Fig. 3(b)).

- What is the packet loss ratio of the system? (That is, the ratio of packets that are not delivered in the end.)

- What is the stability region ($\lambda_{max}(r)$)?

- What is the throughput of the system ($\lambda_o$ on the figure)?

- What is the average end-to-end delay, as a function of $\lambda$?

Assume $r = 4$, $\mu_{DF} = 1$ and $\mu_{AF} = \mu_{MS} = 2\mu_{DF}$, $p_{e,j} = 0.1$ with $j = \{1, 2, ..., r + 1\}$. Please plot:

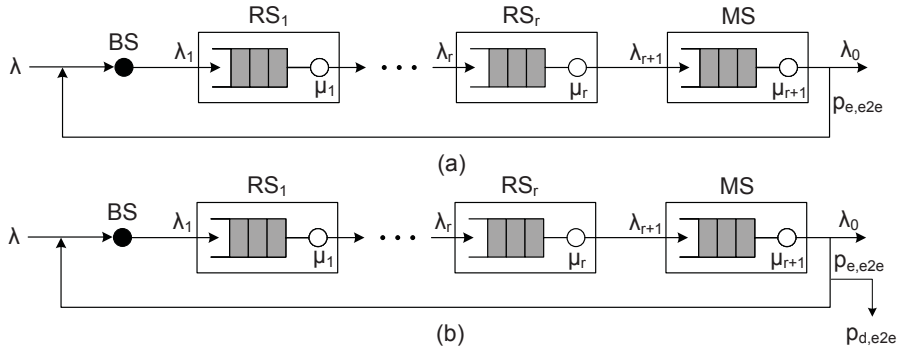- the packet loss ratio when $N$ increases from 3 to 8 for $\lambda = 0.5\lambda_{max}(r)$;

Figure 3: Queuing networks for AF relaying with end-to-end ARQ in the cases that (a) the maximum number of retransmission $= \infty$ and (b) the maximum number of retransmission $N < \infty$.

- the throughput-delay curve when $\lambda < \lambda_{max}(r)$ for $N = 4$.

Discuss the effect of limited number of retransmissions on the stability region and on the end-to-end delay.

# 5   Submission Instructions and Grading

- Submission deadline: **Friday, January 4, 2013**. Electronic submission to **lipingw at kth.se**, or printed copy to STEX. Do not forget to add your name.

- You are allowed to solve the problem in groups, however, you have to prepare a project report on your own. Reports including the same text will be disqualified.

- You need to submit a report of a maximum of 4 printed pages. The report needs to contain a description of the solution of the problems, including detailed derivation of the formulas that used to calculate the results, and discussions on the insight from your results.

- You need to use software tools to get the results, we propose Matlab, but we accept all solutions, e.g., you can even program everything in C. You should not include your codes in the report.

- Check the grammar of the report. There are good tools available to do that. Make sure that performance graphs are possible to interpret. Give the dimensions and units of the axes.

Grading: pass or fail. To pass the moment, you need to show that you made a serious attempt to solve the problems. The best 20% of the submissions receive 5 extra points on the exam in December. Extra points will not be considered at the make-up exam or at later exams. These best 20% reports will be published on the course web.

Would you have any questions, contact Liping at **lipingw at kth.se**.