SOLUTIONS:

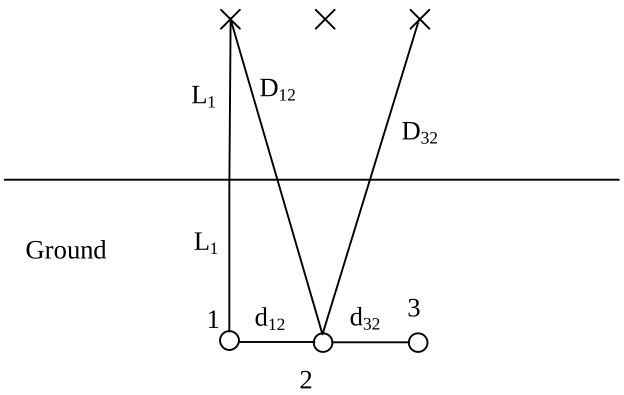
EXAMINATION IN 2C1134 Electrotechnical design December 2003-12-16

Solution to problem 1:

The temperature rise of straight conductors buried in ground is given by

$$\Delta T_i = P_i \frac{1}{2\pi\lambda_g} \ln\frac{2L_i}{r_i} + \sum_{k\neq i} P_k \frac{1}{2\pi\lambda_g} \ln\frac{D_{ik}}{d_{ik}}$$

where L_i is their depth under ground, P_i the heat dissipation per unit length of the conductors, and λ_g is the heat conductivity of the ground. The distances d_{ik} and D_{ik} are illustrated in the figure below.



The temperature rise is maximal of the middle conductor, because it has two neighbour conductors that contributes the temperature rise. We then can concentrate on to treat the middle conductor.

By using the given values one obtains

$$20 = 20 \frac{1}{2\pi\cdot 1.0} \ln\frac{2\cdot 0.5}{0.035} + 2\cdot 20 \frac{1}{2\pi\cdot 1.0} \ln\frac{D_{12}}{d_{12}}$$

From the figure it is evident that

$$D_{12}^2 = (2L_1)^2 + d_{12}^2$$

We then get

$$\ln\frac{\sqrt{(2\cdot 0.5)^2 + d_{12}^2}}{d_{12}} = 1.46\,53$$

and

$$\frac{\sqrt{(2\cdot 0.5)^2 + d_{12}^2}}{d_{12}} = 4.329$$

or

$$1 + d_{12}^2 = 4.329^2 \cdot d_{12}^2 \quad \Rightarrow \quad \cdot d_{12} = 0.237$$

ANSWER: The minimal allowed horizontal separation is 23.7 cm

Solution to problem 2:

$$\tilde{\varepsilon}_r(\omega) = \varepsilon_r'(\omega) - j\varepsilon_r''(\omega) = \varepsilon_\infty + \frac{\Delta\varepsilon}{1 + j\left(\omega/\omega_p\right)} = \varepsilon_\infty + \frac{\Delta\varepsilon}{1+\left(\omega/\omega_p\right)^2} - j\frac{\Delta\varepsilon\,\omega/\omega_p}{1+\left(\omega/\omega_p\right)^2}$$

$$\tan\delta(\omega) = \frac{\varepsilon_r''(\omega)}{\varepsilon_r'(\omega)} = \frac{\dfrac{\Delta\varepsilon\,\omega/\omega_p}{1+\left(\omega/\omega_p\right)^2}}{\varepsilon_\infty + \dfrac{\Delta\varepsilon}{1+\left(\omega/\omega_p\right)^2}} = \frac{\Delta\varepsilon\,\omega/\omega_p}{\varepsilon_\infty + \Delta\varepsilon + \varepsilon_\infty\left(\omega/\omega_p\right)^2} = \frac{\Delta\varepsilon}{\varepsilon_\infty}\cdot\frac{\omega/\omega_p}{1+\dfrac{\Delta\varepsilon}{\varepsilon_\infty}+\left(\omega/\omega_p\right)^2}$$

The maximum is found when $\dfrac{d\tan\delta(\omega)}{d\omega} = 0$

$$\frac{d\tan\delta(\omega)}{d\omega} = \frac{\Delta\varepsilon}{\varepsilon_\infty}\cdot\frac{\left(1+\dfrac{\Delta\varepsilon}{\varepsilon_\infty}+\left(\omega/\omega_p\right)^2\right)\cdot\dfrac{1}{\omega_p} - \dfrac{\omega}{\omega_p}\dfrac{2\omega}{\omega_p^2}}{\left(1+\dfrac{\Delta\varepsilon}{\varepsilon_\infty}+\left(\omega/\omega_p\right)^2\right)^2} = 0$$

, which gives

$$f_{\max} = f_p\sqrt{1+\frac{\Delta\varepsilon}{\varepsilon_\infty}} = 1\,kHz\cdot\sqrt{1+\frac{6}{2}} = 2\,kHz$$

$$\tan\delta_{\max} = \frac{\Delta\varepsilon}{\varepsilon_\infty}\cdot\frac{f_{\max}/f_p}{1+\dfrac{\Delta\varepsilon}{\varepsilon_\infty}+\left(f_{\max}/f_p\right)^2} = \frac{6}{2}\cdot\frac{2/1}{1+\dfrac{6}{2}+\left(2/1\right)^2} = 0.75$$

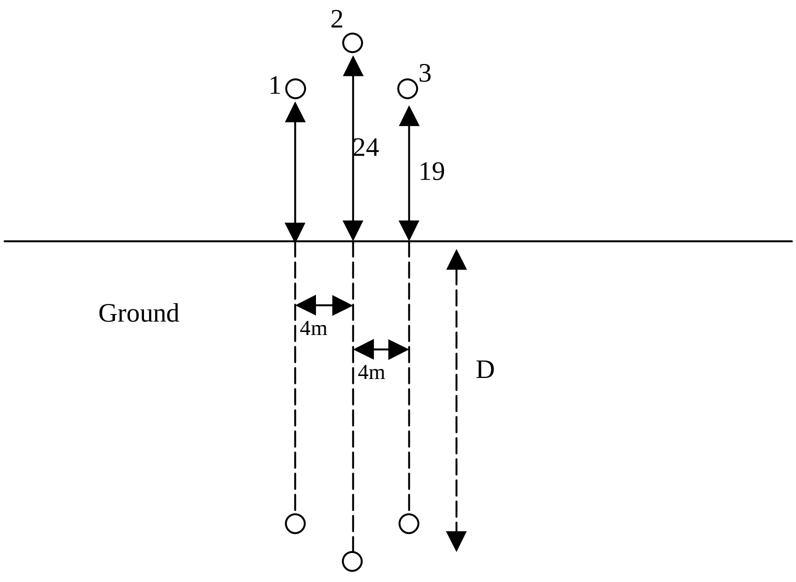
ANSWER: At 2kHz the loss tangent is at its maximum. The value is 0.75

Solutions to problem 3

First estimate the equivalent radius of each phase conductors. For 3 10mm diameter conductors that can be circumscribed by a circle of radius 20cm we have

$$r_{eq} = \sqrt[3]{3\cdot 0.005\cdot 0.2^{3-1}} = 0.08434$$

The configuration of power line can be drawn according to the figure below



The distance to the mirror image currents, which corresponds to the real phase currents can be calculated according to Carson's formula

$$D = 658\sqrt{\frac{\rho}{f}} = \sqrt{\frac{2000}{50}} = 4162 \text{ m}$$

The distances d_{12} , d_{23} and d_{13} between the conductors are calculated according to

$$d_{12} = \sqrt{(24-19)^2 + \left(0-(-4)^2\right)} = 6.403$$

$$d_{21} = d_{12}$$

$$d_{23} = d_{32} = d_{12} = 6.403$$

$$d_{13} = d_{31} = 8m$$

The components of the impedance matrix can now be calculated. The self and mutual inductances of the phases is first calculated as

$$L_{11} = L_{22} = L_{33} = \frac{\mu_0}{2\pi}\left(\frac{1}{4} + \ln\left(\frac{4162}{0.08434}\right)\right) = 2.211\,\mu\text{H/m}$$

$$L_{12} = L_{21} = \frac{\mu_0}{2\pi}\ln\frac{4162}{6.403} = 1.295\cdot 10^{-6}\,[\text{H/m}]$$

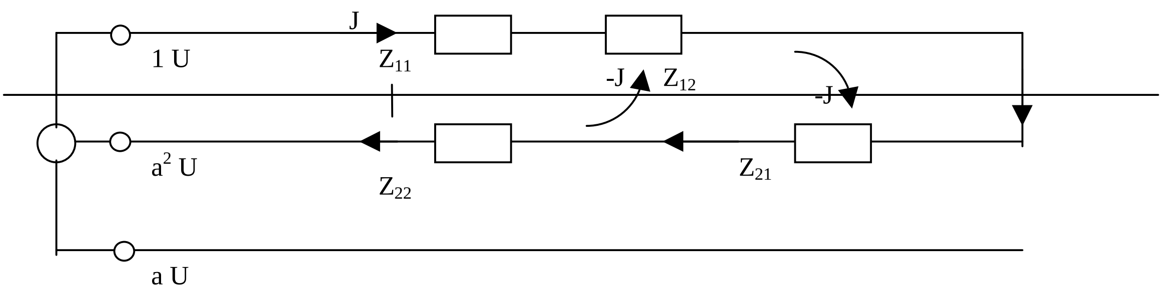
$$L_{13} = L_{31} = \frac{\mu_0}{2\pi}\ln\frac{4162}{8} = 1.251\cdot 10^{-6}\,[\text{H/m}]$$

$$L_{23} = L_{32} = \frac{\mu_0}{2\pi}\ln\frac{4162}{6.403} = 1.295\cdot 10^{-6}\ \ [\text{H/m}]$$

The impedance matrix will the be

$$Z = 10^{-6}\cdot\omega\, i\cdot\begin{pmatrix} 2.211 & 1.295 & 1.251 \\ 1.295 & 2.211 & 1.295 \\ 1.251 & 1.295 & 2.211 \end{pmatrix}\cdot 200\cdot 10^3 \quad \Omega$$

If a short circuit occurs between adjacent phases we have



$$U - a^2U = Z_{11}J + Z_{12}(-J) + Z_{21}(-J) + Z_{22}J$$

$$J = \frac{(1-a^2)U}{Z_{11} + Z_{22} - Z_{12} - Z_{21}} =$$

$$|J| = \left|\frac{\left(1-a^2\right)\frac{420}{\sqrt{3}}\cdot 10^3}{(2\cdot 2.211 - 2\cdot 1.295)i\omega\cdot 0.2}\right| = \frac{\sqrt{3}\,\frac{420}{\sqrt{3}}\cdot 10^3}{1.832\cdot 2\pi\cdot 50\cdot 0.2} = 3.649 \text{ A}$$

ANSWER: The short-circuit current is 3649 A

Solution to problem 4

First name the parts of the transmission such that these are 1, 2, and 3 from left to right.
The transmission coefficients then are
α_{12}=0.5 and α_{23}=0.5
The voltage at both sides of the reflection points A and B must be the same i. e. $u_i+u_r=u_t$.
Definition of transmission and reflection coefficients gives

at A $\quad u_r = \beta_{12} u_i \quad$ and $\quad u_t = \alpha_{12} u_i$ ..; $\alpha_{12} = \frac{2Z_2}{Z_1+Z_2}$, $\quad \beta_{12} = \frac{Z_2 - Z_1}{Z_1 + Z_2}$

$u_r=u_t-u_i \Rightarrow \quad \beta_{12} u_i = \alpha_{12} u_i - u_i = (\alpha_{12} - 1) u_i \quad \Rightarrow \quad \beta_{12} = -0.5$

at B $\quad u_r=u_t-u_i \Rightarrow \quad \beta_{23} u_i = \alpha_{23} u_i - u_i = (\alpha_{23} - 1) u_i \quad \Rightarrow \quad \beta_{23} = -0.5$

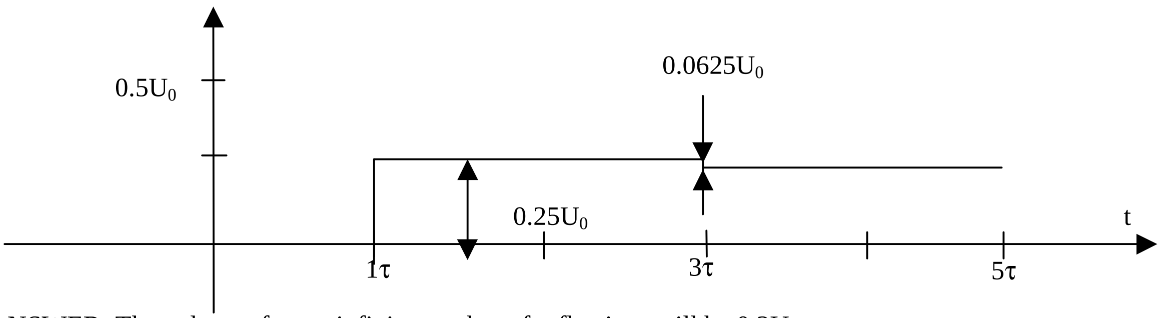
Further we by definition have β_{12}= - β_{21}

Now we can use the reflection diagram method with
α_{12}=0.5 , $\alpha_{23} = 0.5$, $\beta_{21} = 0.5$ $\beta_{23} = -0.5$
With H(s) being the Heavside's step fuction the signal at B then will be

$$(\alpha_{23}\alpha_{12} H(t-\tau) +$$
$$\beta_{21}\beta_{23}\alpha_{23}\alpha_{12} H(t-3\tau) +$$
$$(\beta_{21}\beta_{23})^2 \alpha_{23}\alpha_{12} H(t-5\tau) +$$
$$(\beta_{21}\beta_{23})^3 \alpha_{23}\alpha_{12} H(t-7\tau) +$$
$$+++$$
$$(\beta_{21}\beta_{23})^n \alpha_{23}\alpha_{12} H(t-(2n+1)\tau))U_0$$

After an infinite number of reflections we get

$$u = \alpha_{23}\alpha_{12}\left(1+\sum_{n=1}^{\infty}(\beta_{23}\beta_{21})^n\right)U_0 = \alpha_{23}\alpha_{12}\left(1+\frac{\beta_{23}\beta_{21}}{1-\beta_{23}\beta_{21}}\right)U_0 = 0.5\cdot 0.5\left(1+\frac{(-0.5)\cdot 0.5}{1-(-0.5)\cdot 0.5}\right)U_0 = 0.2U_0$$



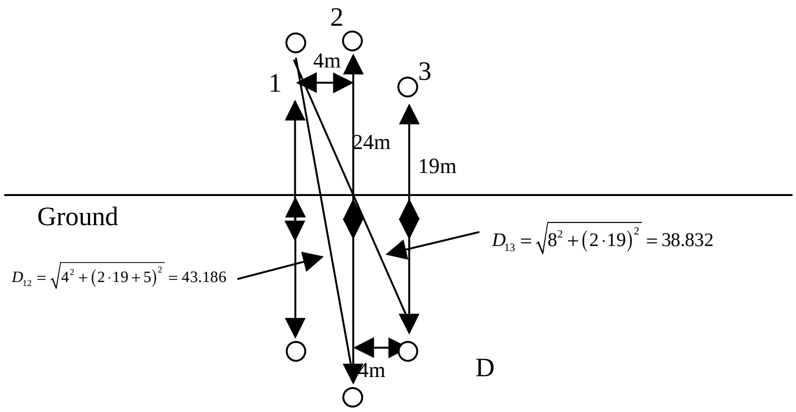
ANSWER: The voltage after an infinite number of reflections will be $0.2U_0$

Solution to problem 5

First estimate the equivalent radius of each phase conductors. For 3 10mm diameter conductors that can be circumscribed by a circle of radius 20cm we have

$$r_{eq} = \sqrt[3]{3 \cdot 0.005 \cdot 0.2^{3-1}} = 0.08434$$

The configuration of power line can be drawn according to the figure below



The distances d_{12} , d_{23} and d_{13} between the conductors are calculated according to

$$d_{12} = \sqrt{(24-19)^2 + \left(0-(-4)^2\right)} = 6.403$$

$$d_{21} = d_{12}$$

$$d_{23} = d_{32} = d_{12} = 6.403$$

$$d_{13} = d_{31} = 8m$$

The coupling coefficients are then calculated according to

$$k_{11} = k_{33} = \frac{1}{2\pi\varepsilon_0}\left(\ln\left(\frac{2 \cdot H_1}{0.08434}\right)\right) = \frac{1}{2\pi \cdot 8.854 \cdot 10^{-12}}\left(\ln\left(\frac{2 \cdot 19}{0.08434}\right)\right) = 1.098 \cdot 10^{11}\ [m/F]$$

$$k_{22} = \frac{1}{2\pi\varepsilon_0}\left(\ln\left(\frac{2 \cdot H_2}{0.08434}\right)\right) = \frac{1}{2\pi \cdot 8.854 \cdot 10^{-12}}\left(\ln\left(\frac{2 \cdot 24}{0.08434}\right)\right) = 1.140 \cdot 10^{11}\ [m/F]$$

$$k_{12} = k_{21} = k_{23} = k_{32}\ \frac{1}{2\pi\varepsilon_0}\ln\frac{D_{12}}{d_{12}} = 3.43 \cdot 10^{10}\ \ [m/F]$$

$$k_{13} = k_{31} = \frac{1}{2\pi\varepsilon_0}\ln\frac{38.832}{8} = 2.84 \cdot 10^{10}\ [m/F]$$

To get capacitances one needs to invert the coupling matrix to obtain the coefficients of capacitance and the coefficients of induction.

$$K = \begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{22} \end{pmatrix} \text{ such that } K^{-1} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{22} \end{pmatrix} \quad \text{e. i. } K^{-1}K = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Now the capacitances between the phases can be calculated as

ANSWER:

$C_{12} = -c_{12}$ capacitance per meter between phase 1 and 2

$C_{23} = -c_{23}$ capacitance per meter between phase 2 and 3

$C_{13} = -c_{13}$ capacitance per meter between phase 1 and 2

$C_{10} = c_{11} + c_{12} + c_{13}$ Capacitance between phase 1 and ground

$C_{20} = c_{22} + c_{12} + c_{23}$ Capacitance between phase 2 and ground

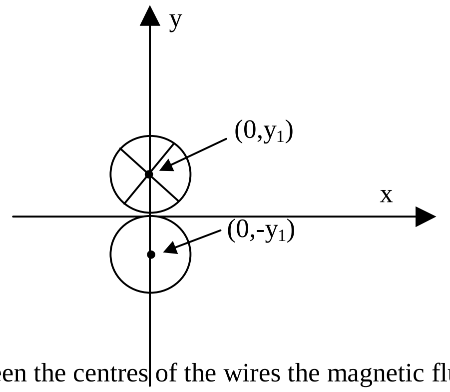
$C_{10} = c_{33} + c_{13} + c_{23}$ Capacitance between phase 3 and ground

Solutions to problem 6

The single Thomson coil turn has a mean radius of R_0. The wire radius is r_1 . The turns can be straightened out. The effective length of the Thomson coil wire then is

$$l = N \cdot 2\pi R_0$$

Introduce a xyz Cartesian co-ordinate system according to the figure below with y_1=r_1.



In a point (0,y) between the centres of the wires the magnetic flux density that originates from a current J in point (0,r_0) is

$$B_{+J} = \frac{\mu_0}{2\pi} \frac{J}{(-y + y_1)}$$

In a point (0,y) between the centres of the wires the magnetic flux density that originates from a current -J in point (0,-r_0) is

$$B_{-J} = \frac{\mu_0}{2\pi} \frac{(-J)}{(y_1 + y)}$$

It is now possible to calculate the generated flux per length generated by a loop comprising the two thin conductors of radiuses y_0<<r_1. End effects are neglected.

$$\phi = \phi_J + \phi_{-J} = \frac{J\mu_0}{2\pi}\int_{y_1-y_0}^{-y_1+y_0} \frac{1}{(-y+y_1)}dx + \frac{(-J)\mu_0}{2\pi}\int_{-y_1+y_0}^{y_1-y_0} \frac{1}{(-y+y_1)}dx$$

$$= \frac{J\mu_0}{2\pi}\left(\left[\ln(-y+y_1)\right]_{y_1-y_0}^{-y_1+y_0} + \left[\ln(-y+y_1)\right]_{-y_1+y_0}^{y_1-y_0}\right)$$

$$= -\frac{J\mu_0}{\pi}\ln\left(\frac{2y_1+y_0}{y_0}\right)$$

According to the thumb rule the flux is directed in the negative direction.
The outer inductance now can be calculated.

$$L_{outer} = \frac{\mu_0}{\pi}\ln\left(\frac{2y_1-y_0}{y_0}\right) \qquad \left[\frac{H}{m}\right]$$

The force per length unit of the Thomson coil wire can be calculated by use of the principle of virtual motion. The inner inductance is independent of the position of the wires. This means that a derivation of L with respect to y_1 only will give non zero contributions from the outer inductance.

$$F = -\left.\frac{\partial W}{\partial y_1}\right|_\phi = -\left.\frac{\partial\left(\frac{1}{2}LJ^2\right)}{\partial y_1}\right|_\phi = -\left.\frac{\partial\left(\frac{1}{2}L\left(\frac{\phi}{L}\right)^2\right)}{\partial y_1}\right|_\phi = -\left.\frac{\partial\left(\frac{1}{2}\frac{\phi^2}{L}\right)}{\partial y_1}\right|_\phi$$

With $\left.\frac{\partial}{\partial y_1}\left(\frac{1}{L}\right)\right|_\phi = \left.\frac{\partial}{\partial y_1}\left(\frac{1}{\frac{\mu_0}{\pi}\ln\left(\frac{2y_1-y_0}{y_0}\right)}\right)\right|_\phi = \left.\left(\frac{\pi}{\mu_0}\frac{(-1)}{\ln^2\left(\frac{2y_1-y_0}{y_0}\right)}\frac{y_0}{2y_1-y_0}\frac{2}{y_0}\right)\right|_\phi$

One gets

$$F = -\frac{1}{2}\left(\frac{J\mu_0}{\pi}\right)^2\ln^2\left(\frac{2y_1-y_0}{y_0}\right)\left.\left(\frac{\pi}{\mu_0}\frac{(-1)}{\ln^2\left(\frac{2y_1-y_0}{y_0}\right)}\frac{2}{(2y_1-y_0)}\right)\right| = J^2\frac{\mu_0}{\pi}\cdot\frac{1}{(2y_1-y_0)}$$

y0<<y1 gives $F = J^2\frac{\mu_0}{\pi}\cdot\frac{1}{(2y_1-y_0)} = J^2\frac{\mu_0}{2\pi}\frac{1}{y_1}$

Initially $y_1=r_1$. With $l=2\pi R_0$ we finally get the initial force according to

$$F == J^2\mu_0\frac{R_0}{r_1}$$

For $J=5\cdot 10^3$, $R_0=0.005$, N=1, and $r_1=0.00025$ we get F=628N

ANSWER: 628N