Särtryck ur 7:e upplagan av "Elementary Linear Algebra".

EXERCISE SET 9.7

1. Find the quadratic forms associated with the following quadratic equations.
(a) $x^2 + 2y^2 - z^2 + 4xy - 5yz + 7x + 2z = 3$
(b) $3x^2 + 7z^2 + 2xy - 3xz + 4yz - 3x = 4$
(c) $xy + xz + yz = 1$
(d) $x^2 + y^2 - z^2 = 7$
(e) $3z^2 + 3xz - 14y + 9 = 0$
(f) $2z^2 + 2xz + y^2 + 2x - y + 3z = 0$

2. Find the matrices of the quadratic forms in Exercise 1.

3. Express each of the quadratic equations given in Exercise 1 in the matrix form $\mathbf{x}^T A\mathbf{x} + K\mathbf{x} + j = 0$.

4. Name the following quadrics.
(a) $36x^2 + 9y^2 + 4z^2 - 36 = 0$
(b) $2x^2 + 6y^2 - 3z^2 = 18$
(c) $6x^2 - 3y^2 - 2z^2 - 6 = 0$
(d) $9x^2 + 4y^2 - z^2 = 0$
(e) $16x^2 + y^2 = 16z$
(f) $7x^2 - 3y^2 + z = 0$
(g) $x^2 + y^2 + z^2 = 25$

5. In each part determine the translation equations that will put the quadric in standard position. Name the quadric.
(a) $9x^2 + 36y^2 + 4z^2 - 18x - 144y - 24z + 153 = 0$
(b) $6x^2 + 3y^2 - 2z^2 + 12x - 18y - 8z = -7$
(c) $3x^2 - 3y^2 - z^2 + 42x + 144 = 0$
(d) $4x^2 + 9y^2 - z^2 - 54y - 50z = 544$
(e) $x^2 + 16y^2 + 2x - 32y - 16z - 15 = 0$
(f) $7x^2 - 3y^2 + 126x + 72y + z + 135 = 0$
(g) $x^2 + y^2 + z^2 - 2x + 4y - 6z = 11$

6. In each part find a rotation $\mathbf{x} = P\mathbf{x}'$ that removes the cross-product terms. Name the quadric and give its equation in the $x'y'z'$-system.
(a) $2x^2 + 3y^2 + 23z^2 + 72xz + 150 = 0$
(b) $4x^2 + 4y^2 + 4z^2 + 4xy + 4xz + 4yz - 5 = 0$
(c) $144x^2 + 100y^2 + 81z^2 - 216xz - 540x - 720z = 0$
(d) $2xy + z = 0$

In Exercises 7–10 translate and rotate the coordinate axes to put the quadric in standard position. Name the quadric and give its equation in the final coordinate system.

7. $2xy + 2xz + 2yz - 6x - 6y - 4z = -9$

8. $7x^2 + 7y^2 + 10z^2 - 2xy - 4xz + 4yz - 12x + 12y + 60z = 24$

9. $2xy - 6x + 10y + z - 31 = 0$

10. $2x^2 + 2y^2 + 5z^2 - 4xy - 2xz + 2yz + 10x - 26y - 2z = 0$

11. Prove Theorem 9.7.1.

Svar till uppgifter

EXERCISE SET 9.7 (page 499)

1. (a) $x^2+2y^2-z^2+4xy-5yz$ (b) $3x^2+7z^2+2xy-3xz+4yz$
(c) $xy+xz+yz$ (d) $x^2+y^2-z^2$
(e) $3z^2+3xz$ (f) $2z^2+2xz+y^2$

2. (a) $\begin{bmatrix}1 & 2 & 0\\ 2 & 2 & -\frac{5}{2}\\ 0 & -\frac{5}{2} & -1\end{bmatrix}$ (b) $\begin{bmatrix}3 & 1 & -\frac{3}{2}\\ 1 & 0 & 2\\ -\frac{3}{2} & 2 & 7\end{bmatrix}$ (c) $\begin{bmatrix}0 & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & 0 & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2} & 0\end{bmatrix}$ (d) $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -1\end{bmatrix}$

(e) $\begin{bmatrix}0 & 0 & \frac{3}{2}\\ 0 & 0 & 0\\ \frac{3}{2} & 0 & 3\end{bmatrix}$ (f) $\begin{bmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 2\end{bmatrix}$

3. (a) $[x\ \ y\ \ z]\begin{bmatrix}1 & 2 & 0\\ 2 & 2 & -\frac{5}{2}\\ 0 & -\frac{5}{2} & -1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}+[7\ \ 0\ \ 2]\begin{bmatrix}x\\ y\\ z\end{bmatrix}-3=0$

(b) $[x\ \ y\ \ z]\begin{bmatrix}3 & 1 & -\frac{3}{2}\\ 1 & 0 & 2\\ -\frac{3}{2} & 2 & 7\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}+[-3\ \ 0\ \ 0]\begin{bmatrix}x\\ y\\ z\end{bmatrix}-4=0$

(c) $[x\ \ y\ \ z]\begin{bmatrix}0 & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & 0 & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2} & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}-1=0$ (d) $[x\ \ y\ \ z]\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}-7=0$

(e) $[x\ \ y\ \ z]\begin{bmatrix}0 & 0 & \frac{3}{2}\\ 0 & 0 & 0\\ \frac{3}{2} & 0 & 3\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}+[0\ \ -14\ \ 0]\begin{bmatrix}x\\ y\\ z\end{bmatrix}+9=0$

(f) $[x\ \ y\ \ z]\begin{bmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}+[2\ \ -1\ \ 3]\begin{bmatrix}x\\ y\\ z\end{bmatrix}=0$

4. (a) Ellipsoid
(b) Hyperboloid of one sheet
(c) Hyperboloid of two sheets
(d) Elliptic cone
(e) Elliptic paraboloid
(f) Hyperbolic paraboloid
(g) Sphere

5. (a) $9x'^2+36y'^2+4z'^2=36$, ellipsoid
(b) $6x'^2+3y'^2-2z'^2=18$, hyperboloid of one sheet
(c) $3x'^2-3y'^2-z'^2=3$, hyperboloid of two sheets
(d) $4x'^2+9y'^2-z'^2=0$, elliptic cone
(e) $x'^2+16y'^2-16z'=0$, elliptic paraboloid
(f) $7x'^2-3y'^2+z'=0$, hyperbolic paraboloid
(g) $x'^2+y'^2+z'^2=25$, sphere

6. (a) $25x'^2-3y'^2-50z'^2-150=0$, hyperboloid of two sheets
(b) $2x'^2+2y'^2+8z'^2-5=0$, ellipsoid
(c) $9x'^2+4y'^2-36z'=0$, elliptic paraboloid
(d) $x'^2-y'^2+z'=0$, hyperbolic paraboloid

7. $x''^2+y''^2-2z''^2=-1$, hyperboloid of two sheets **8.** $x''^2+y''^2+2z''^2=4$, ellipsoid

9. $x''^2-y''^2+z''=0$, hyperbolic paraboloid **10.** $6x''^2+3y''^2-8\sqrt{2}z''=0$, elliptic paraboloid