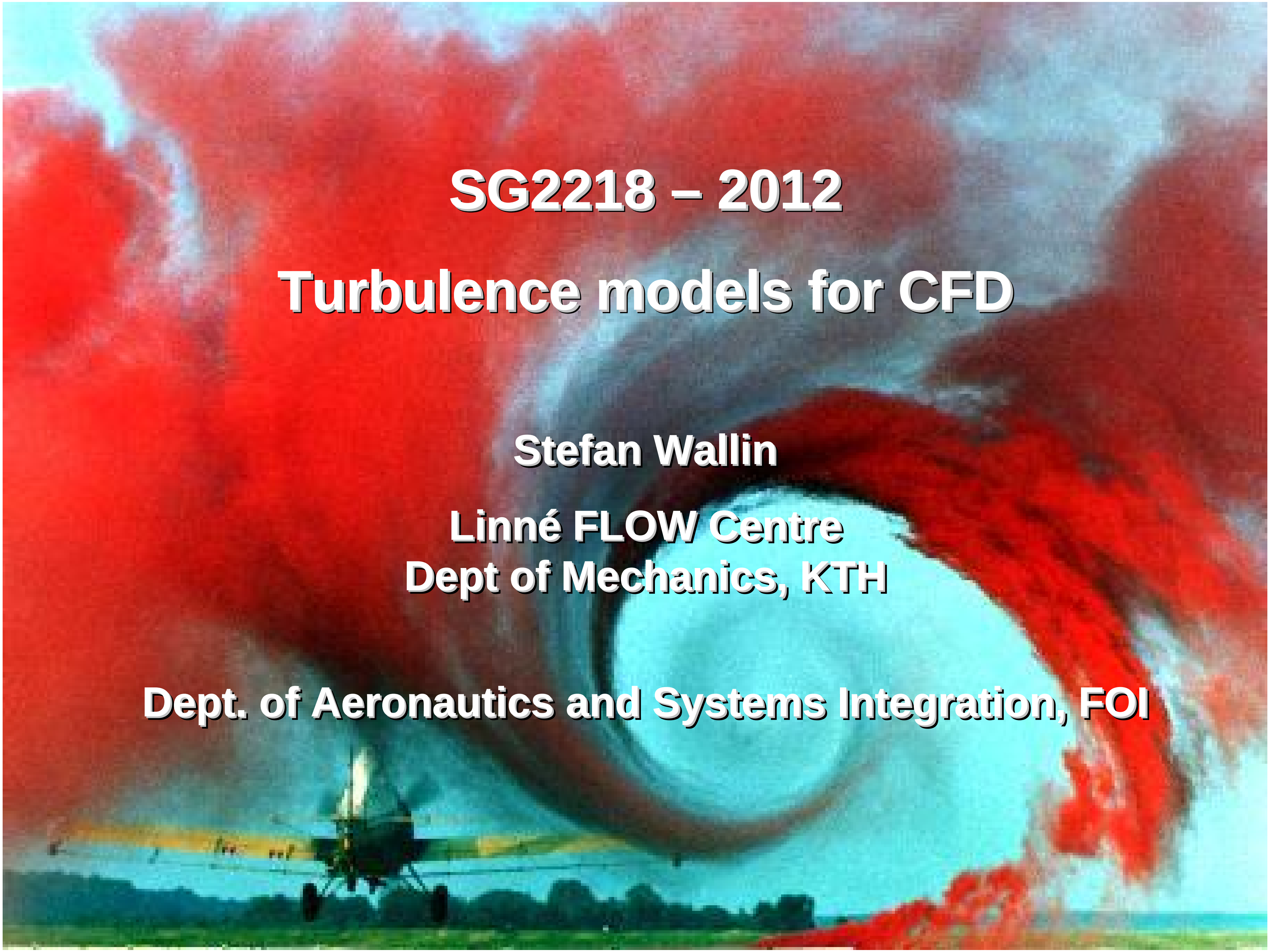

SG2218 – 2012

Turbulence models for CFD

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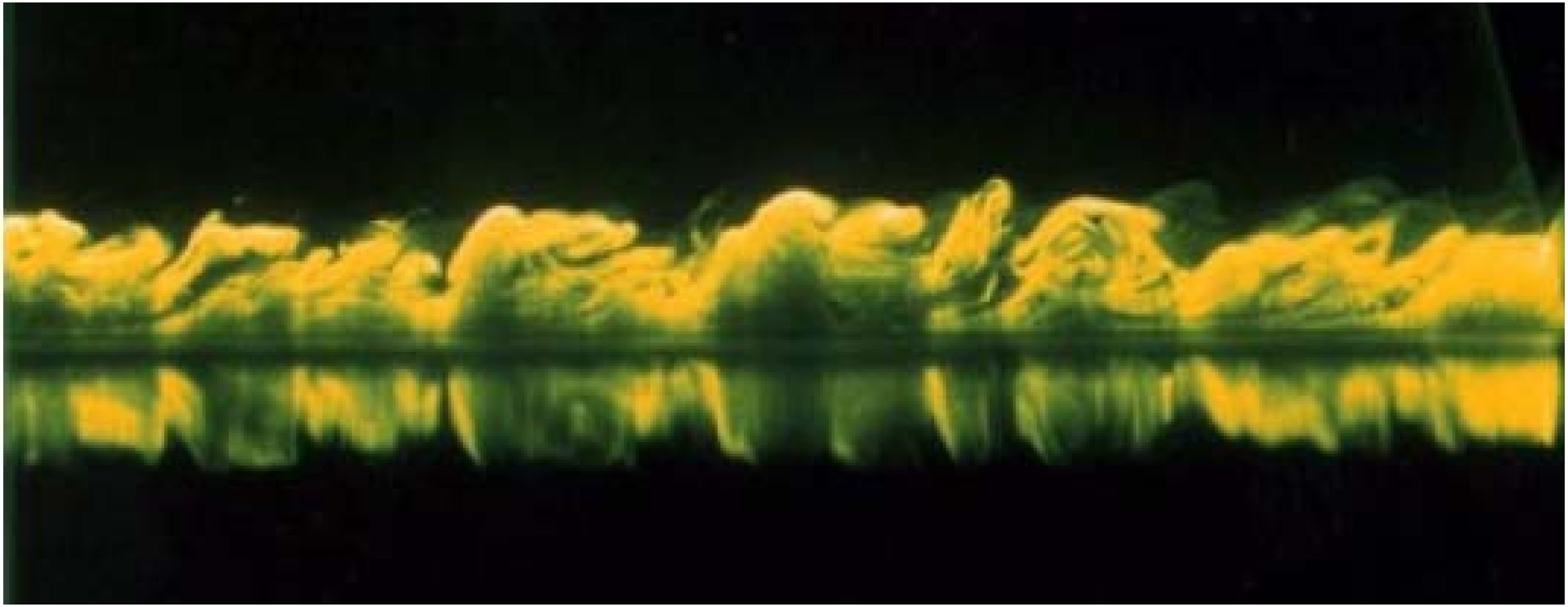
There are no “simple” turbulent flows

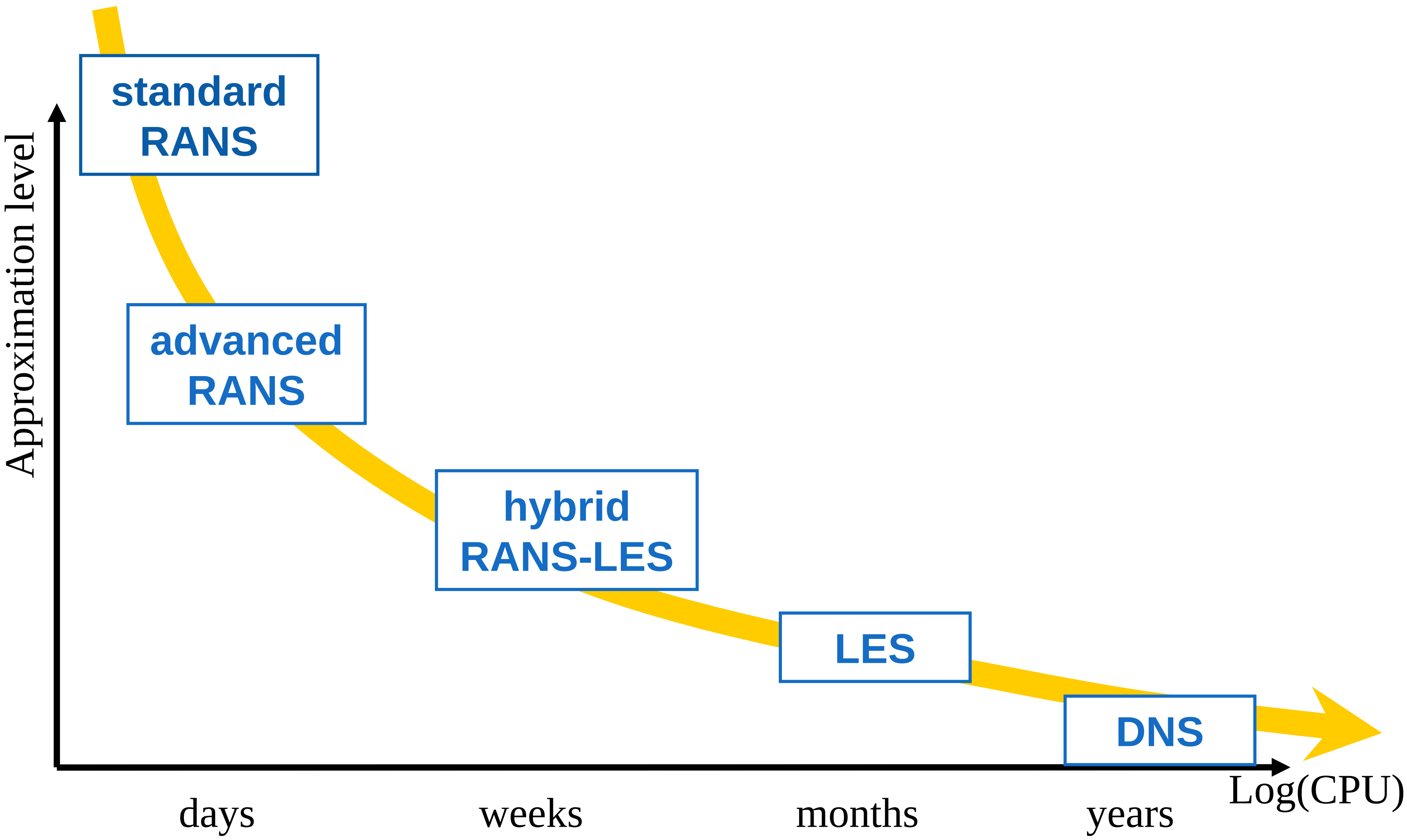
Turbulent boundary layer:

- **Instantaneous velocity field (snapshot)** $u_i(\boldsymbol{x}, t)$

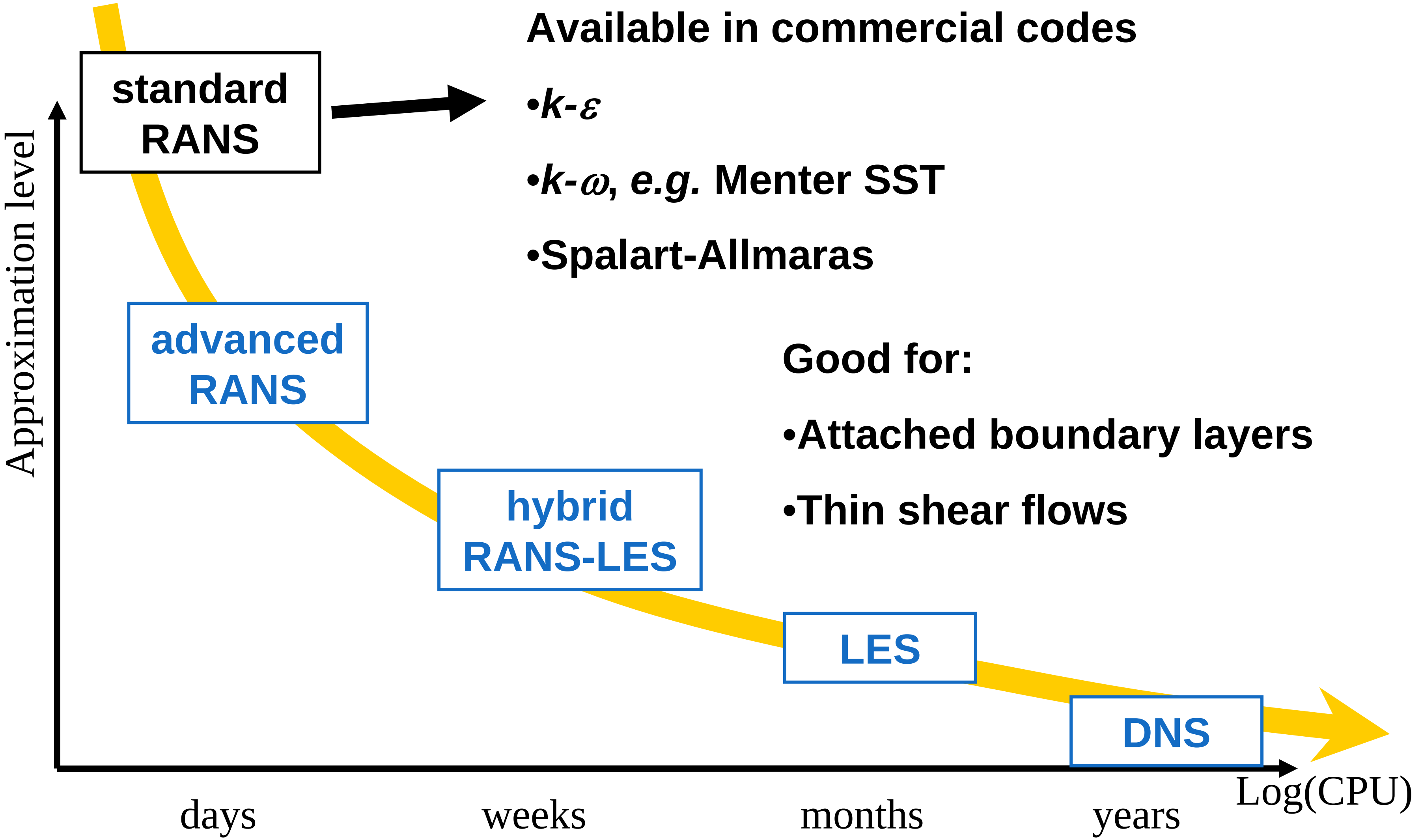


Ref: Prof. M. Gad-el-Hak, University of Notre Dame

Prediction of turbulent flows



Standard RANS models



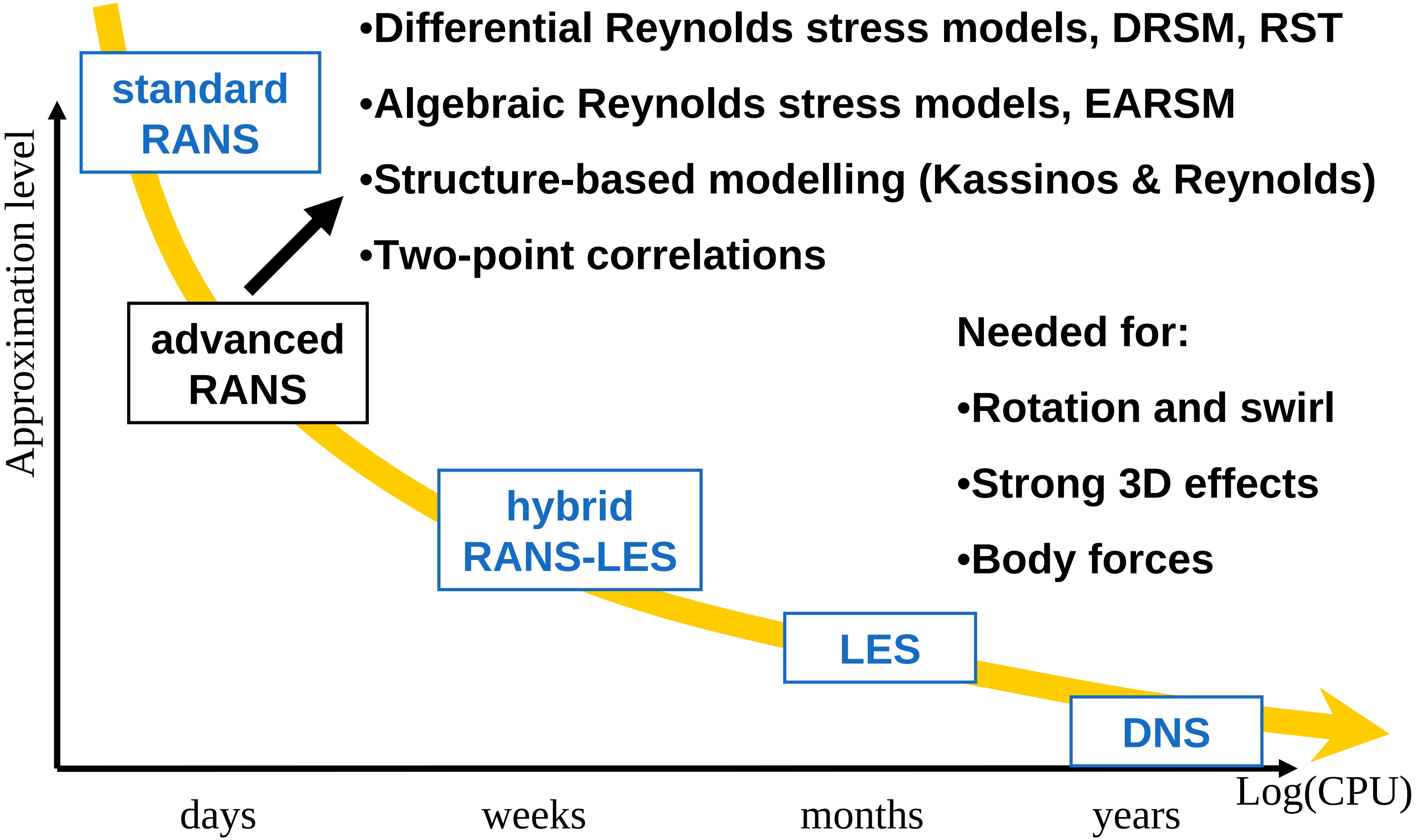
Available in commercial codes

- ***k-ε***
- ***k-ω*, *e.g.* Menter SST**
- **Spalart-Allmaras**

Good for:

- **Attached boundary layers**
- **Thin shear flows**

Advanced RANS models



- Differential Reynolds stress models, DRSM, RST
- Algebraic Reynolds stress models, EARSM
- Structure-based modelling (Kassinos & Reynolds)
- Two-point correlations

Needed for:

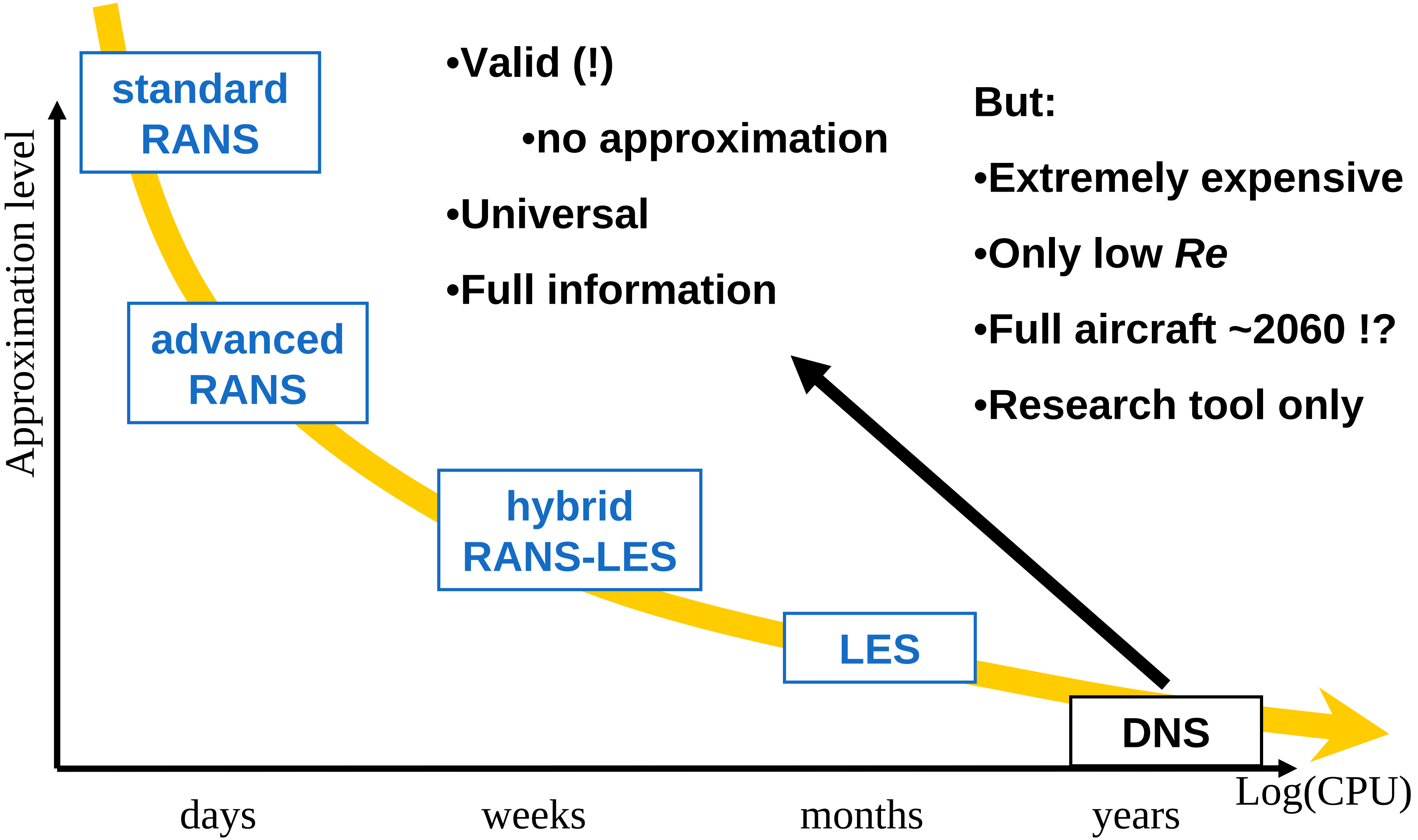
- Rotation and swirl
- Strong 3D effects
- Body forces

Direct Numerical Simulation – DNS

- Valid (!)
  - no approximation
- Universal
- Full information

But:

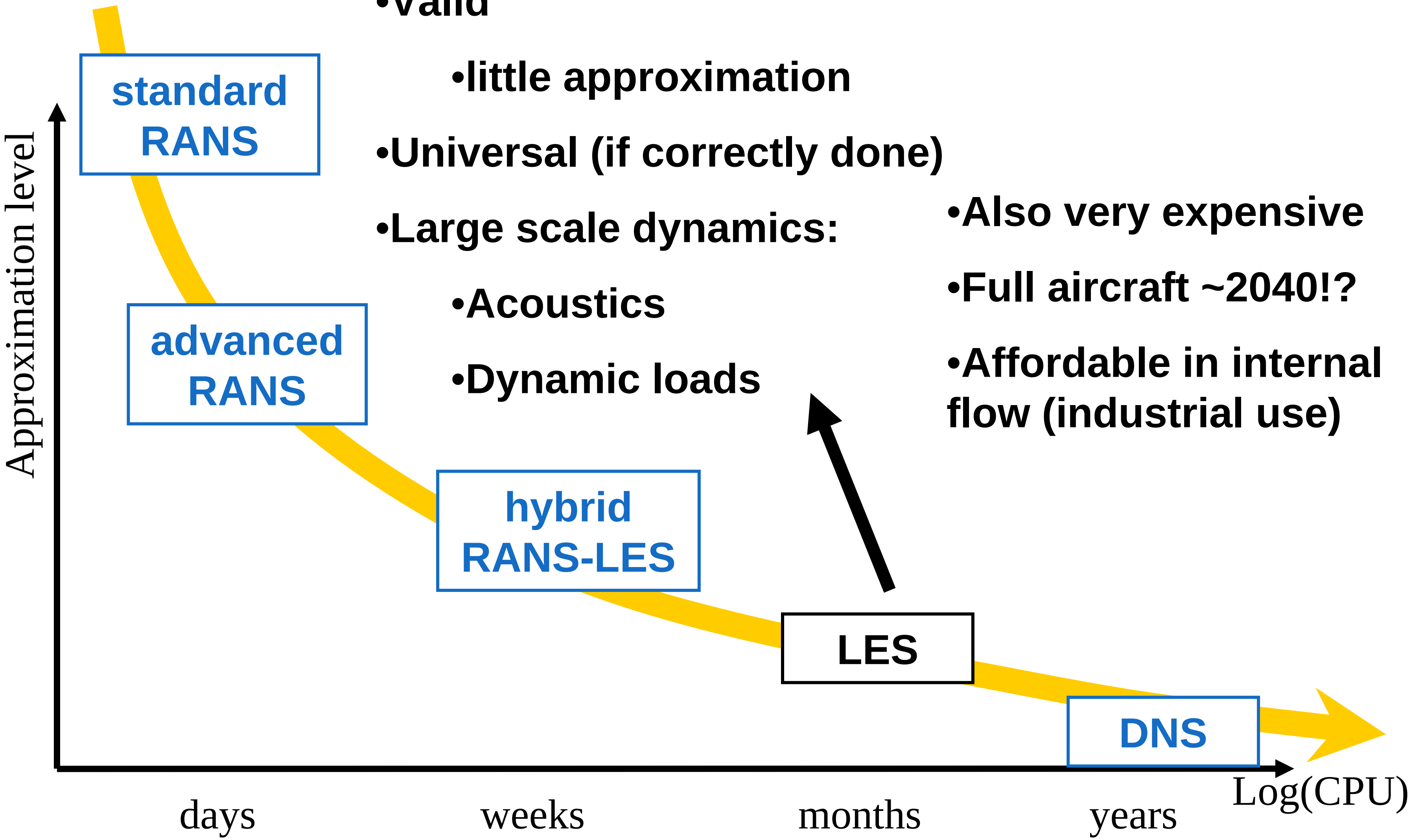
- Extremely expensive
- Only low *Re*
- Full aircraft ~2060 !?
- Research tool only



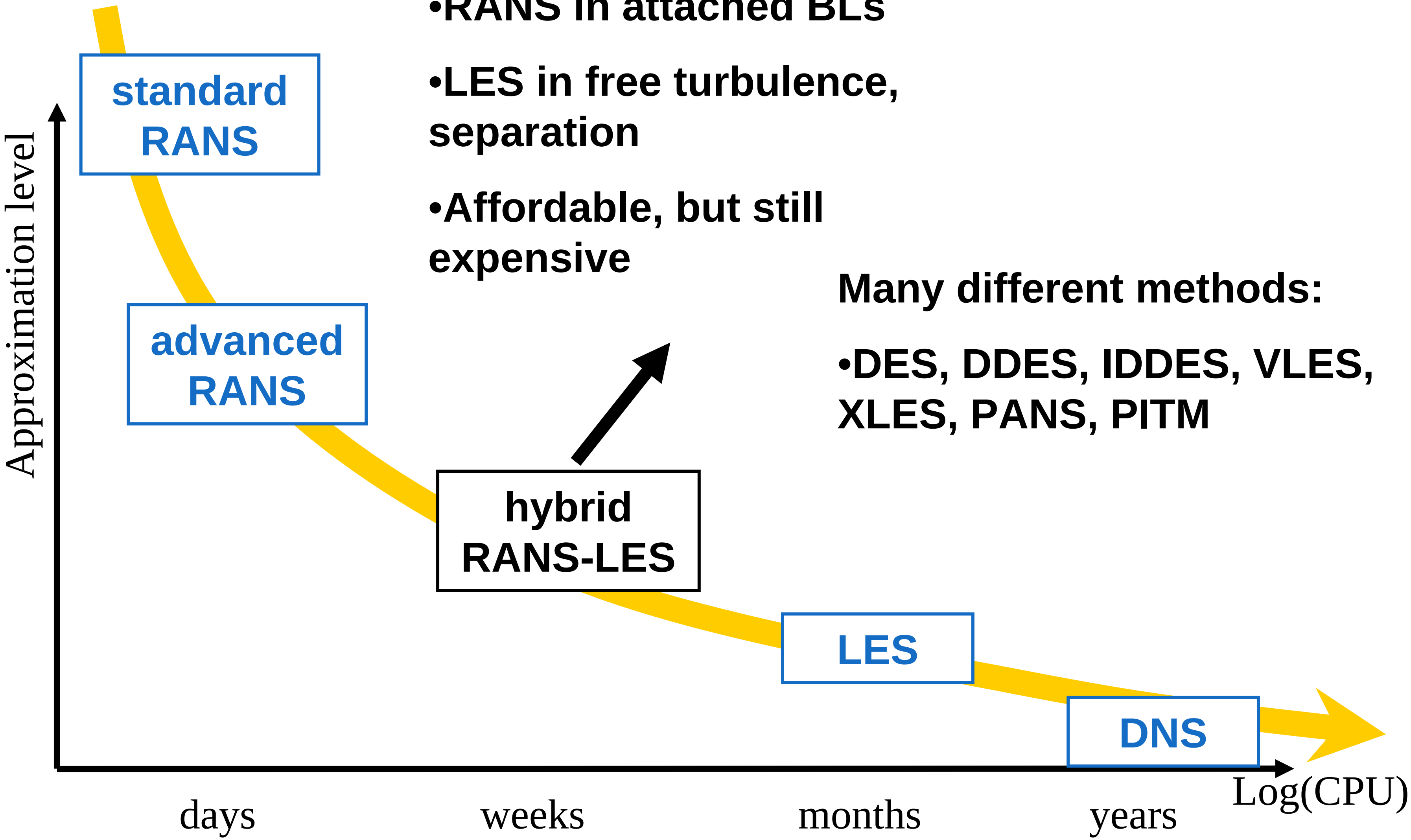
Large Eddy Simulation – LES

- Valid
  - little approximation
- Universal (if correctly done)
- Large scale dynamics:
  - Acoustics
  - Dynamic loads

- Also very expensive
- Full aircraft ~2040!?
- Affordable in internal flow (industrial use)



Hybrid RANS – LES methods

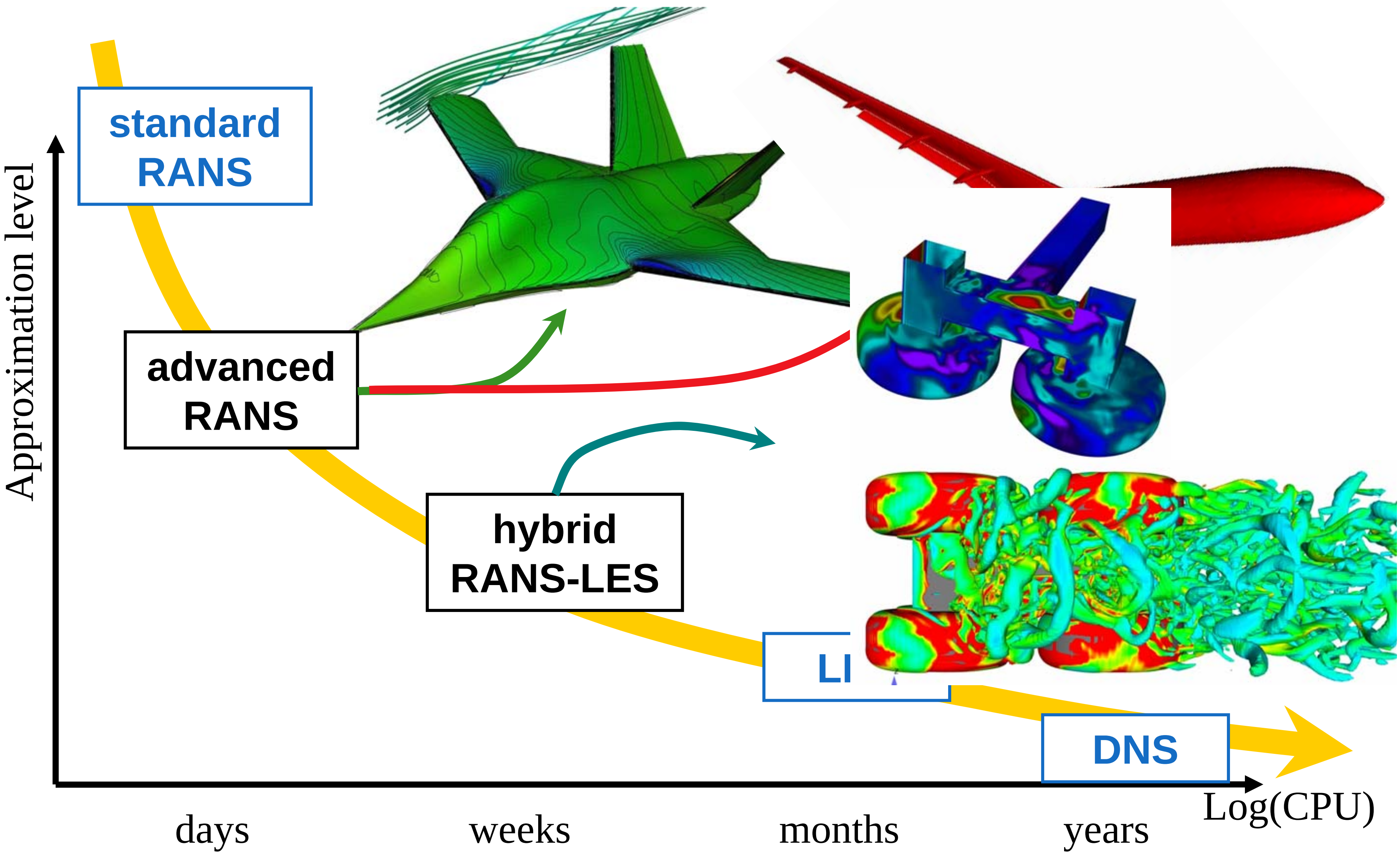


- RANS in attached BLs
- LES in free turbulence, separation
- Affordable, but still expensive

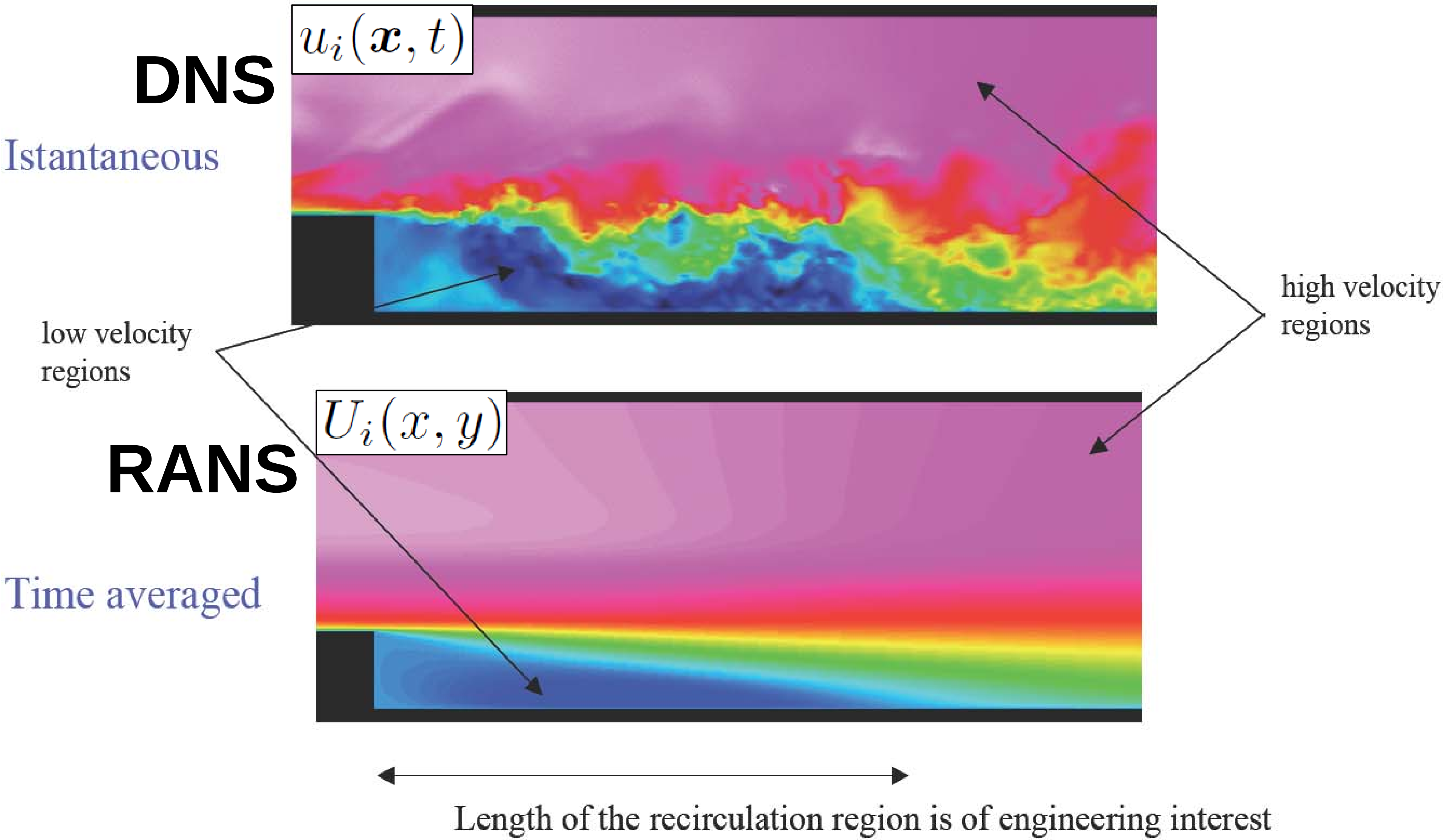
Many different methods:

- DES, DDES, IDDES, VLES, XLES, PANS, PITM

Prediction of turbulent flows



DNS – RANS

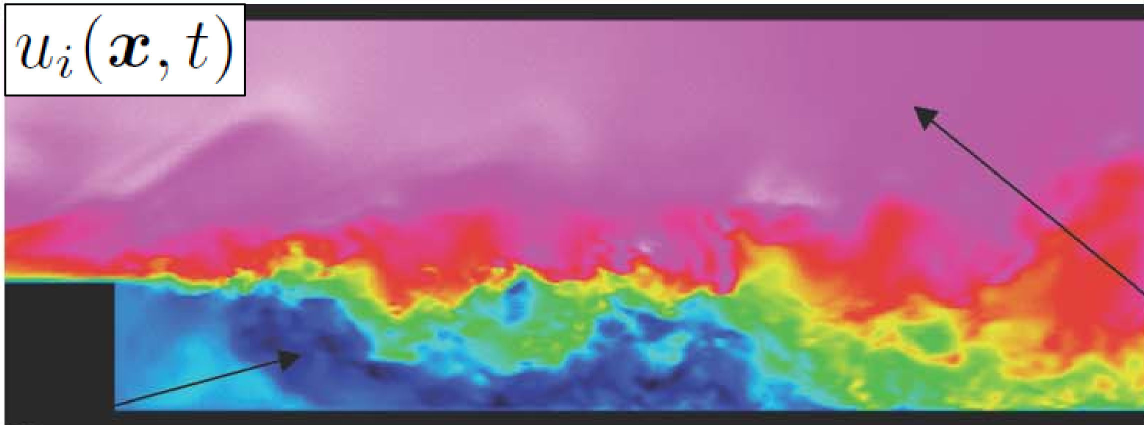


DNS
Istantaneous
$u_i(\boldsymbol{x}, t)$
low velocity regions
high velocity regions
RANS
Time averaged
$U_i(x, y)$
Length of the recirculation region is of engineering interest

DNS – RANS

DNS

Istantaneous



DNS (and also LES):

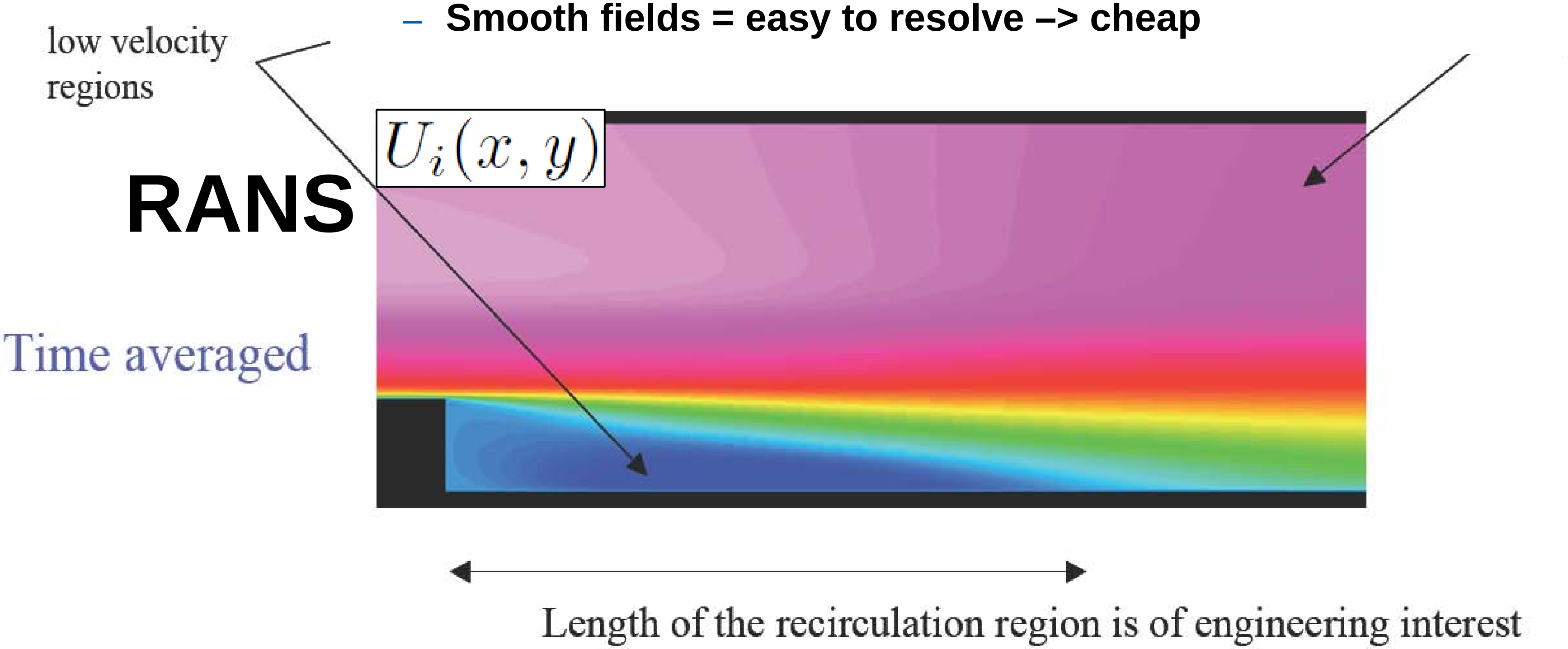
- **3D**
- **Time dependent**
  - **Full information of turbulence scales**
  - **Acoustics and dynamic loads**
- **Huge Reynolds number dependency (al least Re^3)**

Expensive!

DNS – RANS

RANS:

- **Reduction of dimensions –> cheap**
  - **Here: 2D and steady**
- **Only statistical information of turbulence scales:**
  - **Time and length scales**
  - **rms values**
  - **Smooth fields = easy to resolve –> cheap**



How expensive is DNS?

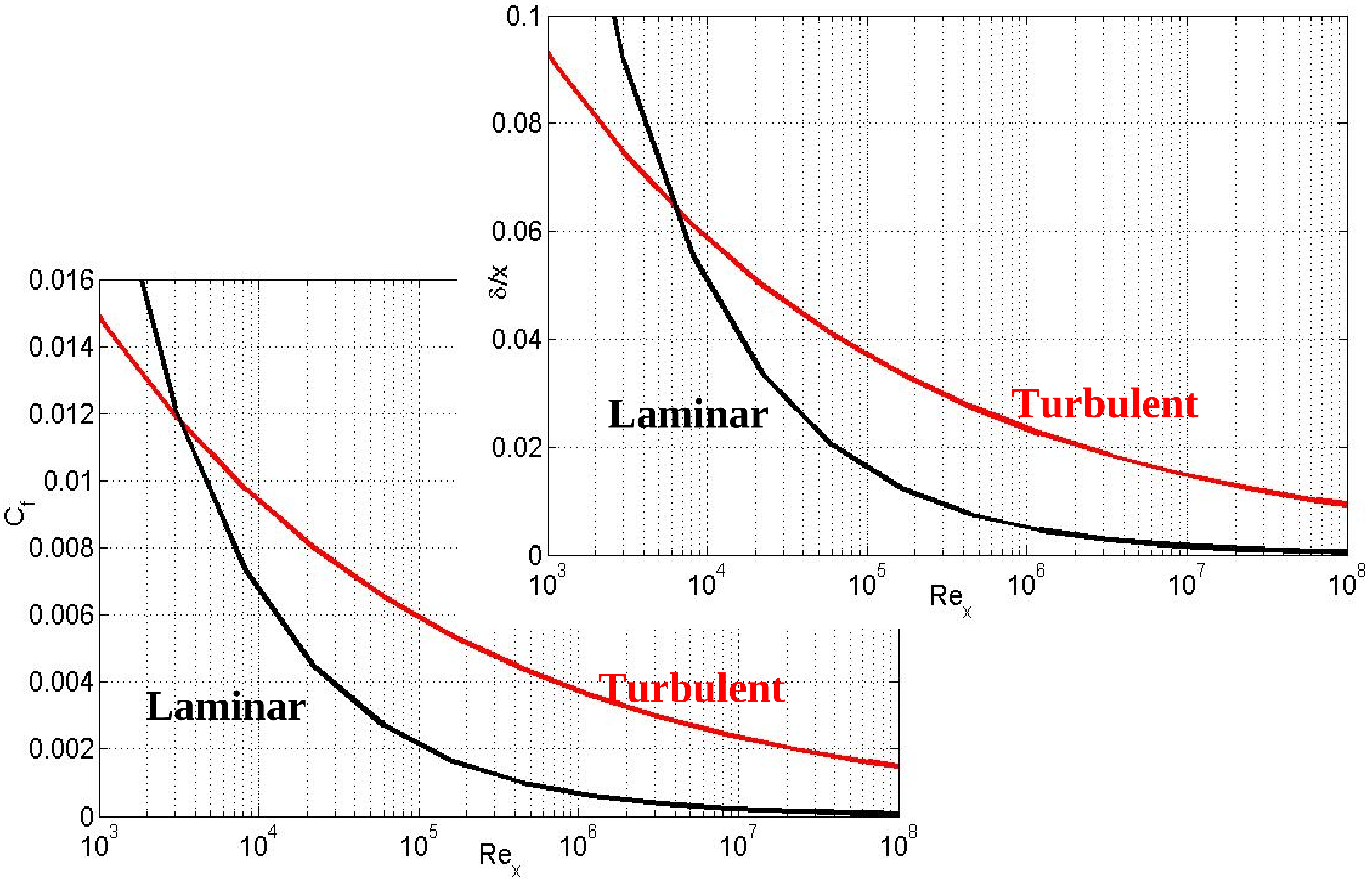
- **DNS of flat plate turbulent boundary layer**
  - **Schlatter, et al., KTH, Dept. of Mechanics**
  - **APS meeting 2010: http://arxiv.org/abs/1010.4000**
  - **http://www.youtube.com/watch?v=4KeaAhVoPIw**
  - **http://www.youtube.com/watch?v=zm9-hSP4s3w**
  - $Re_\theta = 4300$, $Re_\tau = 1400$
  - **8192×513×768 = 3.2×10^9 spectral modes (7.5×10^9 nodes)**
  - $\Delta x^+ = 9$, $\Delta z^+ = 4$ **–>** **box:** $L^+ = 70\,000$, $H^+ = W^+ = 3\,000$
  - **BL relations:** $Re_x = 1.4\times10^6$
  - **CPU time: 3 months @ 4000 CPU cores = 1 unit**
- **DNS of Airplane, same Reynolds number (**$Re_x = 1.4\times10^6$**;** c**=0.5m,** U**=40m/s)**
  - **Only a narrow stripe – wing requires about 1 000 stripes**
  - $N_{nodes} = 10^{13}$
  - **CPU = 10^3 units**

Empirical turbulent BL relations

- **Skin friction coefficient:** $\frac{C_f}{2} = \frac{\tau_w}{\rho U_\infty^2} = \left(\frac{Re_\tau}{Re_\delta}\right)^2 \approx 0.0296 Re_x^{-1/5}$
- **Boundary layer thickness:** $\frac{\delta}{x} = \frac{Re_\delta}{Re_x} \approx 0.37 Re_x^{-1/5}$
- **Boundary layer momentum thickness:** $Re_\tau \approx 1.13 Re_\theta^{0.843}$
- **Reynolds numbers:**

$$Re_\tau \equiv \frac{\delta u_\tau}{\nu} \qquad Re_\theta \equiv \frac{\theta U_\infty}{\nu} \qquad Re_\delta \equiv \frac{\delta U_\infty}{\nu} \qquad Re_x \equiv \frac{x U_\infty}{\nu}$$

Empirical relations plotted

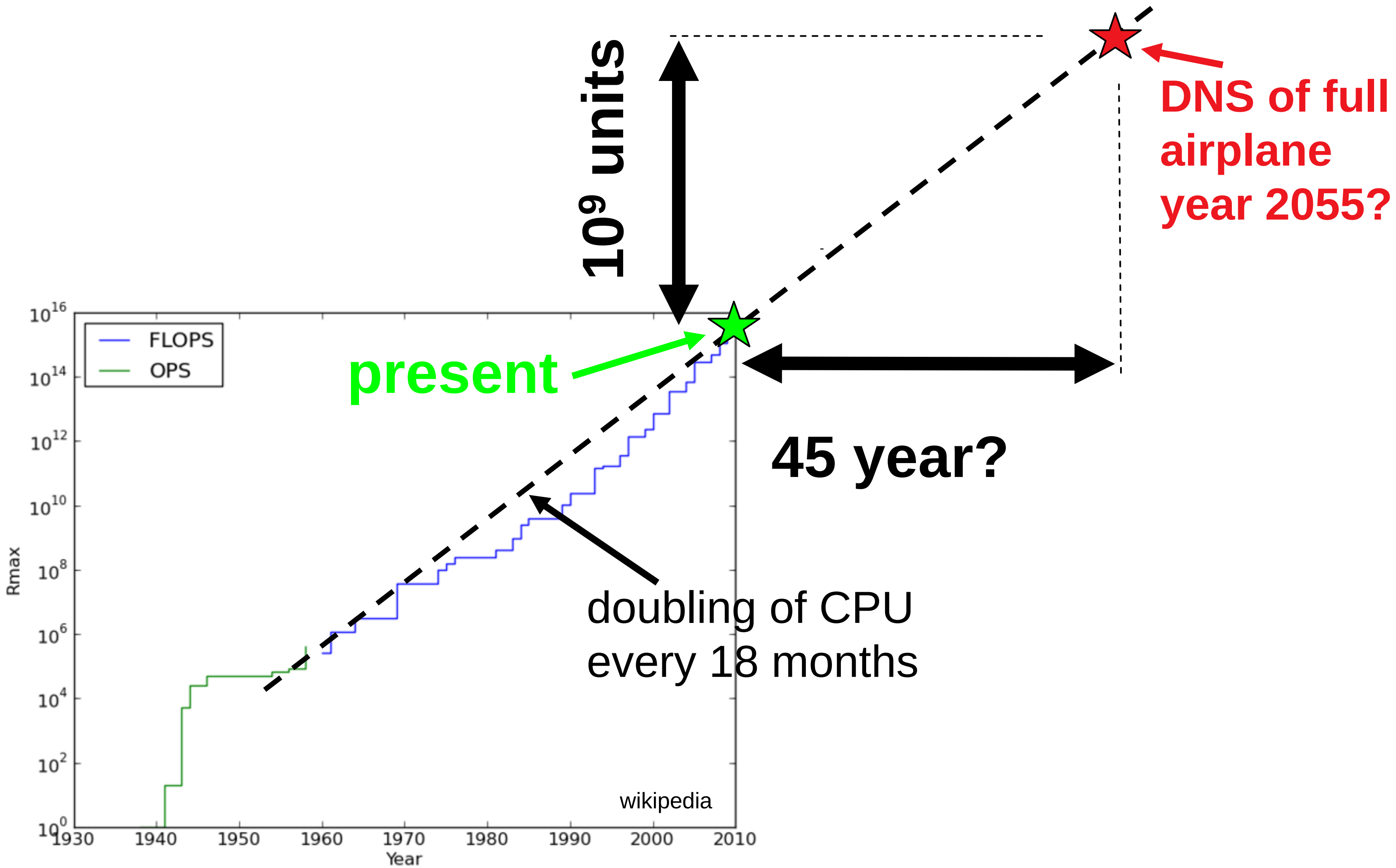


DNS – full scale airplane

- **Re scaling – wall bounded flow**
  - **Nodes:** $N_{nodes} \sim \dfrac{L \times B \times H}{\Delta x \Delta z \Delta y} \sim L^{+2} H^{+} \sim Re_x^{5/2}$
  - **Time steps:** $N_{\Delta T} \sim \dfrac{T}{\Delta T} \sim T^{+} \sim Re_x^{4/5}$
  - **CPU time:** $N_{CPU} \sim N_{nodes} \times N_{\Delta T} \sim Re_x^{33/10}$

- **DNS of Airplane (Re_x = 70×10^6) (factor of 50)**
  - **N_{nodes} = 10^{17}**
  - **CPU = 10^9 units**

Supercomputer development



Computational effort – different approaches

Name	Aim	Unsteady	*Re*-dependence	3/2D	Empiricism	Grid	Steps	Ready
2DURANS	Numerical	Yes	Weak	No	Strong	10^5	$10^{3.5}$	1980
3DRANS	Numerical	No	Weak	No	Strong	10^7	10^3	1990
3DURANS	Numerical	Yes	Weak	No	Strong	10^7	$10^{3.5}$	1995
DES	Hybrid	Yes	Weak	Yes	Strong	10^8	10^4	2000
LES	Hybrid	Yes	Weak	Yes	Weak	$10^{11.5}$	$10^{6.7}$	2045
QDNS	Physical	Yes	Strong	Yes	Weak	10^{15}	$10^{7.3}$	2070
DNS	Numerical	Yes	Strong	Yes	None	10^{16}	$10^{7.7}$	2080

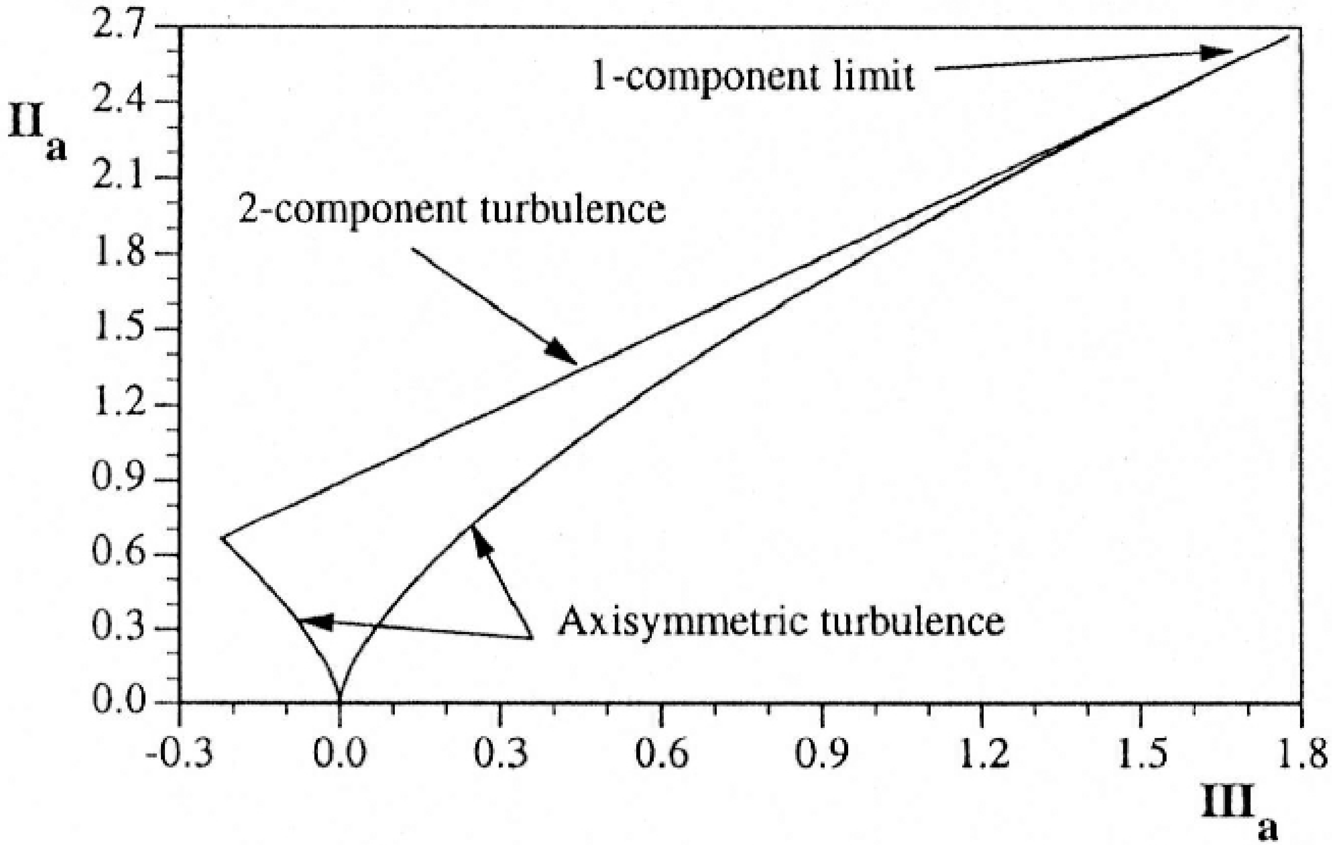
From Spalart, Int. J. Heat and Fluid Flow, 2000

- **RANS: Reynolds Averaged Navier-Stokes**
- **URANS: Unsteady RANS – slowly in time**
- **DES: Detached Eddy Simulation**
- **LES: Large eddy simulation**
- **QDNS: Quasi DNS, or wall resolved LES**
- **DNS: Direct Numerical Simulation (of the Navier-Stokes eq’s)**
- **“Ready”: When first results can be expected**

RANS

- Reynolds stresses $\overline{u'_i u'_k}$
  - Appears because of the non-linear term $u_k \dfrac{\partial u_i}{\partial x_k}$
  - Not "small"
  - Significant effects on the flow
- RANS modelling
  - Model Reynolds stresses in terms of "known" quantities
  - Reduces the problem to steady (or slowly varying)
  - 2D assumptions possible
- Closure problem
  - Equation for $\overline{u'_i u'_k}$ can be derived from the N-S equation
  - Contains higher order correlations, e.g. $\overline{u'_i u'_j u'_k}$
  - All equations of the statistical properties contain additional correlations.

Reynolds stress anisotropy map



anisotropy map in channel flow