Homework Set #7

The intention is that you do the exercises yourself. Oral discussion (without using pen/paper) between students is allowed, but the solution should be written down individually.
The homework must be submitted one day before each tutorial session either on paper (before 6 PM) or via email (before mid night).
Every correctly solved problem gives 1 point, partially correct gives 0.5 point, mostly wrong 0 point.

Numbers below refer to problems in the text book: Amos Lapidoth, "A Foundation in Digital Communication".

1. Exercise 26.4
2. Exercise 26.5
3. Exercise 26.6 i), ii), and iii)
4. Exercise 26.6 iv). Compare your results to the previous exercise.
5. Exercise 26.10
6. Consider the system in Figure 1. In this problem we investigate the degradation in performance that results from using a filter other than the optimum receiver filter.

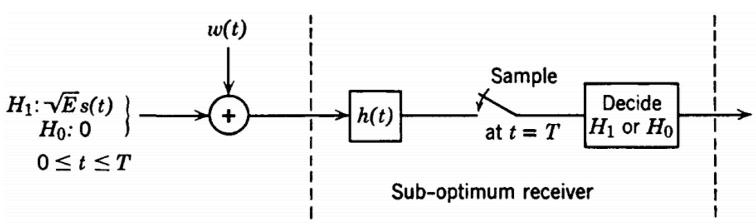

Figure 1: Exercise 5.

$$\int_0^T s^2(t)dt = 1,$$
$$E[w(t)w(\tau)] = \delta(t-\tau)$$

Solution due December 12, 2012

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The received signal is

$$\begin{array}{lll} H_1: & r(t) = \sqrt{E}s(t) + w(t), & -\infty < t < \infty \\ H_0: & r(t) = w(t), & -\infty < t < \infty \end{array}$$

with transmitted waveform

$$s(t) = \begin{cases} \sqrt{\frac{1}{T}}, & \text{for } 0 \le t \le T, \\ 0, & \text{elsewhere} \end{cases} .$$

The optimum filter is given by

$$h_{opt}(t) = \begin{cases} s(T-t), & \text{for } 0 \le t \le T, \\ 0, & \text{elsewhere} \end{cases}$$

and results in

$$\text{SNR}_{opt} = \frac{2E}{N_0}.$$

Suppose that instead of $h_{opt}(t)$ we use the following filter:

$$h(t) = e^{-at}u_{-1}(t) \quad \text{for } -\infty < t < \infty$$

where $u_{-1}(t)$ is the unit step function and a is a design parameter.

(a) Choose the parameter a to maximize the output signal-to-noise ratio.

(b) Compute the resulting SNR and compare with SNR_{opt}. How many dB must the transmitter energy be increase to obtain the same performance?

Hint: The Fourier transform of $h(t)$ is

$$H(f) = \frac{1}{a + j2\pi f}.$$

This problem corresponds to Exercise 4.2.6 in H. Van Trees' *"Detection, Estimation, and Modulation Theory." (Part I).*

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