SOLUTIONS:

EXAMINATION IN 2C1134 Electrotechnical design December 2004-12-16

Solution to problem 1:

The thermal resistance between conductor and lead jacket for 1 m of the cable is:

$$R_{th1} = \frac{1}{\lambda}\int_{x_1}^{x_2}\frac{dx}{A} = \frac{1}{\lambda}\int_{15}^{15+0.3+7+0.25}\frac{dx}{2\pi x\cdot 1} = \frac{1}{2\pi\cdot 0.2}\ln\left(\frac{15+0.3+7+0.25}{15}\right) = 0.324\,[K/W]$$

The thermal resistance between the lead jacket and the surroundings for 1 m of the cable is (lead is considered to be an ideal good thermal conductor):

$$R_{th2} = \frac{1}{\lambda}\int_{x_1}^{x_2}\frac{dx}{A} = \frac{1}{\lambda}\int_{15+0.3+7+0.25+2.5}^{15+0.3+7+0.25+2.5+4.4}\frac{dx}{2\pi x\cdot 1} = \frac{1}{2\pi\cdot 0.2}\ln\left(\frac{15+0.3+7+0.25+2.5+4.4}{15+0.3+7+0.25+2.5}\right) = 0.129\,[K/W]$$

The derived expression for the over temperature relative ground temperature of a surface of temperature is (see Eq. (4.308) in "Electrotechnical modelling):

$$\Delta T = P\cdot\left(\frac{1}{2\pi\lambda}\ln\frac{2L_1}{r_1}\right)$$

For 1 m of the cable we can identify the corresponding thermal resistance for L_1=0.9m and the given radius of the cable by considering the thermal Ohm's law $\Delta T = T_2 - T_1 = R_{th}\cdot P$.

We get

$$R_{th3} = \frac{1}{2\pi\cdot 0.9}\ln\left(\frac{2\cdot 800}{15+0.3+7+0.25+2.5+4.4}\right) = 0.706[K/W]$$

The sum of the thermal resistances are

0.324+0.129+0.706=1.159 [K/W]

The cable power dissipation P for a current J is per meter and at 50° C (30° C temperature rise)

$$P = 0.0287\cdot 10^{-3}\left(1+30\cdot 0.00393\right)J^2 = 3.21\cdot 10^{-5}J^2$$

Now $\Delta T = T_2 - T_1 = R_{th}\cdot P$ gives

$$\left(50-20\right) = 3.21\cdot 10^{-5}J^2\cdot 1.159 \;\Rightarrow\; J = \sqrt{\frac{30}{1.159\cdot 3.21\cdot 10^{-5}}} = 898\,[A]$$

ANSWER: The maximal load current is 898 A

Solution to problem 2:

To determine the reflection coefficient β one needs to find the characteristic impedance $Z = \sqrt{\frac{l}{c}}$.,where l and c are the inductance and capacitance per meter of the cable. The inductance is given and is $\frac{\mu_0}{2\pi}\left(\ln\frac{r_1}{r_0}+\frac{1}{4}\right) \quad [H/m]$. The capacicitance per meter of a coaxial is according to the compendium $\frac{2\pi\varepsilon_r\varepsilon_0}{\ln\left(\frac{r_1}{r_0}\right)}$ - It is now straight-forward to obtain an expression of the characteristic impedance

$$Z = \sqrt{\frac{l}{c}} = \sqrt{\frac{\frac{\mu_0}{2\pi}\left(\ln\frac{r_1}{r_0}+\frac{1}{4}\right)}{\frac{2\pi\varepsilon_r\varepsilon_0}{\ln\left(\frac{r_1}{r_0}\right)}}} = \frac{1}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon_r\varepsilon_0}\left(\ln^2\frac{r_1}{r_0}+\frac{1}{4}\ln\frac{r_1}{r_0}\right)}$$

The inner and outer radius of are given and 2 and 10 mm respectively, which yields a cable impedance of 73.34 Ω . Now the reflection coefficient can be determined according to

$$\beta_{12} = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{50-73.34}{50+73.34} = -0.18926 \approx -0.189$$

The wave propagation time from point A to the 50Ω termination can be designated τ . The wave propagation speed v in the cable is $v = \frac{1}{\sqrt{lc}}$. With the cable data one gets

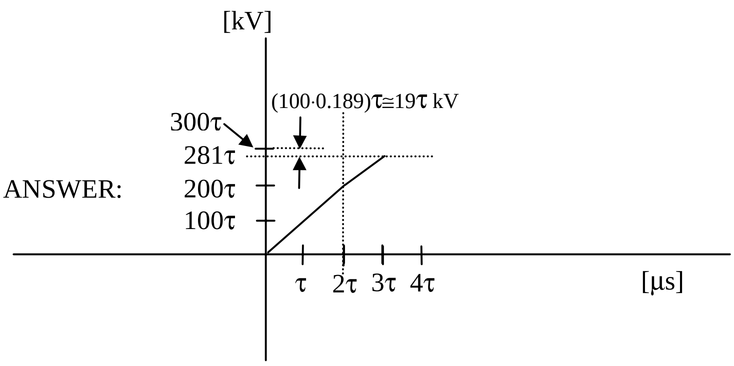
$$v = \frac{1}{\sqrt{\mu_0\varepsilon_r\varepsilon_0\frac{\left(\ln\frac{r_1}{r_0}+\frac{1}{4}\right)}{\ln\left(\frac{r_1}{r_0}\right)}}} = \frac{c}{\sqrt{\varepsilon_r}\cdot 1.155} = \frac{3\cdot 10^8}{\sqrt{2}} = 1.97\cdot 10^8 \ \ [m/s]$$

The wave propagation time τ from point A to the cable termination then is given by

$$30 = v\cdot\tau \quad \Rightarrow \quad \tau = \frac{30}{1.97\cdot 10^8} = 0.152\mu s$$

A reflection diagram then gives the voltage variation at A when a ramp of steepness 100 kV/μs arrives directed towards the termination.

ANSWER:



Solutions to problem 3

The hysteresis loss density during each cycle in the core material is $\oint H dB$, which is the area circumscribed by the the hysteresis loop. According to the given magnetization curve this area is 2·(100·1.7)/2=100· $\hat{B}$ =170 [J/m^3].

At the frequency f the eddy current loss density is $p_e(t) = \frac{\omega^2 \hat{B}^2 d^2}{12\rho} \quad \left[\frac{W}{m^3}\right]$,where ω=2π·f , d is the lamination thickness, $\hat{B}$ the peak magnetic flux density and ρ is the core material resistivity.

In this case we will identify the frequencies where hysteresis and eddy current losses are the same for the different lamination thicknessnes 0.5 , 0.3 and 0.1 mm.

We then get f_1 , f_2, and f_3 according to

$$f_1 \cdot \hat{B} \cdot 100 = \frac{(2\pi f_1)^2 \hat{B}^2 d_1^2}{12\rho} \quad \Rightarrow \quad f_1 = \frac{12 \cdot 100 \cdot \rho}{(2\pi)^2 \hat{B} d_1^2} = \frac{100 \cdot 12 \cdot 0.5 \cdot 10^{-6}}{(2\pi)^2 \cdot 1.7 \cdot (0.5 \cdot 10^{-3})^2} = 35.76$$

$$f_2 \cdot \hat{B} \cdot 100 = \frac{(2\pi f_2)^2 \hat{B}^2 d_2^2}{12\rho} \quad \Rightarrow \quad f_2 = \frac{12 \cdot 100 \cdot \rho}{(2\pi)^2 \hat{B} d_2^2} = \frac{100 \cdot 12 \cdot 0.5 \cdot 10^{-6}}{(2\pi)^2 \cdot 1.7 \cdot (0.3 \cdot 10^{-3})^2} = 99.33$$

$$f_3 \cdot \hat{B} \cdot 100 = \frac{(2\pi f_3)^2 \hat{B}^2 d_3^2}{12\rho} \quad \Rightarrow \quad f_3 = \frac{12 \cdot 100 \cdot \rho}{(2\pi)^2 \hat{B} d_3^2} = \frac{100 \cdot 12 \cdot 0.5 \cdot 10^{-6}}{(2\pi)^2 \cdot 1.7 \cdot (0.1 \cdot 10^{-3})^2} = 894.00$$

ANSWER: The frequencies corresponding to 0.5, 0.3 and 0.1 mm are 36, 99, and 894 Hz.

Solution to problem 4

The charges (rms-value) on the conductors are determined by the phase voltages and the capacitance matrix

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 7.83 & -1.67 & -0.79 \\ -1.67 & 8.11 & -1.67 \\ -0.79 & -1.67 & 7.83 \end{bmatrix} \cdot 10^{-12} \cdot \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} \cdot 145000 \cdot \frac{1}{\sqrt{3}} = \begin{bmatrix} 7.58 + j0.64 \\ -4.09 - j7.09 \\ -3.24 + j6.89 \end{bmatrix} \cdot 10^{-7} \quad C/m$$

$$a = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

Formula (3.7) in the compendium gives

$$E = 2\sum_{1}^{3} \frac{q_i}{2\pi\varepsilon_0} \cdot \frac{H_i}{H_i^2 + x_i^2}$$

By insertion of charges, H=12 and x-values 10, 14 and 18 the electric field adds up to:

E = -393+j378 V/m with absolute value 545 V/m (rms).

ANSWER: The electric field at ground at 10 m horizontal distance from the outer phase is 545 V/m (rms).

Solution to problem 5

The voltage across the two-layered capacitor can be written as

$$U = E_1 d_1 + E_2 d_2$$

E_1 is the field-strength in the oil and E_2 is the field-strength in the oil-impregnated paper.
The continuity of the displacement field across the boundary between the oil and the paper yields

$$\varepsilon_{r1} E_1 = \varepsilon_{r2} E_2$$

d_1 = 2 mm, d_2 = 8 mm
ε_{r1} = 2.1, ε_{r1} = 3.8

The field in the paper is $E_2 = \frac{2.1}{3.8} E_1 = 0.553 E_1$. This means that the oil will breakdown first because when the field in the oil reach the breakdown strength 15 kV/mm, the field in the paper will be 8.3 kV/mm which is below the breakdown field-strength of the paper.
Thus breakdown of the oil will occur at a voltage of

$$U_{bd} = E_{1bd} d_1 + \frac{\varepsilon_{r1}}{\varepsilon_{r2}} E_{1bd} d_2 = 15 \cdot 2 + 0.553 \cdot 15 \cdot 8 = \underline{\underline{96.3\,kV : Ans}}$$

Note! After the breakdown of the oil, the full voltage will fall above the paper and the field-strength in the paper will be 96.3/8 = 12 kV/mm > the breakdown strength and the whole insulation will breakdown.

In the laminated case with 100 laminates the breakdown strength of the oil and the paper will be:

$$E_{b0il} = \frac{15}{\left(\frac{2/100}{2}\right)^{0.3}} = 59.7 \quad kV/mm$$

$$E_{bPaper} = \frac{10}{\left(\frac{8/100}{8}\right)^{0.3}} = 39.8 \quad kV/mm$$

$$U_{bd} = E_{1bd} d_1 + \frac{\varepsilon_{r1}}{\varepsilon_{r2}} E_{1bd} d_2 = 59.7 \cdot 2 + 0.553 \cdot 59.7 \cdot 8 = \underline{\underline{383.5\,kV : Ans}}$$

ANSWER: The breakdown voltage will increase from 93.6 kV to 383.5kV with oil barriers in-between each layer.

Solutions to problem 6

The equations that relate voltage-drops along conductors to phase currents are given by:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{13} & Z_{23} & Z_{33} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

The impedances are given by the formulas below. The conductors are designated 1,2,3 from left to right.

$$Z_{11} = Z_{22} = Z_{33} = j\omega \frac{\mu_0}{2\pi}\left(\frac{1}{4} + \ln\left(\frac{D}{a}\right)\right)L = j2\pi \frac{4\pi 10^{-7}}{2\pi}\left(\frac{1}{4} + \ln\left(\frac{5097}{0.015}\right)\right)\cdot 70\cdot 10^3 = j57.2\Omega$$

$$Z_{12} = Z_{21} = Z_{23} = Z_{32} = j\omega \frac{\mu_0}{2\pi}\ln\left(\frac{D}{d_{12} = d_{23}}\right)L = j2\pi \frac{4\pi 10^{-7}}{2\pi}\ln\left(\frac{5097}{4}\right)\cdot 70\cdot 10^3 = j31.4\Omega$$

$$Z_{31} = Z_{13} = j\omega \frac{\mu_0}{2\pi}\ln\left(\frac{D}{d_{13} = d_{31}}\right)L = j2\pi \frac{4\pi 10^{-7}}{2\pi}\ln\left(\frac{5097}{8}\right)\cdot 70\cdot 10^3 = j28.4\Omega$$

D is the equivalent depth given by Carson's expression: $D = 658\sqrt{\frac{\rho}{f}} = 658\sqrt{\frac{3000}{50}} = 5097m$

In this problem the voltage drops along the conductors 1 and 2 are equal to the phase voltages, because the lines are short-circuited to ground. (This is not the case when a phase-to-phase fault without ground connection occurs.) The (inductive) current in conductor 3 is equal to zero since that line is unconnected.

The voltage at the end connected to the stiff network:

$$U_{01} = \frac{220}{\sqrt{3}}kV, U_{02} = \frac{220}{\sqrt{3}}a^2 kV,\ U_{03} = \frac{220}{\sqrt{3}}akV, \qquad a = e^{j2\pi/3}$$

The currents I_1 and I_2 can be obtained from this simplified equation system.

$$\begin{bmatrix} U_{01} \\ U_{02} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\cdot\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Cramer's rule, e.g. gives:

$$I_1 = \frac{\begin{vmatrix} U_{01} & Z_{12} \\ U_{02} & Z_{22} \end{vmatrix}}{\begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}} = \frac{U_{01}Z_{22} - U_{02}Z_{12}}{Z_{11}Z_{22} - Z_{12}Z_{21}} = (1.52 - j4.07)kA \Rightarrow |I_1| = 4.34kA$$

$$I_2 = \frac{\begin{vmatrix} Z_{11} & U_{01} \\ Z_{21} & U_{02} \end{vmatrix}}{\begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}} = \frac{U_{02}Z_{11} - U_{01}Z_{21}}{Z_{11}Z_{22} - Z_{12}Z_{21}} = (-2.76 + j3.35)kA \Rightarrow |I_2| = 4.34kA$$

The current in the fault location is given by $|I_1 + I_2| = 1.43kA$

ANSWER: The currents in the two short-circuited conductors are 4.34 kA and the current to ground is 1.43 kA