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SG2218
Large Eddy Simulation

Basic equations

- Navier-Stokes equations (u, p denote instantaneous values)

$$\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_k}\left(\nu \frac{\partial u_i}{\partial x_k}\right) \qquad (1)$$

- RANS - ensemble or time averaging: $u_i(\boldsymbol{x}, t) = U_i(\boldsymbol{x}) + u'_i(\boldsymbol{x}, t)$

$$U_i(\boldsymbol{x}) = \frac{1}{T}\int_0^T u_i(\boldsymbol{x}, t')\mathrm{d}t' \qquad (2)$$

- LES - spatial filtering: $u_i(\boldsymbol{x}, t) = \bar{u}_i(\boldsymbol{x}, t) + u'_i(\boldsymbol{x}, t)$ (in the following, over-bar denotes a filtered quantity)

$$\bar{u}_i(\boldsymbol{x}, t) = \int_V u_i(\boldsymbol{x}', t) G(\boldsymbol{x}, \boldsymbol{x}')\mathrm{d}\boldsymbol{x}' \qquad (3)$$

Basic equations ...

- Filtered Navier-Stokes equations

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} - \frac{\partial \tau_{ij}^{\mathrm{SGS}}}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\nu \frac{\partial \bar{u}_i}{\partial x_j}\right) \qquad (4)$$

- Subgrid-scale (SGS) stress tensor

$$\tau_{ij}^{\mathrm{SGS}} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j = L_{ij} + C_{ij} + R_{ij} \qquad (5)$$

where the decomposition according to Leonard (1974) is

- $L_{ij} = \overline{\bar{u}_i \bar{u}_j} - \bar{u}_i \bar{u}_j$ - Leonard stresses (interaction between resolved scales)
- $C_{ij} = \overline{\bar{u}_i u'_j} + \overline{u'_i \bar{u}_j}$ - cross terms (interaction between resolved and unresolved scales)
- $R_{ij} = \overline{u'_i u'_j}$ - SGS Reynolds stress (interaction between unresolved small scales)

LES v.s. RANS equations

Filtered N-S (LES)	RANS
Most of the turbulent energy resolved by $\bar{u}_i$	no turbulence resolved in U_i
u'_i represents only the smallest SGS scales	u'_i represents all turbulent fluctua-tio
Solution of $\bar{u}_i$ always 3D and time dependent	solution of U_i steady (or slowly varying) and may be homoge-neous (or slowly varyin) in space

Modelling of SGS turbulence

- SGS represents only small part of turbulence energy
  - Modelling not that critical
  - ”resolving at least 80% of turbulence kinetic energy”
- Small scale turbulence more isotropic and in equilibrium
  - Modelling more simple
- Eddy viscosity models are OK
- Filter usually related to grid
  - This means that models are grid dependent by nature (!)
  - Inhomogeneous grid size –> inhomogeneous filter
  - Commutation errors (?)

Smagorinsky (1963)

- Eddy viscosity model
- SGS eddy viscosity and lenght scale relations
- Assumption: SGS production = dissipation
- Subgrid viscosity: $\nu_T^{\mathrm{SGS}} = (C_s \Delta)^2 S$
- One model constant – Smagorinsky constant
  - Standard value 0.18
  - Not very universal

Other models

- One or two-equation models
  - Solve for SGS turbulence energy
- Dynamic models
  - Evaluate Cs from coarser filtering – no model constant
  - Germano (1991)
- Implicit LES
  - $\nu_{SGS} \sim \Delta^2$ – as in a 2'nd order numerical scheme
  - Let the numerical scheme dissipate the SGS energy
  - MILES – Boris (1992)

Applications for LES and RANS

	LES	RANS
Suitable cases:	free shear flows, blunt bodies, geometry induced separation, low Re number	thin shear layers, weak geometry induced separation, high Re number
Problem cases	thin shear layers (expensive)	massive separation

- But, how to compute a case with both thin boundary layers and massive separation, e.g. a not-so-streamlined car?

Hybrid LES – RANS models

LES and RANS in different regions:

- RANS where LES is to expensive
  - Boundary layers
  - Thin shear layers
- LES where RANS fails
  - Massive separation

- Very active research field
- Many names:
  - DES, PANS, VLES, DDES, IDDES, URANS, XLES, T-RANS, ...

Spalart DES

Spalart et al. 1997

"Detached Eddy Simulation"

- Using the fact that the filtered N-S equation and the RANS equation can be written in the same form - only the modelling of $\overline{u'_i u'_j}$ differs.
- Use a RANS model for $\overline{u'_i u'_j}$ in thin shear layers.
- Use a LES SGS model for $\overline{u'_i u'_j}$ away from walls.

Spalart DES ...

- Starting point is the Spalart-Allmaras 1-eq. model (see notes from RANS modelling).

$$\frac{D\nu_T}{Dt} = c_{b1} S \nu_T - c_{w1} f_w \left(\frac{\nu_T}{d}\right)^2 + \mathcal{D}_{\nu_T}$$

- d is the distance from the wall. If d is replaced with Δ, the filter width, and the balance between production and dissipation is fulfilled (equilibrium hypothesis)

$$c_{b1} S \nu_T \approx c_{w1} f_w \left(\frac{\nu_T}{\Delta}\right)^2$$

or rewritten

$$\nu_T \approx \frac{c_{b1}}{c_{w1} f_w} \Delta^2 S$$

which is similar to the Smagorinsky SGS model

Spalart DES ...

- With $d = \Delta$ the Spalart-Allmaras will work as a SGS model
- Spalart DES

$$d = \min(d, C_{\mathrm{DES}}\Delta)$$

where Δ is the local grid size.

- RANS close to the wall where grid size (in any direction) > wall distance
- LES far from the walls where the grid size < wall distance
- Interface between RANS and LES dependent on the grid
- DES works well in cases with both regions with thin BLs (high Re) and massive separation

Rudimentary landing gear

- Resolved surface pressure (top) and
- Resolved turbulent structures (bottom)
- Hybrid RANS – LES model
  - RANS: here a simple mixing length model

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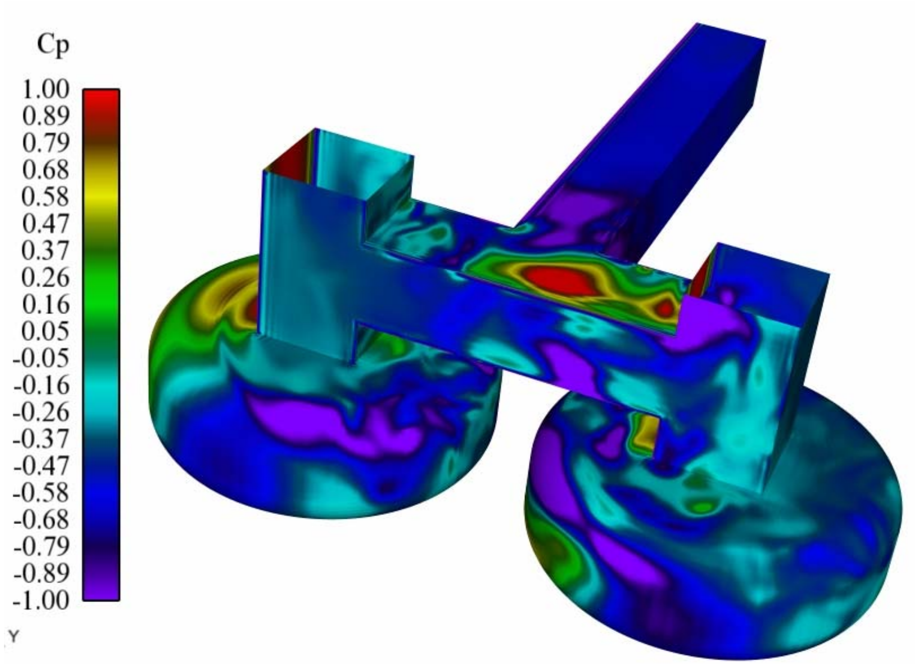

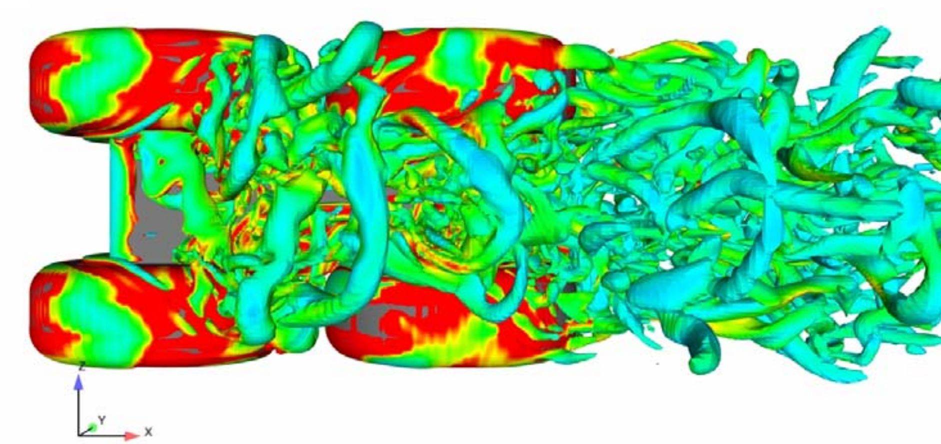

More examples
Hybrid RANS-LES