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Turbulence models for CFD

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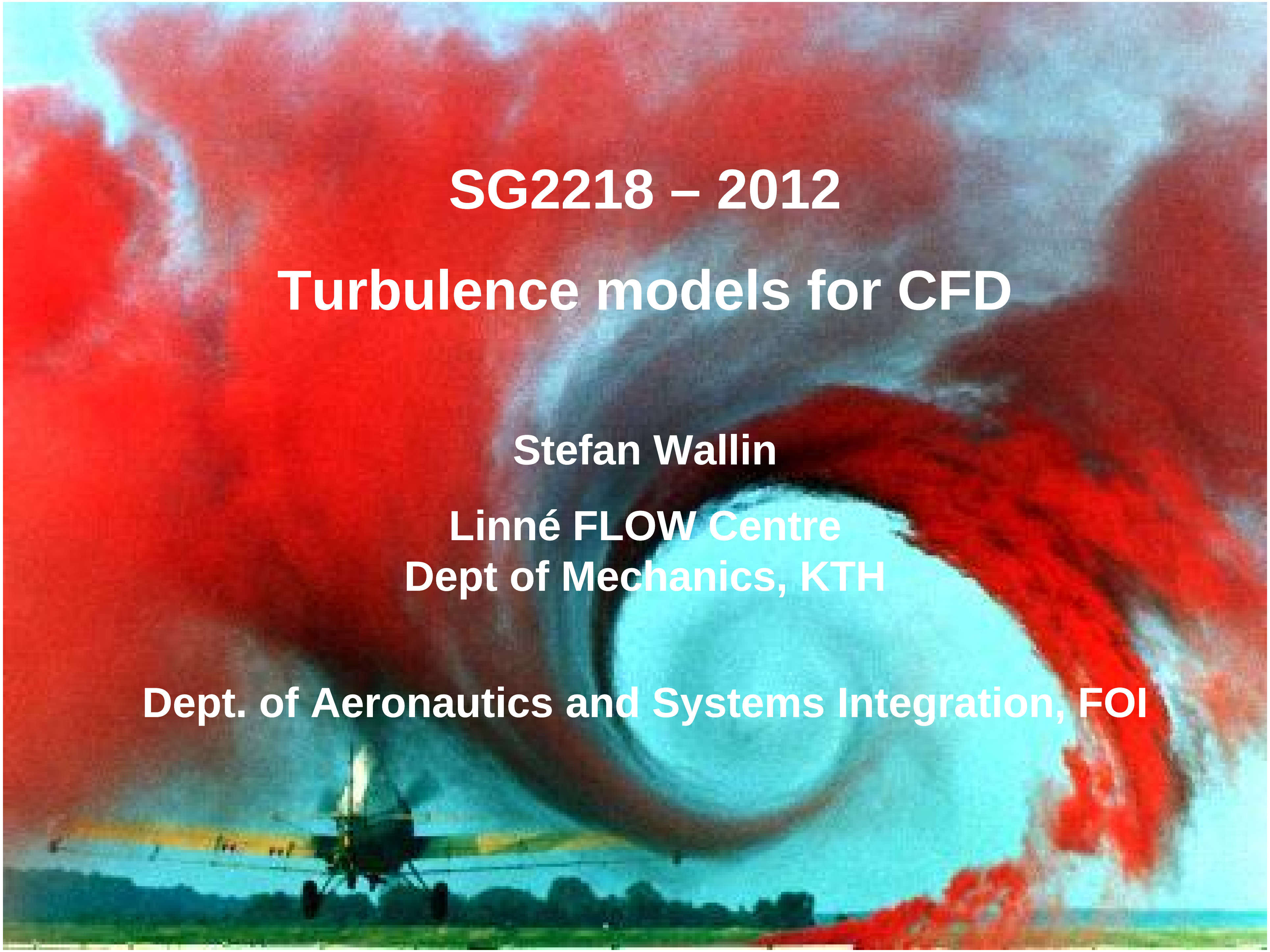


Last time

- **Boussinesq hypothesis (eddy-viscosity assumption)**

$$\overline{u'v'} = -\nu_T \frac{dU}{dy} \qquad a_{ij} = -2\frac{\nu_T}{K} S_{ij}$$

  - **Problems: history effects, alignment, rotation**

- **Eddy-viscosity models**
  - **Algebraic (zero-equation models)**
  - **One-equation models (Spalart-Allmaras)**
  - **Two-equation models (std k–ε, Wilcox k–ω, Menter SST k–ω)**

The $K - \varepsilon$ model

$$\frac{DK}{Dt} = \mathcal{P} - \varepsilon + \frac{\partial}{\partial x_k}\left[\left(\nu + \frac{\nu_T}{\sigma_K}\right)\frac{\partial K}{\partial x_k}\right]$$

$$\frac{D\varepsilon}{Dt} = (C_{\varepsilon 1}\mathcal{P} - C_{\varepsilon 2}\varepsilon)\frac{\varepsilon}{K} + \frac{\partial}{\partial x_k}\left[\left(\nu + \frac{\nu_T}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_k}\right]$$

$$\mathcal{P} = 2\nu_T S_{ij} S_{ji} \qquad \nu_T = C_\mu \frac{K^2}{\varepsilon}$$

Model coefficients (standard values):

$$C_\mu = 0.09\,,\ C_{\varepsilon 1} = 1.44\,,\ C_{\varepsilon 2} = 1.92\,,\ \sigma_K = 1.0\,,\ \sigma_\varepsilon = 1.3$$

The $K - \omega$ model

Kolmogorov (1942), Wilcox (80:s and 90:s)

ω is intepreted as the inverse time scale of the large eddies, and the $K - \omega$ model reads

$$\frac{DK}{Dt} = 2\nu_T S_{ij} S_{ji} - C_\mu K\omega + \frac{\partial}{\partial x_k}\left[\left(\nu + \frac{\nu_T}{\sigma_K}\right)\frac{\partial K}{\partial x_k}\right]$$

$$\frac{D\omega}{Dt} = 2\alpha S_{ij} S_{ji} - \beta\omega^2 + \frac{\partial}{\partial x_k}\left[\left(\nu + \frac{\nu_T}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_k}\right]$$

$$\nu_T = \frac{K}{\omega}$$

The model coefficients proposed by Wilcox (1988) are

$$C_\mu = 0.09, \; \alpha = 5/9 \approx 0.56, \; \beta = 0.075, \; \sigma_K = 2.0, \; \sigma_\omega = 2.0$$

The $K - \omega$ model – problems

Unphysical influence of free stream conditions

- **Calibration of the Schmidt numbers**
- **Introducing a “cross diffusion term”**
- **Such modifications have been proposed by**
  - **Menter SST (1993)**
  - **Kok (1999)**

Boundary conditions

The $K - \varepsilon$ model is singular at the wall

- **Log-law boundary conditions**
  - **Not strictly valid in separated flows**
- **Near-wall (low-Reynolds number) corrections**
  - **Wall damping functions based on**

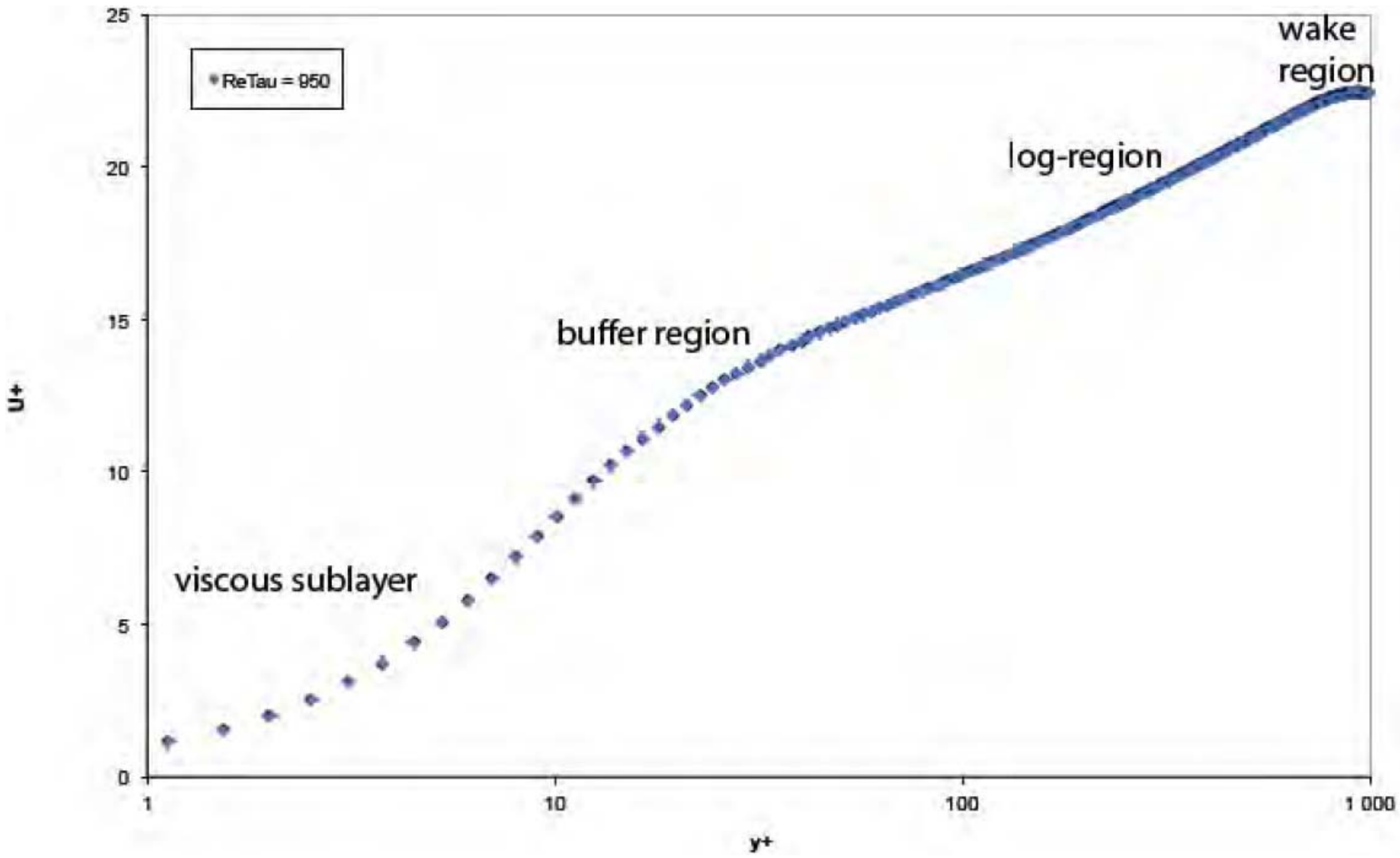
$$y^+ \equiv y u_\tau / \nu \,,\; y^* \equiv y\sqrt{K}/\nu \text{ or } Re_T \equiv K^2/\nu\varepsilon$$

  - **Active up to** $y^+ \sim 50$
  - **Near-wall grid size** $\Delta y^+ \approx 1$

The $K - \omega$ model

- **No such problems – can be integrated to the wall**
- **Near-wall grid size** $\Delta y^+ \approx 1$

Turbulent boundary layer



Modelling of production

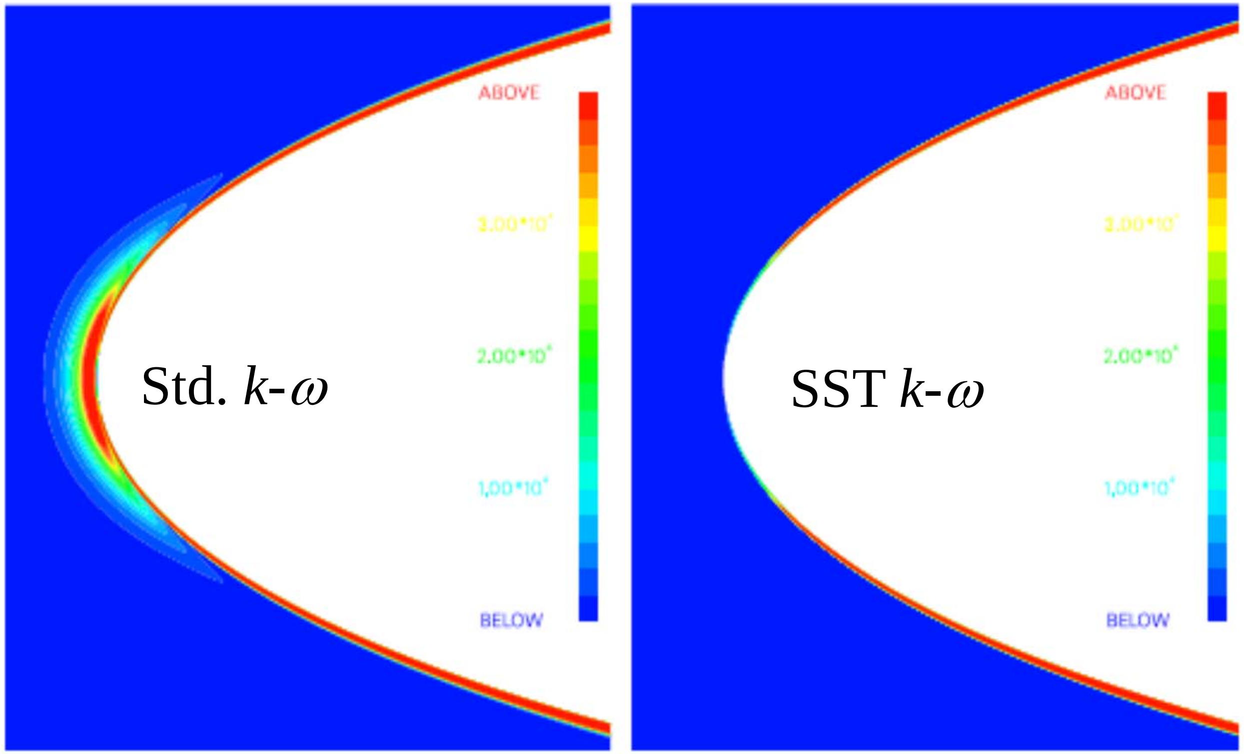
Exact: $\mathcal{P} = -\overline{u_i u_k}\frac{\partial U_i}{\partial x_k}$ **Model:** $\mathcal{P} = -2\nu_T S_{ij} S_{ij}$

- **Strain rate dependency**
  - **Exact – linear dependency:** $\mathcal{P} \sim S$ $S \sim \partial U / \partial y$
  - **Model – quadratic dependency:** $\mathcal{P} \sim S^2$
- **What is the consequence**
  - **No problem in equilibrium flows**
  - **Stagnation flows: turbulence overpredicted -> e.g. heat transfer in stagnation regions**
  - **Separated flows: turbulence overpredicted -> separation size typically underpredicted**
- **How to improve: Menter SST**
  - **Limit turbulence viscosity** $\nu_T = \frac{a_1 K}{\max(a_1 \omega, S)}$

Example – stagnation flow

Flow around a wing profile – leading edge

- Turbulence kinetic energy shown
- Std. Eddy-viscosity models – excessive production of *K*
- Cured by ”SST”



Rotation and flow curvature

- **Eddy-viscosity model e.g. std K-eps model**
- **Dependent on** S_{ij}
  - **Symmetric part of velocity gradient**
  - **Invariant of rotation**
- **No dependence on** Ω_{ij}

$$\frac{DU_i}{Dt} = -\frac{\partial}{\partial x_i}\left(\frac{P}{\rho} + \frac{2}{3}K\right) + \frac{\partial}{\partial x_k}[2(\nu + \nu_T)S_{ij}]$$

$$\frac{DK}{Dt} = \mathcal{P} - \varepsilon + \frac{\partial}{\partial x_k}\left[\left(\nu + \frac{\nu_T}{\sigma_K}\right)\frac{\partial K}{\partial x_k}\right]$$

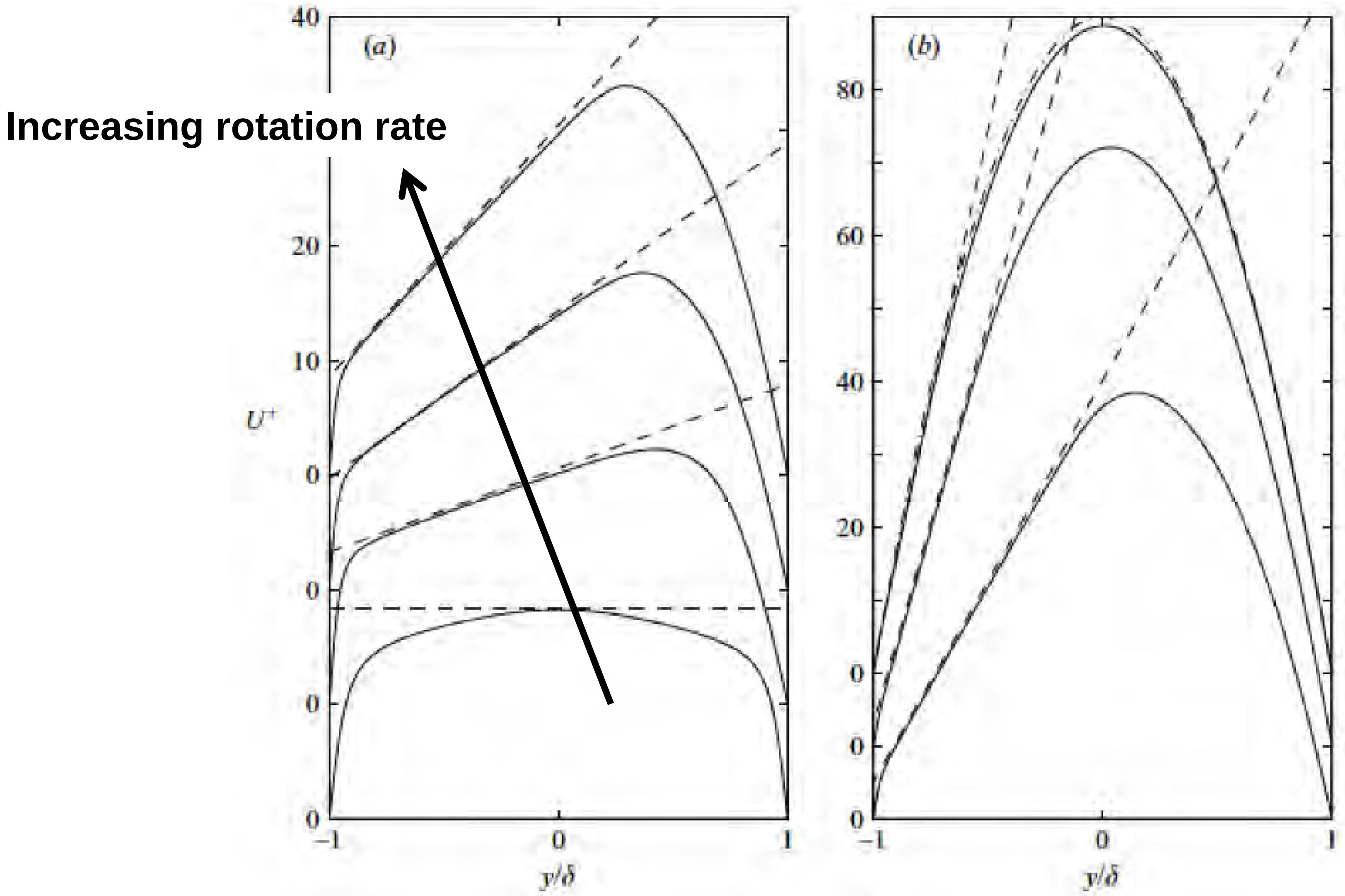
$$\frac{D\varepsilon}{Dt} = (C_{\varepsilon 1}\mathcal{P} - C_{\varepsilon 2}\varepsilon)\frac{\varepsilon}{K} + \frac{\partial}{\partial x_k}\left[\left(\nu + \frac{\nu_T}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_k}\right]$$

$$\mathcal{P} = 2\nu_T S_{ij} S_{ji} \qquad \nu_T = C_\mu \frac{K^2}{\varepsilon}$$

Thus:

- **No model influence on rotation, swirl, or flow curvature**
  - **But turbulence is very dependent**
- **Empirical “fixes”: Different rotation corrections**

Rotating turbulent channel



Milestone – Eddy-viscosity models (EVM)

- 0- and 1-eq models – incomplete (additional information needed)
- 2-eq models (K-ε)
  - Based on N-S equations
  - Model coefficients by calibration/analysis of generic flows
- Popular eddy-viscosity models (EVM):
  - Spalart-Allmaras 1-eq model
  - Menter SST K-ω model
- Good for:
  - Attached thin boundary layers
  - Mainly 2D flows
- EVMs in general not good for:
  - Non-equilibrium flow
  - Swirl, rotation and flow curvature
  - Boundary layer separation
- There are fixes …
- A better way is to get rid of the eddy-viscosity assumption

–> Reynolds stress models

Reynolds stress models (RST or DRSM)

- **Reynolds stress equation**

$$\frac{\mathrm{D}\,\overline{u_i' u_j'}}{\mathrm{D}t} = \mathcal{P}_{ij} + \Pi_{ij} - \varepsilon_{ij} - \frac{\partial T_{ijk}}{\partial x_k}$$

- **Advection by the mean flow (exact) = Transported by the mean flow**

$$\mathrm{D}/\mathrm{D}t \equiv \partial/\partial t + U_k \partial/\partial x_k$$

- **Production (exact) = of energy, taken from mean flow**

$$\mathcal{P}_{ij} \equiv -\overline{u_j' u_k'}\frac{\partial U_i}{\partial x_k} - \overline{u_i' u_k'}\frac{\partial U_j}{\partial x_k}$$

or

$$\mathcal{P}_{ij} = -K\left(\frac{4}{3}S_{ij} + (a_{ik}S_{kj} + S_{ik}a_{kj}) - (a_{ik}\Omega_{kj} - \Omega_{ik}a_{kj})\right)$$

Reynolds stress models …

- **Pressure-strain rate (model) = Redistribution among components**

$$\frac{\Pi_{ij}}{\varepsilon} = -\left(C_1^0 + C_1^1\frac{\mathcal{P}}{\varepsilon}\right)a_{ij} + C_2 S_{ij}^*$$
$$+C_3\left(a_{ik}S_{kj}^* + S_{ik}^* a_{kj} - \frac{2}{3}a_{kl}S_{lk}^*\delta_{ij}\right) - C_4\left(a_{ik}\Omega_{kj}^* - \Omega_{ik}^* a_{kj}\right)$$

  - **LRR: Launder, Reece & Rodi (1975)**
  - **SSG: Speziale, Sarkar & Gatski (1991)**

- **Dissipation rate (isotropic) = Viscous dissipation into heat**

$$\varepsilon_{ij} = \frac{2}{3}\varepsilon\delta_{ij}$$

  - **Plus equation for** ε

- **Turbulent flux = Redistribution in space**
  - **Gradient diffusion** $$T_{ijk} = -\left(\nu + \frac{\nu_T}{\sigma_K}\right)\frac{\partial \overline{u_i' u_j'}}{\partial x_k}$$
  - **Daly & Harlow (GGD)** $$T_{ijk} = -\left(\nu\delta_{kl} + c_s\frac{K}{\varepsilon}\overline{u_k' u_l'}\right)\frac{\partial \overline{u_i' u_j'}}{\partial x_l}$$