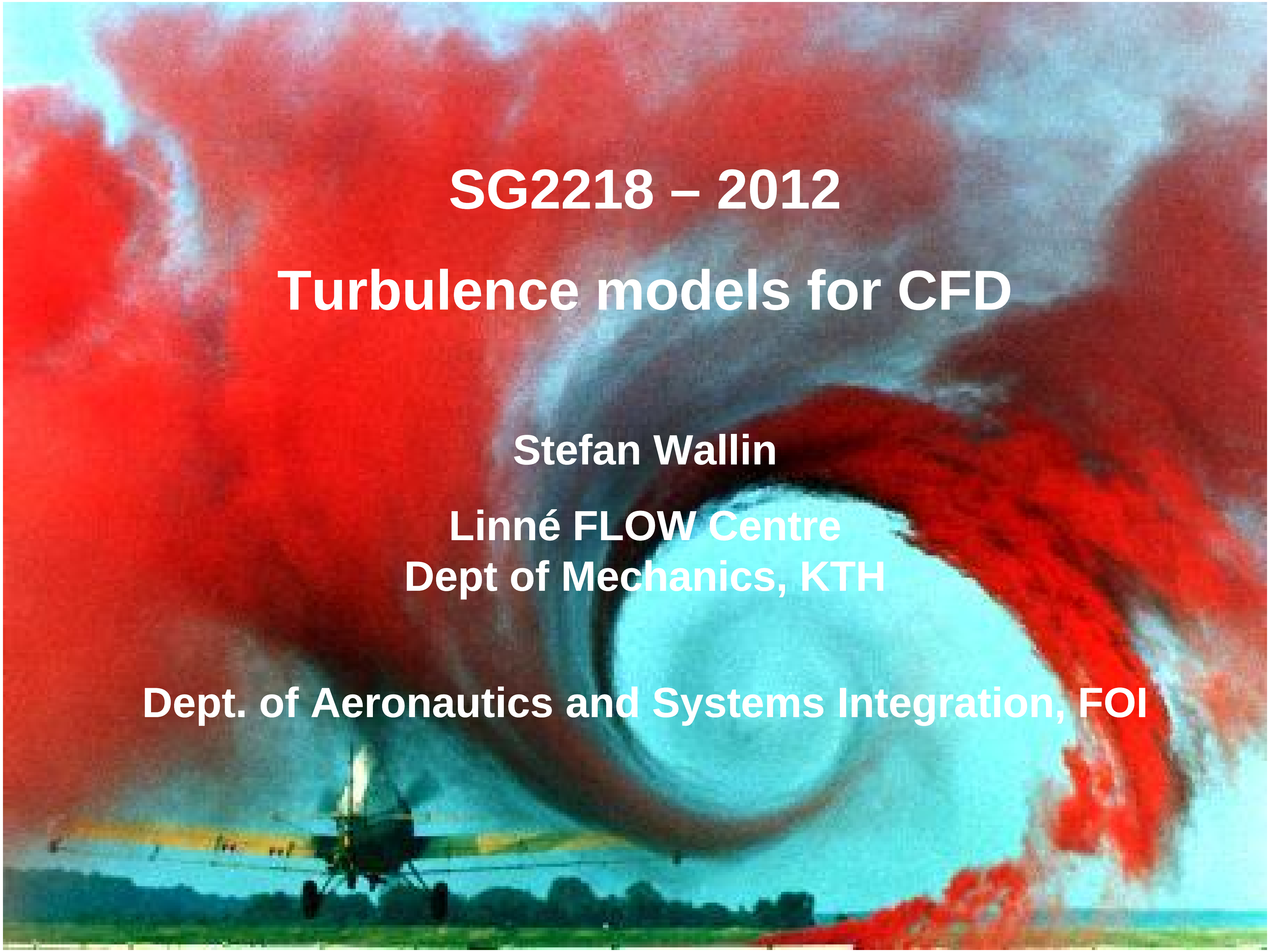

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Turbulence models for CFD

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Last times

- **RANS models: RANS equation**

$$\frac{\mathrm{D}\,U_i}{\mathrm{D}t} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\nu\frac{\partial U_i}{\partial x_j} - \overline{u_i' u_j'}\right)$$

- **Eddy-viscosity models (Boussinesq)**

$$\overline{u_i' u_j'} - \frac{2}{3}K\delta_{ij} = -\nu_T\frac{\partial U_i}{\partial x_j} = -2\nu_T S_{ij} \qquad a_{ij} = -2\frac{\nu_T}{K}S_{ij}$$

  – **Two-equation models (std *k*–*ε*, Wilcox *k*–*ω*, Menter SST *k*–*ω*)**

$$\nu_T = C_\mu\frac{K^2}{\varepsilon} \qquad \nu_T = \frac{K}{\omega}$$

- **Reynolds stress models**

$$\frac{\mathrm{D}\,\overline{u_i' u_j'}}{\mathrm{D}t} = \mathcal{P}_{ij} + \Pi_{ij} - \varepsilon_{ij} - \frac{\partial T_{ijk}}{\partial x_k}$$

  – **Plus equation for *ε***

Differential Reynolds stress models (DRSM, RST)

- **Transport equation for $\overline{u_i u_j}$**

$$\frac{\mathrm{D}\overline{u_i u_j}}{\mathrm{D}t} + \mathcal{D}_{ij} = \mathcal{P}_{ij} - \varepsilon_{ij} - \Pi_{ij}$$

- **Transport equation for a_{ij}**

$$\frac{K}{\varepsilon}\left(\frac{\mathrm{D}a_{ij}}{\mathrm{D}t} + \mathcal{D}_{ij}^{(\mathrm{a})}\right) = -\left(a_{ij} + \frac{2}{3}\delta_{ij}\right)\left(\frac{\mathcal{P}}{\varepsilon} - 1\right) + \frac{\mathcal{P}_{ij} - \varepsilon_{ij} - \Pi_{ij}}{\varepsilon}$$

Algebraic Reynolds stress models

- **Transport equation for $\overline{u_i u_j}$**

$$\frac{\mathrm{D}\overline{u_i u_j}}{\mathrm{D}t} + \mathcal{D}_{ij} = \mathcal{P}_{ij} - \varepsilon_{ij} - \Pi_{ij}$$

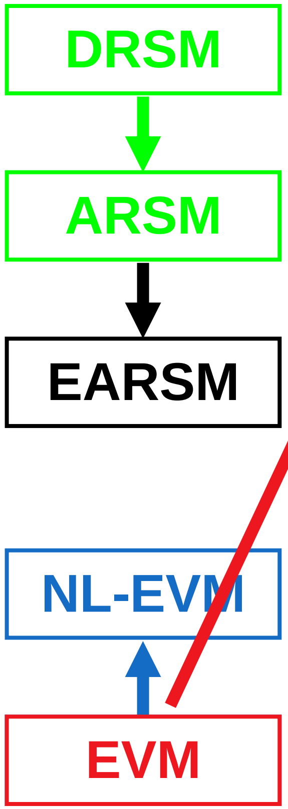
- **Transport equation for a_{ij}**

$$\frac{K}{\varepsilon}\left(\frac{\mathrm{D}a_{ij}}{\mathrm{D}t} + \mathcal{D}_{ij}^{(\mathrm{a})}\right) = -\left(a_{ij} + \frac{2}{3}\delta_{ij}\right)\left(\frac{\mathcal{P}}{\varepsilon} - 1\right) + \frac{\mathcal{P}_{ij} - \varepsilon_{ij} - \Pi_{ij}}{\varepsilon}$$

- **Weak-equilibrium assumption**
  - a_{ij} **constant in time and space**
- **Implicit relation in a_{ij}**

$$0 = -\left(a_{ij} + \frac{2}{3}\delta_{ij}\right)\left(\frac{\mathcal{P}}{\varepsilon} - 1\right) + \frac{\mathcal{P}_{ij} - \varepsilon_{ij} - \Pi_{ij}}{\varepsilon}$$

Explicit algebraic Reynolds stress models



- **Explicit solution for a_{ij}**

$$a_{ij} = \beta_1 S_{ij} + \beta_4 \left(S_{ik}\Omega_{kj} - \Omega_{ik}S_{kj} \right) + \ldots \qquad \text{5-10 terms in 3D}$$

$$\beta_\alpha = \beta_\alpha(\text{invariants of } S_{ij} \text{ and } \Omega_{ij})$$

- **Solved together with transport equations for K and ω**

$$\frac{\mathrm{D}K}{\mathrm{D}t} + \mathcal{D} = \mathcal{P} - \varepsilon$$

$$\frac{\mathrm{D}\omega}{\mathrm{D}t} + \mathcal{D} = \gamma \frac{\omega}{K} \mathcal{P} - \omega^2$$

- **Replaces the eddy-viscosity assumption**
  - **constant β_1**
  - **Insensitive of rotation (global & local)**

$$\mathcal{P} = -\varepsilon a_{ij} S_{ji}$$

Non-linear eddy-viscosity models

- **Explicit solution for a_{ij}**

$$a_{ij} = \beta_1 S_{ij} + \beta_4 \left(S_{ik}\Omega_{kj} - \Omega_{ik}S_{kj} \right) + \ldots$$

$$\beta_\alpha = \beta_\alpha(\text{invariants of } S_{ij} \text{ and } \Omega_{ij})$$

- **Solved together with transport equations for K and ω**

$$\frac{\mathrm{D}K}{\mathrm{D}t} + \mathcal{D} = \mathcal{P} - \varepsilon$$

$$\frac{\mathrm{D}\omega}{\mathrm{D}t} + \mathcal{D} = \gamma \frac{\omega}{K}\mathcal{P} - \omega^2$$

- **Non-linear eddy-viscosity models**
  - **similar form**
  - **Not related to DRSM**

Most general EARSM in 3D flows

$$
\begin{aligned}
a_{ij} = {} & \beta_1 S_{ij} \\
& + \beta_2 \left(S_{ik} S_{kj} - II_S \delta_{ij}/3 \right) \\
& + \beta_3 \left(\Omega_{ik} \Omega_{kj} - II_\Omega \delta_{ij}/3 \right) + \beta_4 \left(S_{ik} \Omega_{kj} - \Omega_{ik} S_{kj} \right) \\
& + \beta_5 \left(S_{ik} S_{kl} \Omega_{lj} - \Omega_{ik} S_{kl} S_{lj} \right) \\
& + \beta_6 \left(S_{ik} \Omega_{kl} \Omega_{lj} + \Omega_{ik} \Omega_{kl} S_{lj} - IV \delta_{ij}/3 \right) \\
& + \beta_7 \left(S_{ik} S_{kl} \Omega_{lp} \Omega_{pj} + \Omega_{ik} \Omega_{kl} S_{lp} S_{pj} - 2V \delta_{ij}/3 \right) \\
& + \beta_8 \left(S_{ik} \Omega_{kl} S_{lp} S_{pj} - S_{ik} S_{kl} \Omega_{lp} S_{pj} \right) \\
& + \beta_9 \left(\Omega_{ik} S_{kl} \Omega_{lp} \Omega_{pj} - \Omega_{ik} \Omega_{kl} S_{lp} \Omega_{pj} \right) \\
& + \beta_{10} \left(\Omega_{ik} S_{kl} S_{lp} \Omega_{pq} \Omega_{qj} - \Omega_{ik} \Omega_{kl} S_{lp} S_{pq} \Omega_{qj} \right)
\end{aligned}
$$

$$
II_S = S_{kl} S_{lk} \qquad II_\Omega = \Omega_{kl} \Omega_{lk}
$$

$$
III_S = S_{kl} S_{lm} S_{mk} \qquad IV = S_{kl} \Omega_{lm} \Omega_{mk} \qquad V = S_{kl} S_{lm} \Omega_{mn} \Omega_{nk}
$$

Explicit algebraic Reynolds stress models

- Model behaviour close to full DRSM
  - production of turbulence
  - non-equilibrium behaviour
  - rotation & curvature
- Convergence and CPU cost close to std. two-eq. models (e.g. K - ω)
- History of EARSM:
  - Pope (1975) – First outline of 3D EARSM, 2D impl. EARSM
  - Taulbee (1992) – 3D solution of simplified RSM
  - Gatski & Speziale (1993) – 3D EARSM
  - Girimaji (1995) – 2D self-consistent EARSM
  - Wallin & Johansson (1996, 2000) – self-consistent EARSM
  - Hellsten (2005) – omega model for EARSM
  - Menter et al. (2009) – WJ-EARSM + Menter BSL
    - Available in Ansys CFX, currently implemented in Fluent
- Used in (aeronautic) industry (Airbus, SAAB, Alenia, Dassault)

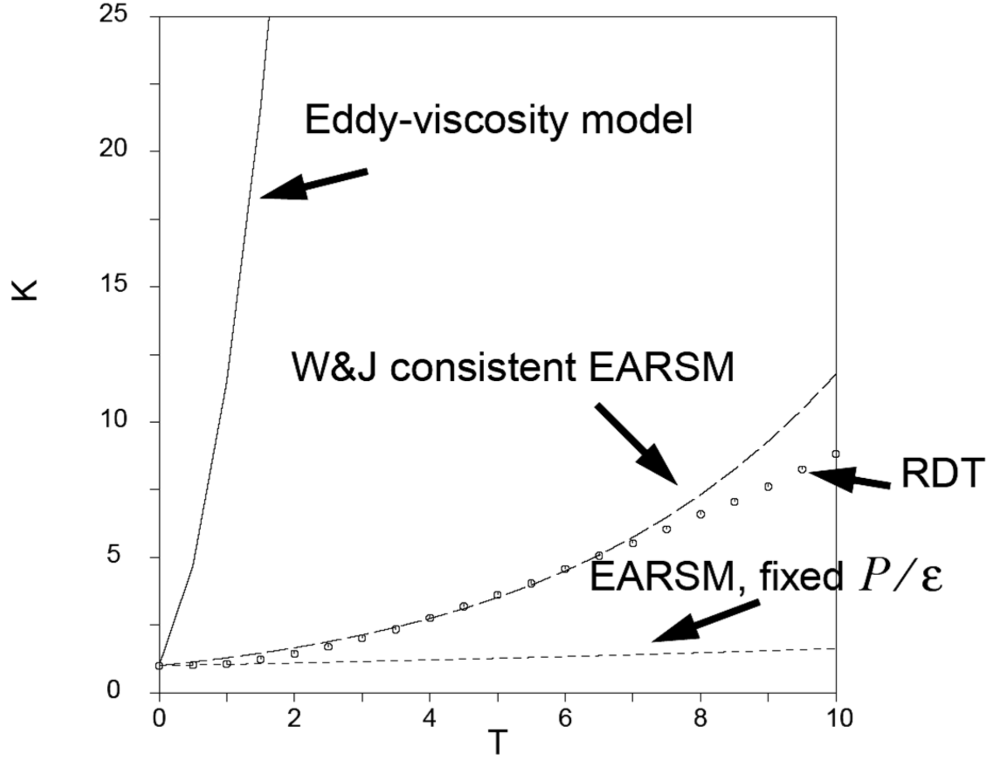
Model prediction of anisotropy

Boundary layers (log layer)	a_{12}	a_{11}	a_{22}	a_{33}
Moser, Kim & Mansour (1998)	-0.29	0.34	-0.26	-0.08
EARSM	-0.30	0.25	-0.25	0.00
std eddy-viscosity model	-0.30	0	0	0

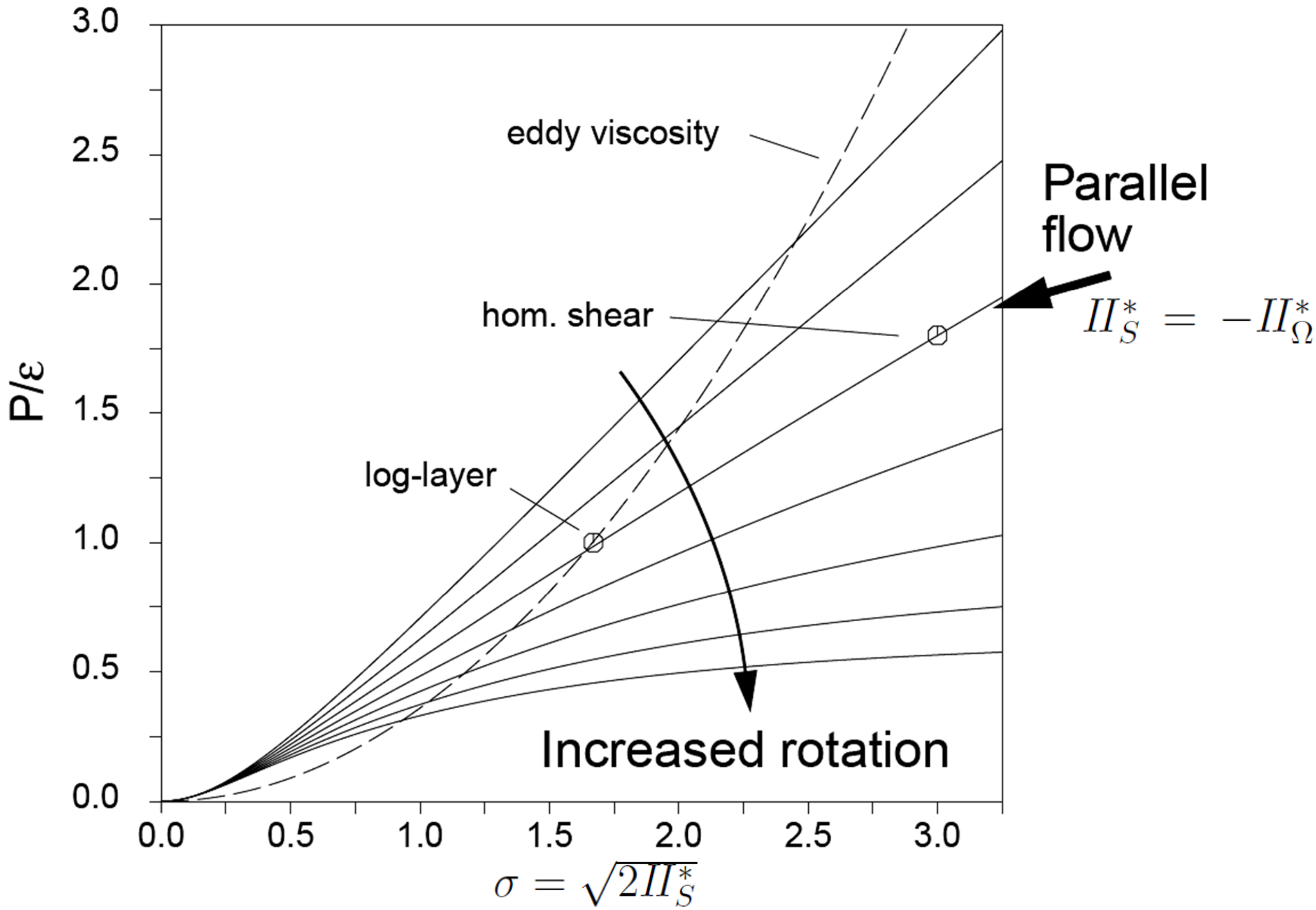
Equilibrium homogeneous shear flow	a_{12}	a_{11}	a_{22}	a_{33}
Tavoularis & Corrsin (1981) expr.	-0.30	0.40	-0.28	-0.12
EARSM	-0.30	0.31	-0.31	0.00
std eddy-viscosity model	-0.40	0	0	0
Menter SST $K - \omega$ model	-0.30	0	0	0

Self consistency

- **Self consistency = no approximations from ARSM to EARSM**
  - **Fulfilled in 2D flows**
- **Example:**
  - **Rapidly sheared homogeneous shear flow** $SK/\varepsilon = 50$
  - **Development of turbulent kinetic energy in time**



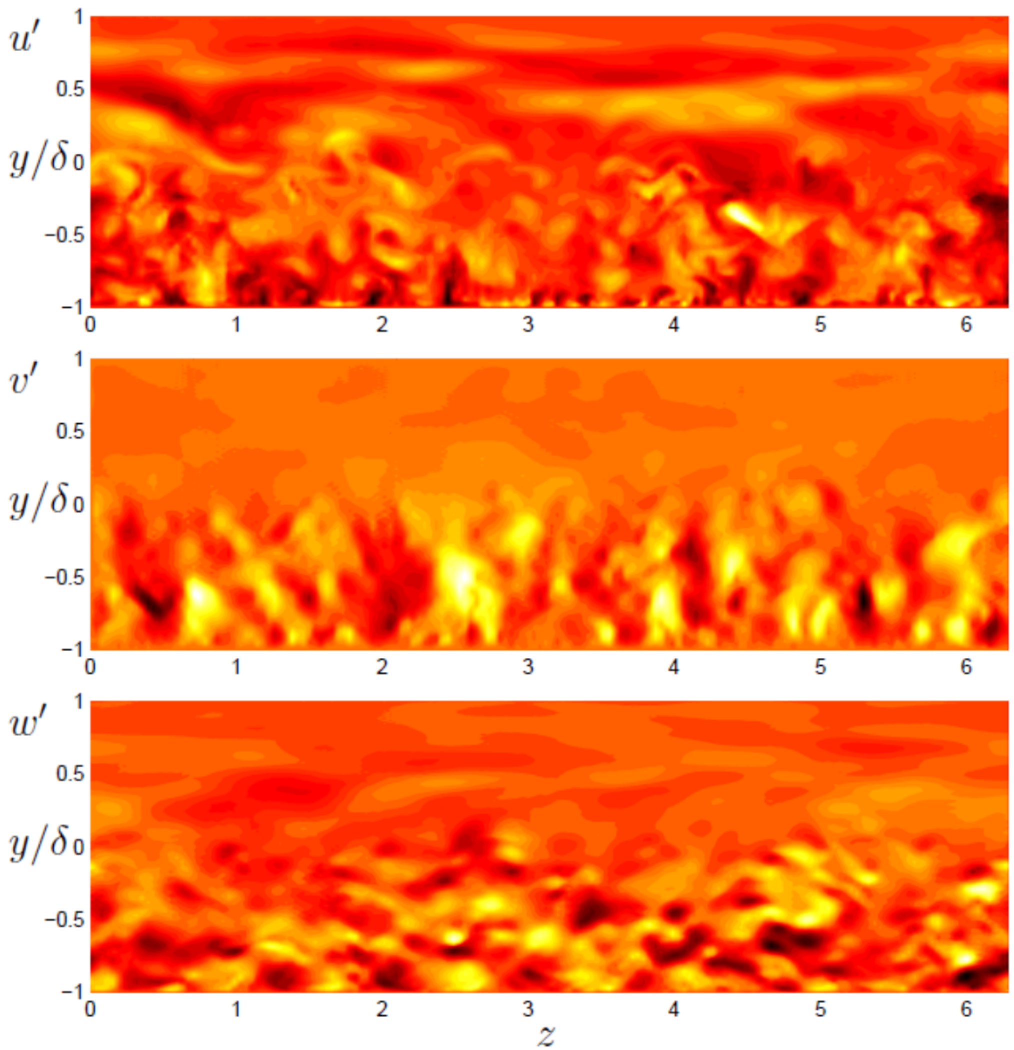
Production of *K*

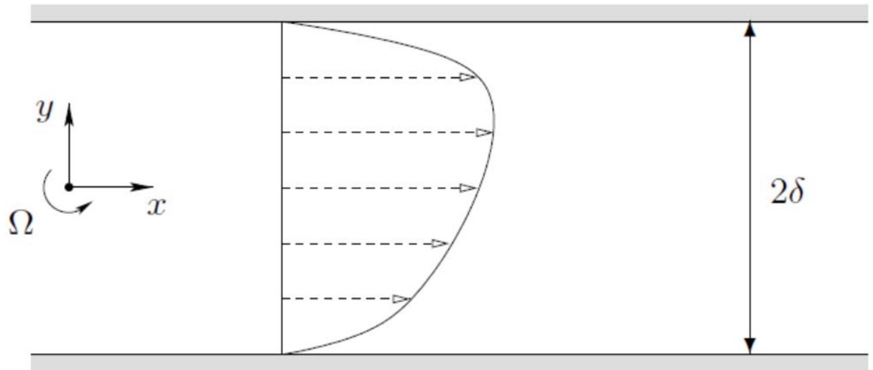


Rotating plane channel flow

- **Upper side stabilized by Coriolis forces**
- **Lower side destabilized**
- **Asymmetric velocity profile**
- **Rapid rotation**
  - **Upper side laminarized**

DNS by Grundestam et al. (2007)

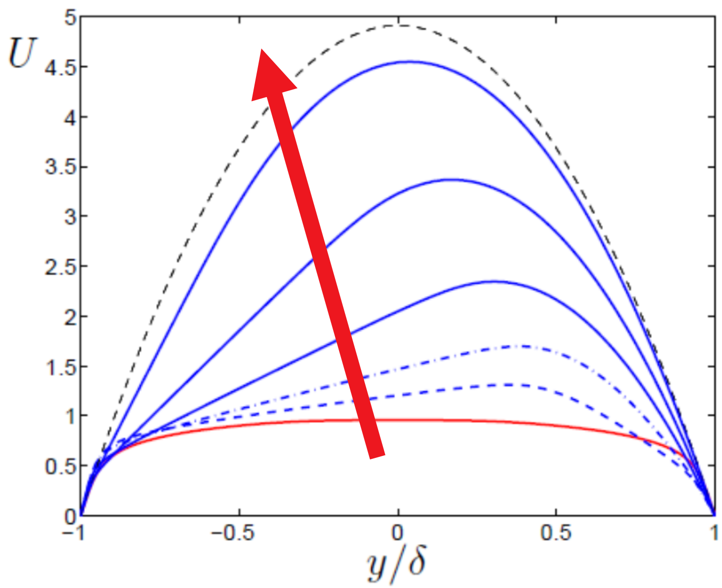




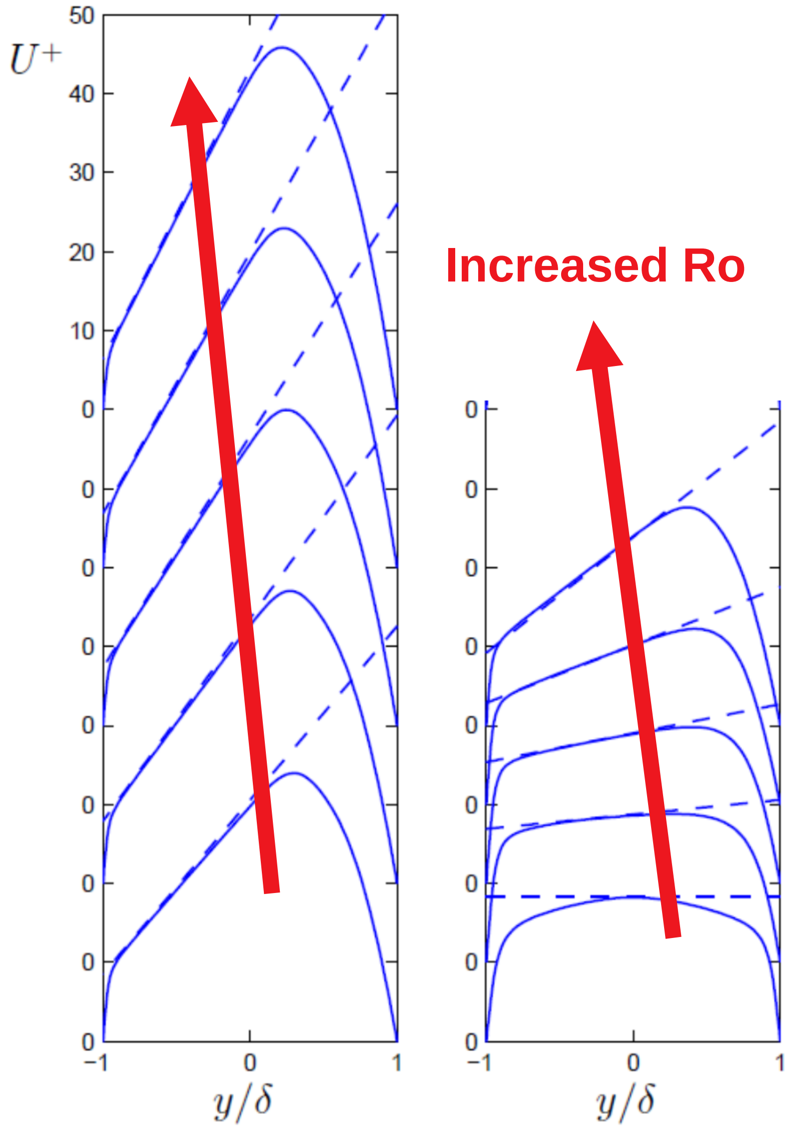
Rotating channel flow – RANS models

- DRSM and EARSM (blue curves)
  - Correct Ro effects
  - EARSM need additional Ro fix(!)
- EVM (red curve)
  - No rotational effects at all

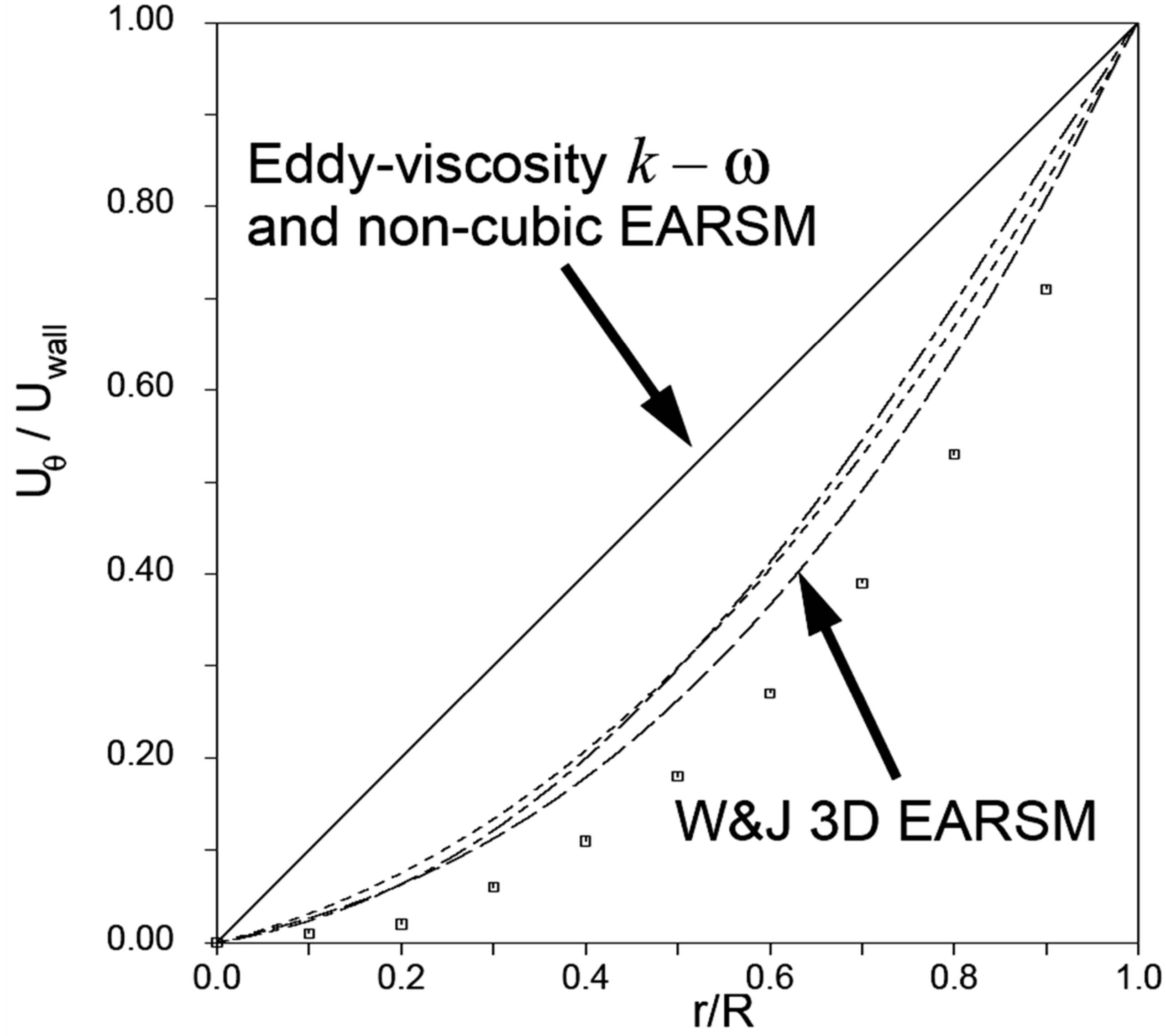
RANS model



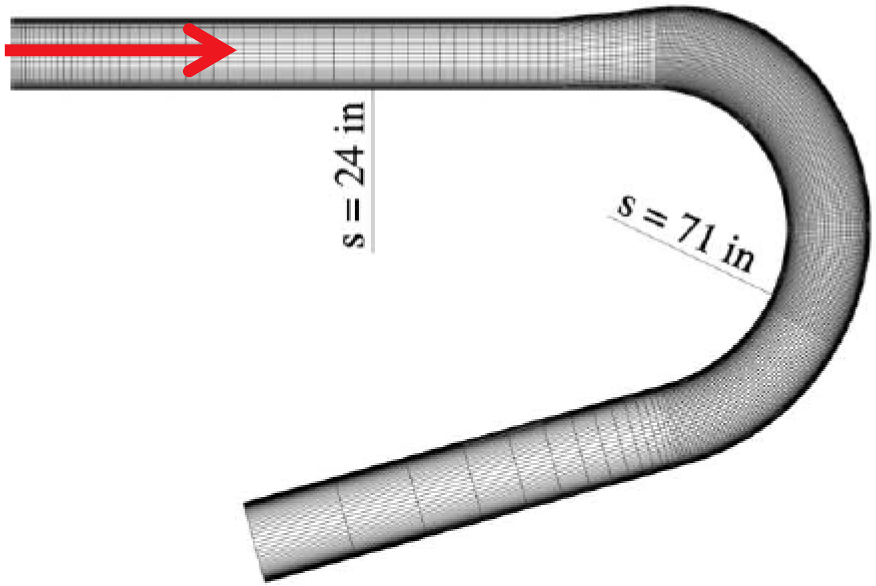
DNS

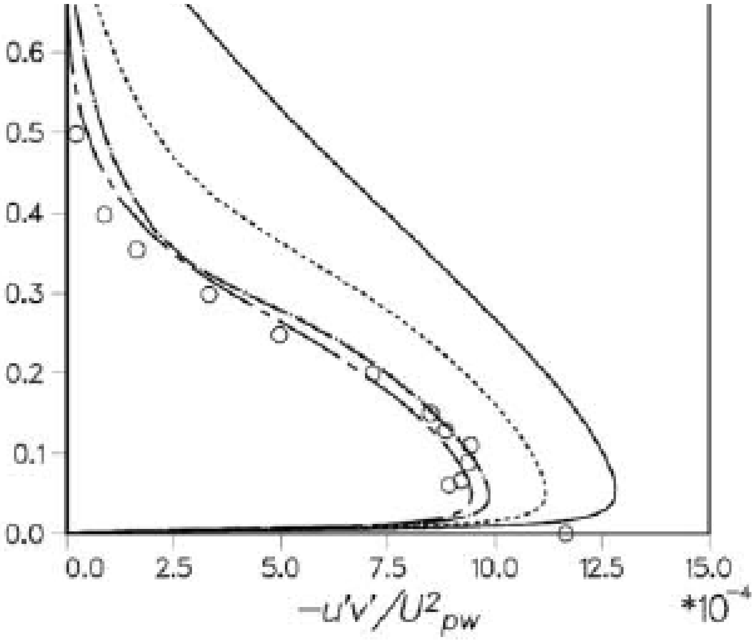


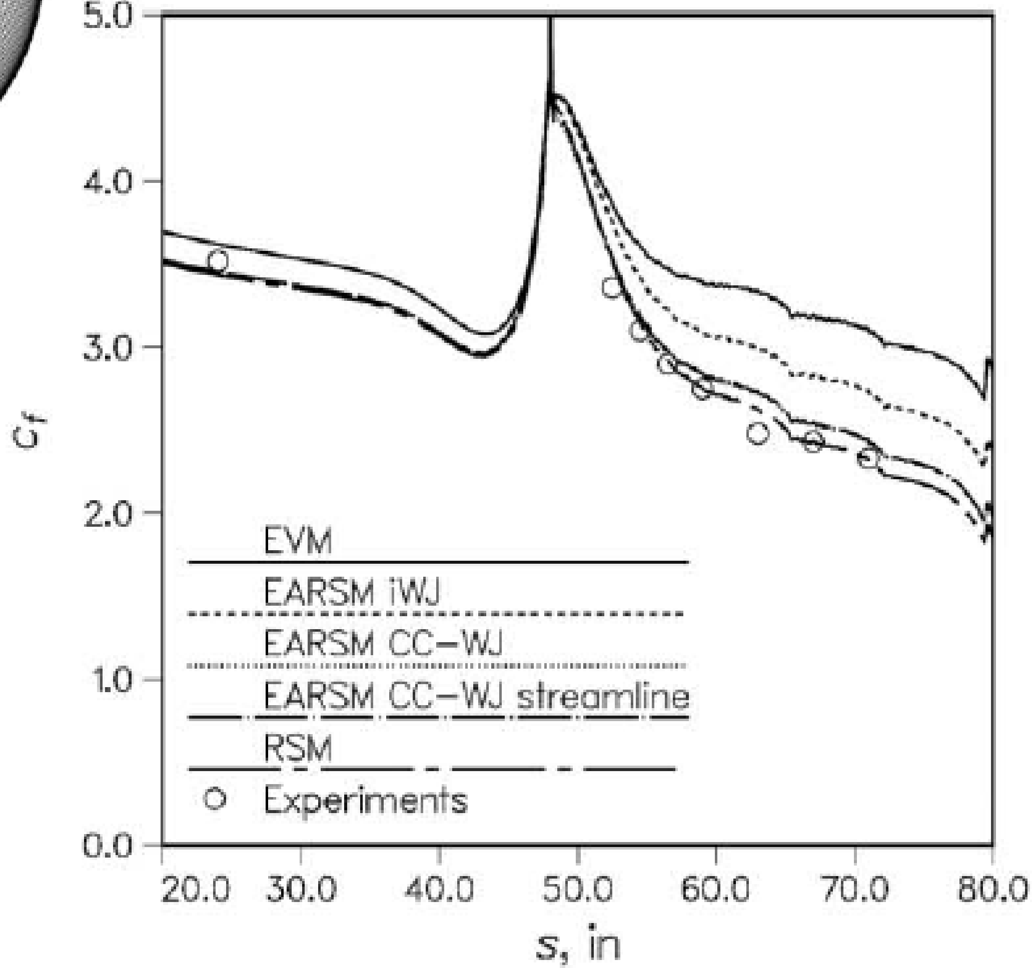
Rotating pipe (3D): Swirl velocity



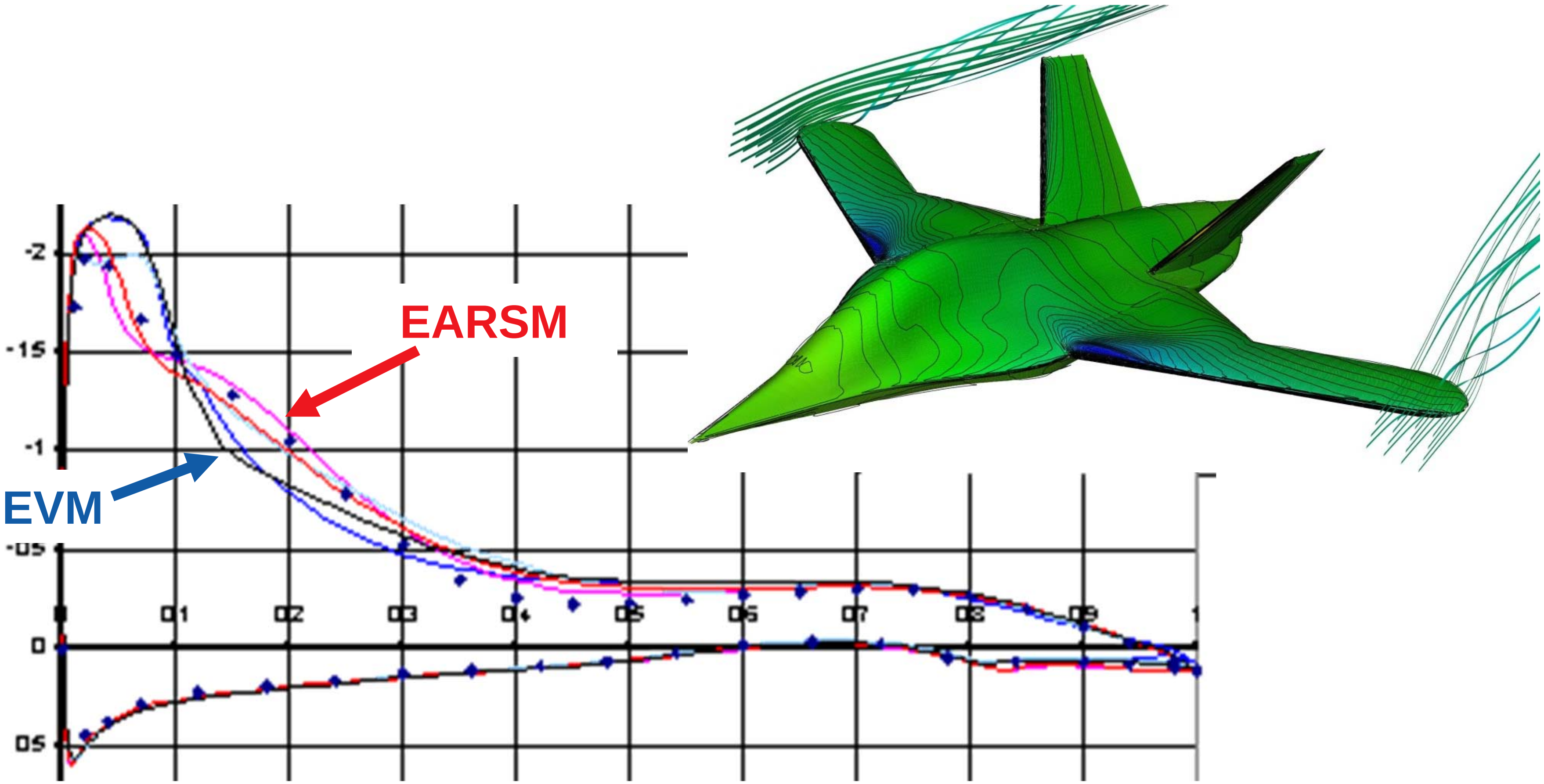
Curved channel flow



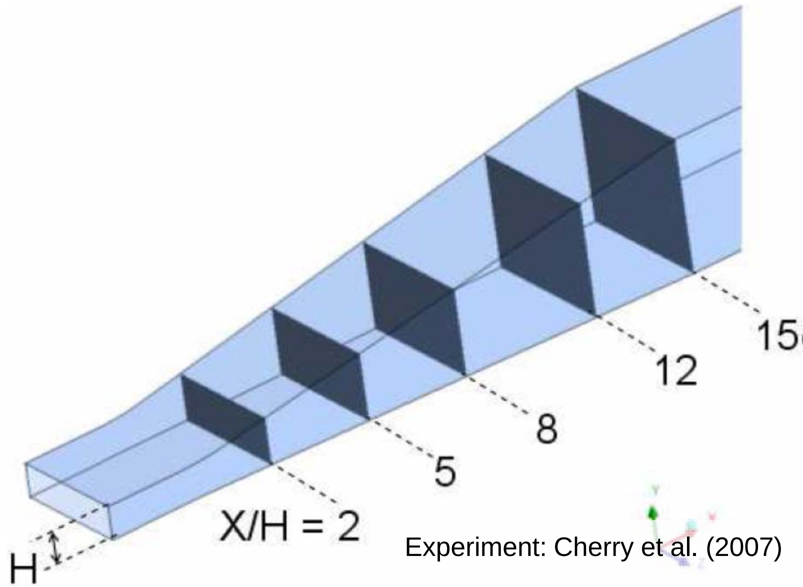




Flying vehicles at high angle of attack

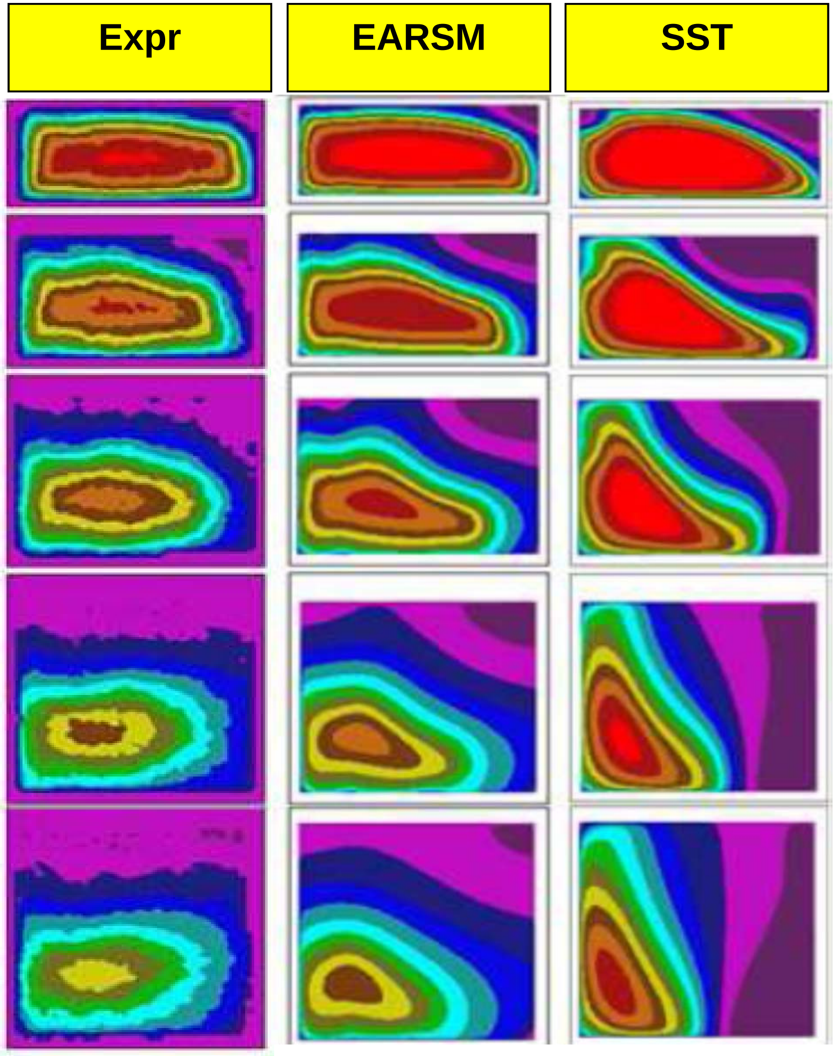


3D diffuser - results



Experiment: Cherry et al. (2007)

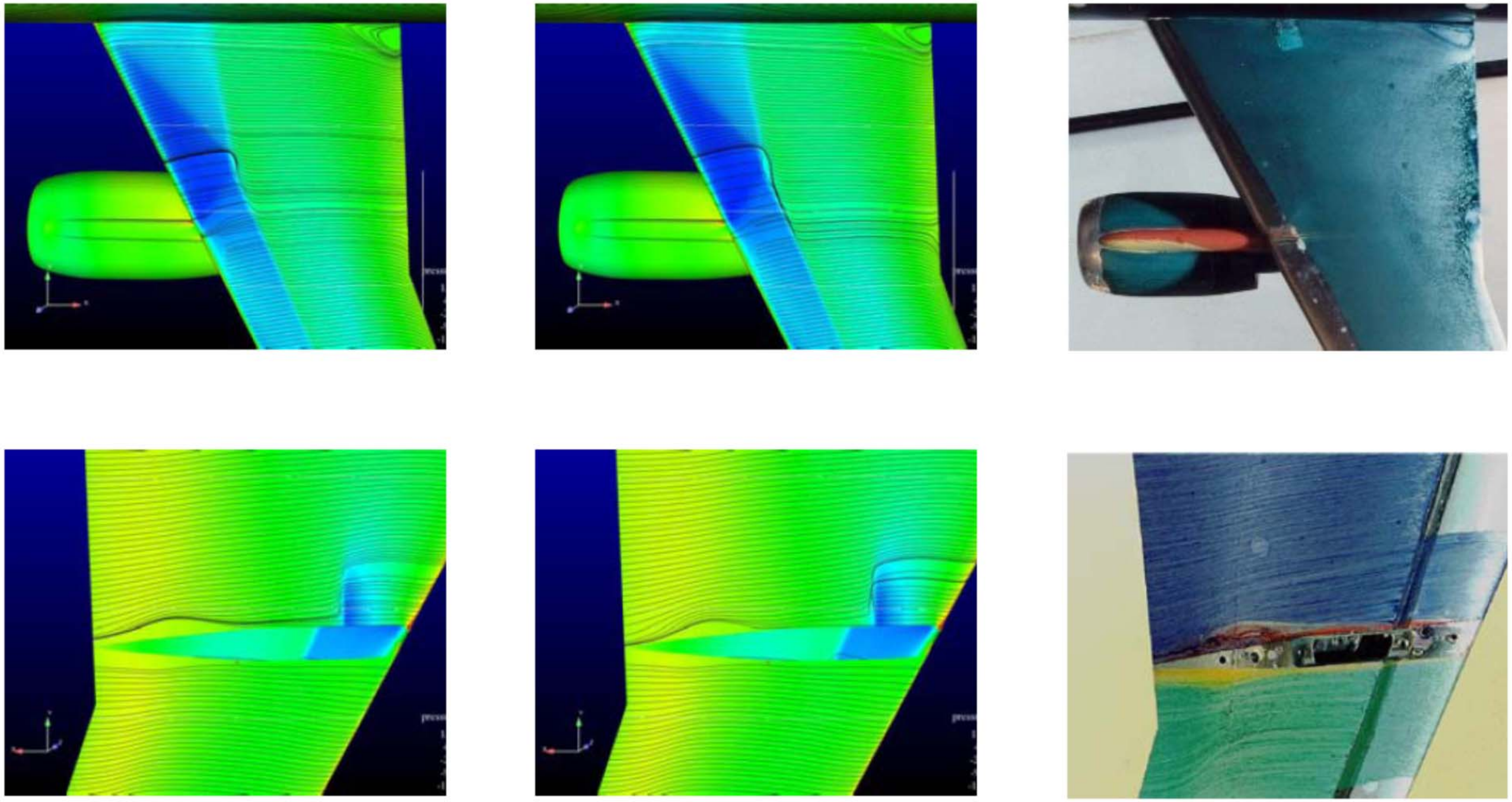
- **Velocity at cross section**
- **EARSM able to reproduce 3D sep.**
- **Incoming fully developed square duct**
  - **Corner secondary flow important**
  - **Cannot be captured by EVM**



Computations by Ansys CFX, Menter et al. (2009)

DLR-F6

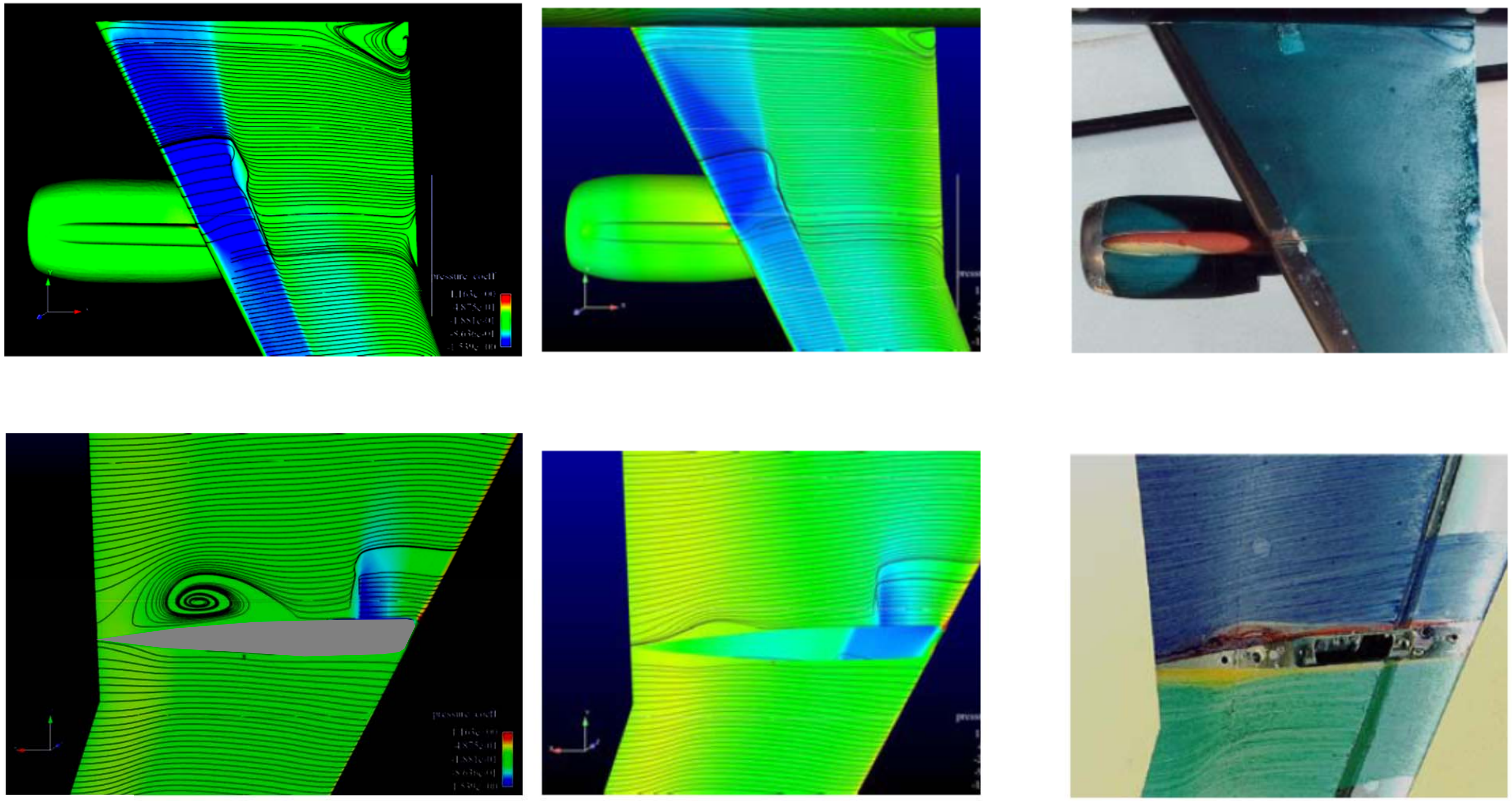
- **EARSM as well as DRSM show good results**
  - **EARSM a good approximation of DRSM**
- **CPU cost: EARSM ~ EVM DRSM ~ 2 times EARSM**



DRSM EARSM-BSL Expr

DLR-F6

- EARSM as well as DRSM show good results
  - EARSM a good approximation of DRSM
- CPU cost: EARSM ~ EVM DRSM ~ 2 times EARSM



EVM EARSM-BSL Expr

Milestone

- EARSM
  - Mostly good results
  - Good convergence and CPU time

- When is full DRSM needed?

- (E)ARSM is not a good approximation in
  - Weakly sheared flows
  - Very rapidly rotating or swirling flows

Shortcomings of RANS

- Flows with massive large-scale separation
  – around blunt bodies
  – internal flows with obstacles
- Capturing unsteadiness
  – turbulence noise
  – turbulence – structure interaction
  – weather forecast
- Combustion