KTH Electrical Engineering

Exam in 2C1118 System Planning, 12 March, 2007, 8:00–13:00, D31-D34

Allowed aids

In this exam you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten**, **single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the exam.

PART I (MANDATORY)

Write all answers on the answer sheet provided. Motivations and calculations do not have to be presented.

Part I can yield 40 points in total. The examinee is guaranteed to pass if the score is at least 35 points. If the result in part I is at least 32 points, then there will be a possibility to complement for passing the exam with the grade 3.

Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

a) (1 p) Which of the following players have the responsibility to continuously maintain the physical balance between generation and consumption?

1. Each producer and consumer.
2. The balance responsible players.
3. The system operator.

b) (1 p) What does a firm power contract mean?

1. The customer must in advance notify the supplier about how much the customer will consume during each trading period.
2. The customer buys the same amount of energy in each trading period as long as the contract is valid.
3. During the time the contract is valid, the customer is allowed to consume as much energy they want each trading period, provided that the maximal power is not exceeded.

c) (2 p) The following applies to a power exchange using a price cross: I) Players who want to sell to the exchange should state in their bid the lowest price for which they are willing to sell, II) Players who want to buy from the exchange should state in their bid the highest price which they are willing to pay, III) All accepted bids trade at the same price (if transmission limitations and losses are neglected).

1. Only I is true.
2. Only II is true.
3. Only III is true.
4. I and II are true but not III.
5. All the statements are true.

Problem 2 (6 p)

Consider the common electricity market in the two nations Rike and Maa. Assume that there is perfect competition, that all players have perfect information and that there are neither reservoir nor capacity limitations in the power plants. The power systems of Rike and Maa are interconnected via an HVDC line and if this line is utilised as much as possible then it can transfer 18 TWh/year. Other data are shown in table 1. The variable production costs are assumed to be linear in the intervals, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

Table 1 Data of the electricity market in Rike and Maa.

Power source	Production capability [TWh/year]		Variable costs [¤/MWh]
	Rike	Maa	
Hydro power	50	10	30–60
Coal condensing	15	15	160–180
Consumption [TWh/year]	35	40	

a) (2 p) What would the price be in the common electricity market if there was *no* transmission limitation between the two countries?

b) (1 p) How much should be traded between the two countries if there was *no* transmission limitation, i.e., which country would export and how much?

c) (2 p) Which electricity price will there be in Rike then the transmission limitation is considered?

d) (1 p) Which electricity price will there be in Maa then the transmission limitation is considered?

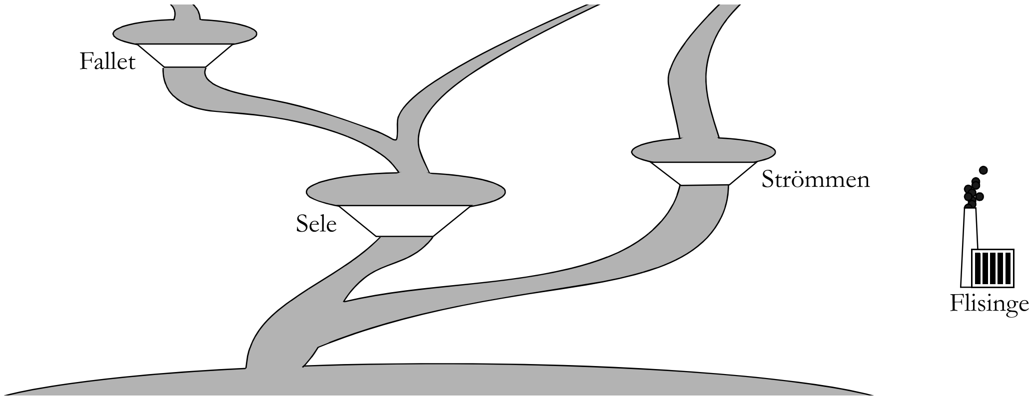
Problem 3 (6 p)

The hydro power plant Forsen is one of the units which participate in the primary control of Rike. At nominal frequency (50 Hz) Forsen is generating 200 MW. The gain in Forsen is set to 100 MW/Hz and it is available within the frequency range 50 ± 0.1 Hz.

a) (3 p) At a certain occasion, the power system in Rike has a total gain (i.e., including the gain in Forsen) of 2 525 MW/Hz and the frequency is 50.06 Hz. How much is generated in Forsen at this occasion?

b) (3 p) At the occasion described in part a, a failure occurs in a transformer at Forsen and the power plant is disconnected from the grid. What will the frequency be when the primary control has restored the balance between generation and consumption? Answer with three decimals!

Problem 4 (12 p)



AB Elkraft owns three hydro power plant located as in the figure above. Besides, the company also owns the biomass-fuelled power plant Flisinge. The following symbols have been introduced in a short-term planning problem for the power plants of the company:

Indices for the hydro power plants: Fallet 1, Sele 2, Strömmen 3.

D_t = contracted load during hour t, $t = 1, \ldots, 24$,
G_t = electricity generation in Flisinge, hour t, $t = 1, \ldots, 24$,
$Q_{i,j,t}$ = discharge in hydro power plant i, segment j, hour t, $i = 1, 2, 3$, $j = 1, 2, 3$, $t = 1, \ldots, 24$,
$V_{i,t}$ = local inflow to reservoir i, hour t, $i= 1, 2, 3$, $t = 1, \ldots, 24$,
β = variable generation cost in Flisinge,
$\mu_{i,j}$ = marginal production equivalent in hydro power plant i, segment j, $i = 1, 2, 3$, $j = 1, 2, 3$.

a) (4 p) Formulate the load balance constraint of AB Elkraft for hour t. Use the symbols defined above.

b) (2 p) The reservoir of the power plant Sele stores 3 600 000 m^3 water at 8:00. Up to 8:30 there is neither any discharge nor spillage from this reservoir, but 180 m^3/s is discharged in the power plant Fallet and the local inflow between Fallet and Sele is 20 m^3/s. How many hour equivalents (HE) water is stored in the reservoir of Sele at 8:30? The water delay time between the power plants can be neglected.

c) (2 p) When creating a piecewise linear model of the electricity generation in a hydro power plant, it is appropriate I) To put the breakpoints between the linear segments at the discharges where there are local maxima in the efficiency, as these discharges will then be more frequent in the solution of the planning problem, II) To let the marginal production equivalents increase with increasing discharge, because there will then be no need for integer variables in the planning problem, III) To assume that the electricity generation is a function of only the discharge (i.e., to neglect how the reservoir level affects the head, etc.).

1. None of the statements is true.
2. Only III is true.
3. I and II are true but not III.
4. I and III are true but not II.
5. All the statements are true.

d) (2 p) Assume that AB Elkraft has decided that Flisinge should not be shut down for a shorter time than four hours, i.e., if the power plant is stopped at 12:00 then it may not be started again before 16:00. Introduce the following symbols:

s_t^+ = start-up variable for hour *t* (1 if Flisinge is starting generation at the beginning of hour *t*, otherwise 0),

s_t^- = stop variable for hour *t* (1 if Flisinge is stopping generation at the beginning of hour *t*, otherwise 0).

How should a linear constraint be formulated in order to describe the relation between s_t^-, s_{t+1}^+, s_{t+2}^+ and s_{t+3}^+?

1. $s_t^- \cdot s_{t+1}^+ \cdot s_{t+2}^+ \cdot s_{t+3}^+ = 1$.
2. $s_t^- + s_{t+1}^+ + s_{t+2}^+ + s_{t+3}^+ < 1$.
3. $s_t^- + s_{t+1}^+ + s_{t+2}^+ + s_{t+3}^+ \leq 1$.
4. $s_t^- + s_{t+1}^+ + s_{t+2}^+ + s_{t+3}^+ = 1$.
5. $s_t^- + s_{t+1}^+ + s_{t+2}^+ + s_{t+3}^+ \geq 1$.

e) (2 p) Introduce the following parameters in the planning problem of AB Elkraft:

$\overline{G}$ = maximal generation in Flisinge when the power plant is committed,

G_t = generation in Flisinge during hour *t*,

u_t = unit commitment in Flisinge during hour *t* (1 if the power plant is committed during hour *t*, otherwise 0).

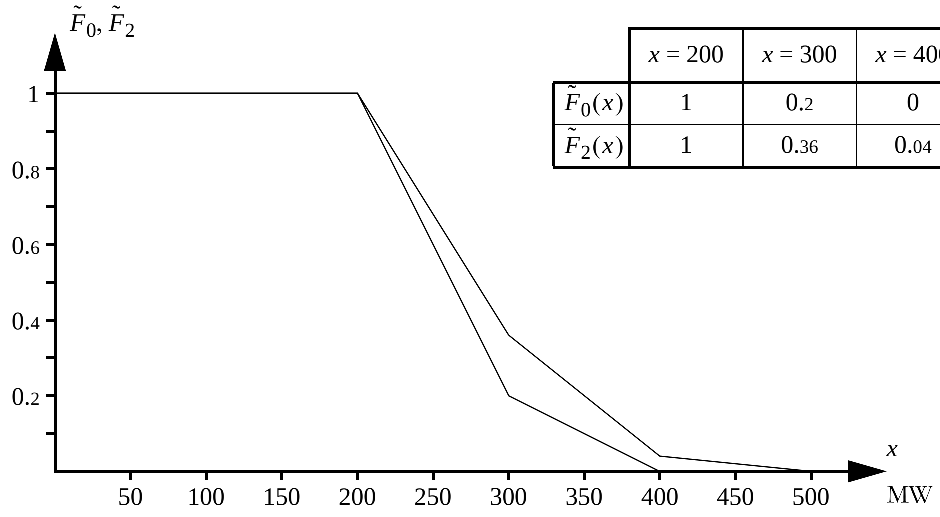
How should a linear constraint be formulated in order to describe the relation between $\overline{G}$, G_t and u_t for hour *t*?

1. $G_t + \overline{G} + u_t \leq 0$.
2. $G_t + \overline{G} \cdot u_t \leq 0$.
3. $G_t + \overline{G} \cdot u_t = 0$.
4. $G_t + \overline{G} \cdot u_t \geq 0$.
5. $G_t - \overline{G} \cdot u_t \leq 0$.

Problem 5 (12 p)

The national grid in Ensi is supplied by a number of hydro power plants and a thermal power plant in Ebbuga. The hydro power plants have a total capacity of 300 MW and can be considered 100% available. The thermal power plant has an installed capacity of 100 MW, the availability is 80% and the variable operation cost is 1 000 ¤/MWh.

The figure below shows the duration curve of the total load in Ensi, $\tilde{F}_0(x)$, as well as the duration curve of the equivalent load including outages in the thermal power plant, $\tilde{F}_2(x)$.



	$x = 200$	$x = 300$	$x = 400$	$x = 500$
$\tilde{F}_0(x)$	1	0.2	0	0
$\tilde{F}_2(x)$	1	0.36	0.04	0

a) (3 p) What is the *ETOC* of the system?

b) (3 p) What is the *LOLP* of the system?

c) (2 p) In a Monte Carlo simulation of Ensi it is necessary to generate random numbers for the available generation capacity in the thermal power plant, $\overline{G}$. Assume that the random number $U_0 = 0.3$ is obtained from a $U(0, 1)$-distribution. This value is transformed to $\overline{G} = 100$ using the inverse transform method. What is the complementary random number of this value for $\overline{G}$?

d) (2 p) Table 2 shows a compilation of the results of a Monte Carlo simulation of Ensi, where the power system of Ensi has been modelled using a multi-area model. The control variate method has been applied in order to get more accurate results. Which estimate of *ETOC* is obtained from this Monte Carlo simulation?

e) (2 p) Which estimate of *LOLP* is obtained from the Monte Carlo simulation?

Table 2 Results from a Monte Carlo simulation of the national power system in Ensi.

Number of scenarios, n	Results from a multi-area model		Results from a single-area model	
	$\sum_{i=1}^{n} toc_i$	$\sum_{i=1}^{n} lolo_i$	$\sum_{i=1}^{n} \tilde{toc}_i$	$\sum_{i=1}^{n} \tilde{lolo}_i$
1 000	9 070 000	77	7 370 000	42

PART II (FOR HIGHER GRADES)

All introduced symbols must be defined. Solutions should include sufficient detail that the argument and calculations can be easily followed.

The answer to each problem must begin on a new sheet, but answers to different parts of the same problem (a, b, c, etc.) can be written on the same sheet. The fields *Namn* (Name), *Blad nr* (Sheet number) and *Uppgift nr* (Problem number) must be filled out on every sheet.

Part II gives a total of 60 points, but this part will only be marked if the candidate has obtained at least 35 points in part I. Then the results of parts I and II and the bonus points will be added together to determine the examination grade (3, 4, 5).

Problem 6 (10 p)

Consider a simplified model of the electricity market in Republiken during one day. The maximal daily production and the variable costs are shown in table 3 below. The variable operation costs are assumed to be linear within the intervals, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

The electricity market in Republiken is organised so that producers and suppliers trade in an electricity market, where we can assume perfect competition, perfect information and that there are neither capacity, transmission nor reservoir limitations. The suppliers then sell the electricity to the consumers. Hence, notice that there are two different prices in the electricity market of Republiken: first there is a *market price*, which is set by the trading between producers and suppliers, and then there is an *electricity price*, which is an agreement between the suppliers and the consumers.

Concerning the electricity price, the suppliers offers two kinds of contracts. A fixed price contract means that the consumers pay a fixed price, namely 210 ¤/MWh. The other kind of contract means that the consumers pay a variable price, which is equal to the market price plus an uplift of 5%. During the day considered in this problem, we consumers will demand 42 000 MWh if they have fixed price contracts. The relation between consumption and variable price is shown in the figure below.

What is most beneficial to the suppliers during this day, if all consumers have fixed price contracts or if all consumers have variable price contracts?

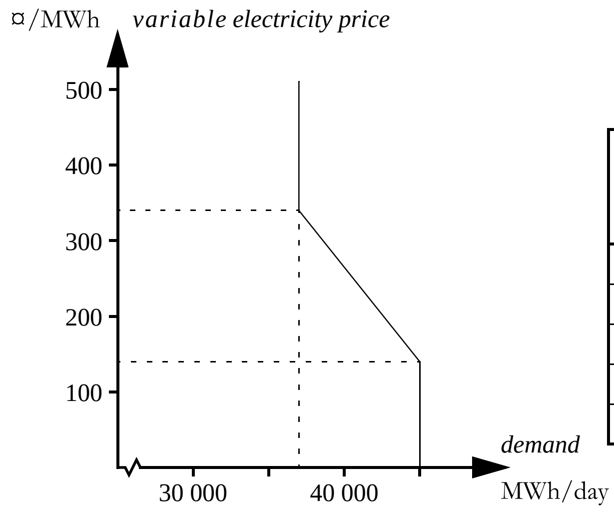


Table 3 Data for the electricity market in Republiken.

Power source	Production capability [MWh/day]	Variable costs [¤/MWh]
Hydro power	15 000	10
Nuclear power	20 000	100–120
Bio mass	3 000	100–200
Coal condensing	10 000	200–300
Gas turbines	1 000	500–600

Problem 7 (10 p)

Consider a power system comprising the four countries Aland, Beland, Celand and Deland. At a certain occasion there is balance production and consumption. The frequency is exactly 50 Hz throughout the entire system. Data for the transmission lines between the countries are shown in table 4. Each transmission line is equipped with a protection system which after a short time delay disconnects the line if the power flow exceeds the maximal capacity of the line. The power flows on the HVDC lines are not affected by the frequency of the system, but can only be controlled manually.

The gain of the system is distributed according to table 5. Assume that there are no limitations to how much the power plants participating in primary control can increase or decrease their generation.

Table 4 Data for the transmission lines.

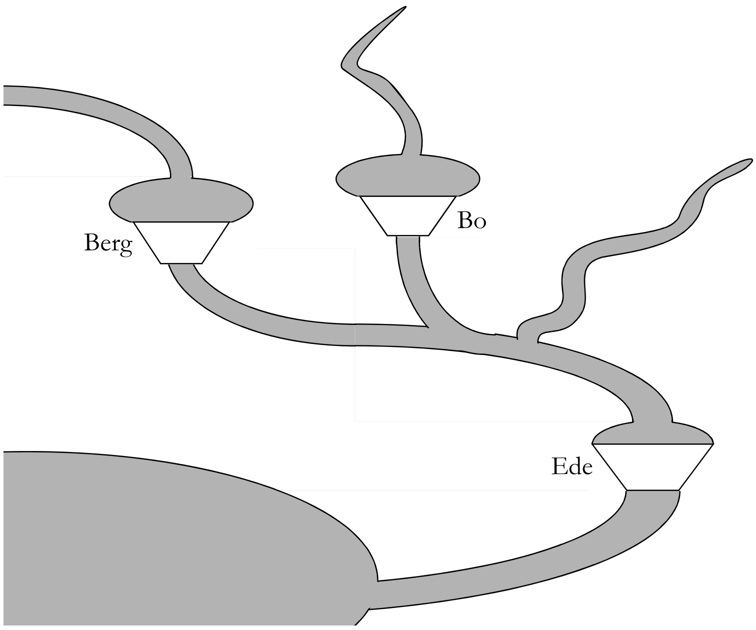
Connection	Type	Current transmission [MW]	Maximal capacity [MW]
Aland ↔ Beland	Direct current (HVDC)	140 MW from Aland to Beland	400
Aland ↔ Celand	Alternating current	125 MW from Celand to Aland	400
Beland ↔ Celand	Alternating current	250 MW from Celand to Beland	300
Celand ↔ Deland	Direct current (HVDC)	1 100 MW from Deland to Celand	1 100

Table 5 Distribution of the gain.

Country	Gain [MW/Hz]
Aland	850
Beland	200
Celand	650
Deland	2 000

Assume that it has been decided that the system should be capable of managing a failure of the HVDC line between Celand and Deland, when Celand is importing as much as possible from Deland, without having a frequency deviation larger than 0.7 Hz from the nominal frequency (50 Hz) and that no transmission lines should be overloaded. How would it be possible to arrange so that the system fulfils this requirement in a simple manner (i.e., without investing in new power plants or increased transmission capacity)?

Problem 8 (20 p)



AB Vattenkraft owns three hydro power plants located as in the figure above. Data of the power plants are given in table 6. The company is planning to sell their generation on the local power exchange, ElKräng. It is assumed that the company can sell as much as they want to the prices stated in table 7. Stored water is assumed to be used for electricity generation at the best marginal production equivalent in each power plant, and future electricity generation is valued 300 SEK/MWh. The water delay time between the power plants can be neglected.

a) (10 p) Formulate the planning problem of AB Vattenkraft as an LP problem. Use the notation in table 8 for the parameters (it is however permitted to add further symbols if you consider it necessary).

NOTICE! The following is required to get full score for this problem:

- The symbols for the optimisation variables must be clearly defined.
- The optimisation problem should be formulated so that it is easy to determine what the objective function is, which constraints there are and which limits there are.
- The possible values for all indices should be clearly stated for each equation.

b) (10 p) Assume that AB Vattenkraft has signed a contract with the system operator, stating that the company should supply a gain of at least 320 MW/Hz during each hour. AB Vattenkraft may freely choose how the gain should be divided between the three power plants. However, each power plant has a limited set of possible states for the gain (see table 6). The gain must be available in the interval 50±0.1 Hz, i.e., if a certain gain is set in a power plant, the base generation must be planned so that there are margins to increase or decrease the electricity generation as much as required for a deviation from the nominal frequency not exceeding 0.1 Hz.

How must the planning problem from part a be reformulated in order to consider the requirement on gain and primary control reserves. Do not forget to define all new variables and parameters that you introduce!

Hint: Use binary variables which are equal to one if a certain gain is set in a certain power plant during a certain hour and which otherwise are equal to zero.

Table 6 Data for the hydro power plants of AB Vattenkraft.

Power plant	Installed capacity [MW]	Start contents of the reservoir [HE]	Maximal contents of the reservoir [HE]	Marginal production equivalents [MWh/HE]		Maximal discharge [HE]		Local inflow [HE]	Possible gain [MW/Hz]
				Segment 1	Segment 2	Segment 1	Segment 2		
Berg	92	850	1 000	0.68	0.60	100	40	80	0, 40, 80, 160
Bo	82.4	1 065	1 400	0.72	0.62	80	40	60	0, 40, 80, 160
Ede	172.5	760	800	0..80	0.70	150	75	10	0, 80, 160, 320

Table 7 Expected electricity prices at ElKräng.

Hour	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Price at ElKräng [SEK/MWh]	246	243	242	243	248	259	266	303	390	405	378	322
Hour	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24
Price at ElKräng [SEK/MWh]	297	294	285	281	288	378	417	300	277	261	253	245

Table 8 Notation for the planning problem of AB Vattenkraft.

Symbol	Explanation	Value
$\overline{H}_i$	Installed capacity in power plant *i*	See table 6
$M_{i,0}$	Star contents of reservoir *i*	See table 6
$\overline{M}_i$	Maximal contents of reservoir *i*	See table 6
$\mu_{i,j}$	Marginal production equivalent in power plant *i*, segment *j*	See table 6
$\overline{Q}_{i,j}$	Maximal discharge in power plant *i*, segment *j*	See table 6
V_i	Local inflow to reservoir *i*	See table 6
λ_t	Expected electricity price at ElKräng hour *t*	See table 7
λ_f	Expected future electricity price	300

Problem 9 (20 p)

The island Ilha is located in the middle of the Pacific Ocean. The island has gradually become depopulated, but there are two small villages and a power company left. One of the villages, Selva, is located in the middle of the island, in the jungle. In Selva there is a 200 kW diesel generator set, which is having a variable operation cost of 1 ¤/kWh and an availability of 75%. The other village, Costa, is located along the coast. Outside Costa there is a wave energy plant, which has an installed capacity of 100 kW. The maximal available capacity of the wave energy plant is random, because it depends on the size of the waves. An approximation of the probability distribution of the available capacity is that there is a 40% chance that it can generate 50 kW and 60% chance that it can generate 100 kW.

The wave energy plant will always generate its maximal available capacity. If the load is less than the available capacity, then the surplus is consumed by a water heater at the small hospital in Costa.

The diesel generator set has a low efficiency when it is generating less than 50 kW, which means that the operation cost per hour is almost the same when 10 kW is generated as when 50 kW is generated. The power company has therefore agreed to set the control systems so that if the diesel generator is operating, it will never generate less than 50 kW and if there is a surplus from the diesel generator then that surplus is consumed in the water heater at the hospital in Costa. However, notice that there will never be a surplus from both the wave energy plant and the diesel generator set at the same time—if the available generation capacity of the wave energy plant exceeds the load then the diesel generator set will automatically be switched of.

The load in Costa is $N(110, 40)$-distributed and the load in Selva is $N(90, 30)$-distributed. The line between the two villages has always sufficient capacity and the losses are negligible.

a) (2 p) One of the factors described above must be neglected in order to apply probabilistic production cost simulation for the power system on Ilha. Which simplification is it that we have to do?

b) (4 p) The villagers are dissatisfied with the poor reliability of supply and thinks that the power company should take actions to reduce the risk of power deficit below 10%. Table 9 shows results from a probabilistic production cost simulation of the power system on Ilha. The duration curve of the equivalent load including outages in the wave energy plant is denoted $\tilde{F}_1(x)$ and the duration curve of the equivalent load including outages in both the wave energy plant and the diesel generator set is denoted $\tilde{F}_2(x)$. How large is the risk of power deficit in the current system?

Table 9 Equivalent load for the power system on Ilha.

	$x = 0$	$x = 50$	$x = 100$	$x = 150$	$x = 200$	$x = 250$	$x = 300$	$x = 350$	$x = 400$	$x = 450$	$x = 500$	$x = 550$
$\int_x^\infty \tilde{F}_1(\xi)d\xi$	220.00	170.01	120.26	72.67	33.63	10.48	1.92	0.18	0.01	0.00	0.00	0.00
$\tilde{F}_2(x)$	1.0000	0.9994	0.9891	0.9205	0.7249	0.4689	0.3032	0.2308	0.1587	0.0730	0.0190	0.0024
$\int_x^\infty \tilde{F}_2(\xi)d\xi$	270.00	220.01	170.20	122.00	80.22	50.36	31.51	18.30	8.41	2.62	0.48	0.05

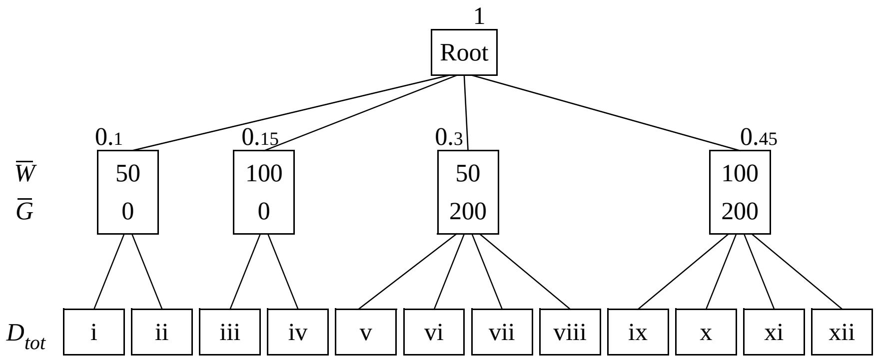
c) (4 p) Some villagers mean that the best way to increase the reliability of supply would be if the power company was ordered to pay a compensation of for example 2 ¤/kWh whenever consumers are disconnected. How much would a compensation for load shedding affect the current system, i.e., how much would the annual costs of the power company increase if load shedding results in a cost of 2 ¤/kWh?

d) (6 p) Monte Carlo simulation can be used to avoid the simplification from part a. It is appropriate to use stratified sampling in order to achieve a good accuracy. The figure below shows a strata tree for the power system of Ilha. State appropriate intervals for the nodes i–xii on the lower level of the tree, and calculate the stratum weights.

Hint: The distribution function of the standardised normal distribution, $\Phi(x)$, is shown in table 10. For $x < 0$ we can calculate $\Phi(x)$ using $\Phi(-x) = 1 - \Phi(x)$.

Table 10 The distribution function of the standardised normal distribution.

x	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
1	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
2	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
3	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000



e) (4 p) The results from a number of scenarios in a Monte Carlo simulation of the Ilha system are shown in table 11. Which estimates of *ETOC* and *LOLP* are obtained from these scenarios?

Table 11 Some scenarios from a Monte Carlo simulation of the power system on Ilha.

Available generation capacity [kW]		Load [kW]		Operation cost, *TOC*	Loss of load, *LOLO*
Wave energy plant, $\overline{W}$	Diesel generator set, $\overline{G}$	Costa	Selva		
50	200	108	102	160	0
50	200	115	127	192	0
100	200	108	75	83	0
100	200	24	147	71	0

KTH Electrical Engineering

Answer sheet for part I

Name: ....

Personal number: ....

Problem 1

a) Alternative .... is correct.

b) Alternative .... is correct.

c) Alternative .... is correct.

Problem 2

a) .... ¤/MWh

b) .... would export .... TWh/year

c) .... ¤/MWh

d) .... ¤/MWh

Problem 3

a) .... MW **b)** .... Hz

Problem 4

a) ....

b) .... HE

c) Alternative .... is correct.

d) Alternative .... is correct.

e) Alternative .... is correct.

Problem 5

a) .... ¤/h **b)** .... %

c) .... MW

d) .... ¤/h **e)** .... %

Problem 1

a) 3, **b)** 2, **c)** 5.

Problem 2

a) Neglecting the transmission limitation, we find that the total consumption of the two nations amounts to 75 TWh. The hydro power can in total produce 60 TWh, which means that half of the coal condensing potential must be used. Hence, 50% of the price interval of the coal condensing is used, i.e., the electricity price must be 170 ¤/MWh.

b) Without the transmission limitation the electricity production would be 57.5 TWh in Rike (50 TWh hydro and half of the coal condensing potential). As the consumption in Rike is only 35 TWh, the export to Maa would be 22.5 TWh.

c) Rike must produce 53 TWh to cover both the domestic consumption and the maximal export to Maa. Out of these 53 TWh it is possible to produce 50 TWh in the hydro power, whereas the remaining 3 TWh must originate from the coal condensing. Thus, 20% of the Rike coal condensing potential is utilised, which means that 20% of the price interval for coal condensing in used. Hence, the electricity price in Rike must be 164 ¤/MWh.

d) The power plants in Maa only need to produce 22 TWh to cover the load of 40 TWh, because 18 TWh are imported from Rike. They hydro power can supply 10 TWh and the remaining 12 TWh must therefore be produced by coal condensing units. In Maa, 80% of the coal condensing potential is used, and consequently the electricity price must be 176 ¤/MWh.

Problem 3

a) The generation in Forsen is calculated according to $G = G_0 - R(f - f_0) = 200 - 100(50.06 - 50) = 194$ MW.

b) There is now a deficit of 194 MW, and at the same time the system gain is reduced to 2 425 MW/Hz. The frequency must then change by $\Delta f = \Delta G/R = 194/2\,425 = 0.08$ Hz. As we have a deficit, the frequency must decrease, i.e., the new frequency will be $50.06 - 0.08 = 49.98$ Hz.

Problem 4

a) $\sum_{i=1}^{3}\sum_{j=1}^{3} \mu_{i,j} Q_{i,j,t} + G_t = D_t$.

b) After half an hour the reservoir holds $3\,600\,000 + 30 \cdot 60 \cdot 200 = 3\,960\,000$ m^3 water. Since 1 HE corresponds to 3 600 m^3 the content is 1 100 HE.

c) 4.

d) 3.

e) 5.

Problem 5

a) First we notice that $\tilde{F}_1(x) = \tilde{F}_0(x)$, because there are no outages in the hydro power plants. The expected generation per hour in the thermal power plant is then

$$EG_2 = EENS_1 - EENS_2 = \int_{300}^{\infty} \tilde{F}_1(x)dx - \int_{400}^{\infty} \tilde{F}_2(x)dx =$$

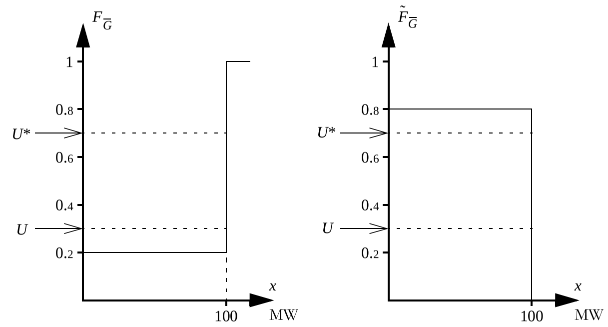
$$= 0.2 \cdot 100/2 - 0.04 \cdot 100/2 = 8 \text{ MWh/h}.$$

Thus, we get $ETOC = 1\,000 \cdot EG_2 = 8\,000$ ¤/h.

b) The risk of power deficit is given by

$$LOLP = \tilde{F}_2(400) = 4\%.$$

c) If we draw the distribution function or the duration curve of $\overline{G}$ it is easy to see that both $U = 0.3$ and $U^* = 1 - U$ are transformed into 100, i.e., $\overline{G}^* = \overline{G} = 100$.



d) We obtain the following estimate of *ETOC*:

$$m_{TOC} = m_{(TOC - \tilde{TOC})} + \mu_{\tilde{TOC}} = \frac{1}{n}\sum_{i=1}^{n}(toc_i - \tilde{toc}_i) + \mu_{\tilde{TOC}} =$$

$$= \frac{1}{n}\sum_{i=1}^{n} toc_i - \frac{1}{n}\sum_{i=1}^{n} \tilde{toc}_i + \mu_{\tilde{TOC}} = 9\,070\,000/1\,000 - 7\,370\,000/1\,000 + 8\,000 = 9\,700 \text{ ¤/h},$$

where $\mu_{\tilde{TOC}}$ is the expectation value of the operation cost in a single-area model, i.e., the value of *ETOC* calculated in part a.

e) In the same manner as in part d, we get the following estimate of *LOLP*:

$$m_{LOLO} = 77/1\,000 - 42/1\,000 + 0.04 = 7.5\%.$$

Problem 6

The electricity demand at a fixed price is 42 000 MWh. Hydro power, nuclear and biomass can in total produce 38 000 MWh. The remaining 4 000 MWh must come from coal condensing, which means that 40% of the potential is utilised. Hence, 40% of the price interval for coal condensing is utilised, which means that the market price of electricity must be 240 ¤/MWh. Thus, the market price is higher than the fixed price the consumers pay, which is obviously not profitable for the suppliers during this day.

With a variable price, it is impossible for the suppliers to make a loss, because they buy electricity for the price λ and sell it for the price 1.05λ. The conclusion must therefore be that during this day it is better for the suppliers if the consumers have variable price contracts.

Problem 7

Deland constitutes a synchronous grid on its own, and it has a gain of 2 000 MW/Hz. A failure on the HVDC line would result in a surplus of 1 100 MW, which causes a frequency increase equal to 1 100/2 000 = 0.55 Hz, which is acceptable.

The other countries form a synchronous grid where the total gain is 1 700 MW/Hz. A failure on the HVDC line would result in a deficit of 1 100 MW in this synchronous grid; thus, we would get a frequency decrease of 1 100/1 700 ≈ 0.64 Hz, which is acceptable. The question is if the transmission lines have sufficient margins. We get the following generation increases in the different countries:

$$\Delta G_A = R_A \Delta f = 850 \cdot 1\,100/1\,700 = 550 \text{ MW},$$

$$\Delta G_B = R_B \Delta f = 200 \cdot 1\,100/1\,700 \approx 129.4 \text{ MW},$$

$$\Delta G_C = R_C \Delta f = 650 \cdot 1\,100/1\,700 \approx 420.6 \text{ MW}.$$

In Aland the load is unchanged, whereas the generation increases by 550 MW. The surplus will be exported on the AC line to Celand (because the transmission on the HVDC line must be changed manually), which means that instead of importing 125 MW from Celand, we now get an export of 425 MW from Aland. This is however, not possible, because the maximal capacity of the line is 400 MW.

Assume that the export from Aland to Beland is increased by P MW, where P obviously cannot be larger than 260 MW (because then the capacity of the HVDC line would be exceeded). The change in the flow between Aland and Beland would cause the flow between Celand and Beland to decrease by P MW and that the flow from Celand to Aland is increasing by P MW. A production increase of 550 MW in Aland would then result in an export from Aland to Celand which amounts to $550 - (125 + P)$. Apparently the connection would not be overloaded if P is at least 25 MW.

The new flow from Celand to Beland would become $250 - P - \Delta G_B$. The capacity of the connection is 300 MW, which obviously is sufficient regardless of which value of P we choose in the interval between 25 and 260 MW.

Thus, the conclusion is that if the flow is increased between Aland and Beland by any value between 25 and 260 MW then the system will be able to manage a failure of the Deland to Celand HVDC line without any larger frequency deviations or disconnected transmission lines.

Problem 8

a) The problem we want to solve is

maximise *income of sold electricity + value of stored water*,

subject to *hydrological balance of hydro power plant*,

limittions in generation capacity, etc.

Indices of the power plants

Berg 1, Bo 2, Ede 3.

Parameters

The parameters are defined in table 8 in the problem text.

Optimisation variables

$Q_{i,j,t}$ = discharge in power plant i, segment j, during hour t, $i = 1, 2, 3, j = 1, 2, t = 1, \ldots, 24$,

$S_{i,t}$ = spillage from reservoir i during hour t, $i = 1, 2, 3, t = 1, \ldots, 24$,

$M_{i,t}$ = contents of reservoir i at the end of hour t, $i = 1, 2, 3, t = 1, \ldots, 24$.

Objective function

$$\text{maximise} \quad \sum_{t=1}^{24} \lambda_t \left(\sum_{i=1}^{3} \sum_{j=1}^{2} \mu_{i,j} Q_{i,j,t} \right) + \lambda_f ((\mu_{1,1} + \mu_{3,1}) M_{1,24} + (\mu_{2,1} + \mu_{3,1}) M_{2,24} + \mu_{3,1} M_{3,24}.$$

Constraints

Hydrological balance for Berg and Bo:

$$M_{i,t} - M_{i,t-1} + Q_{i,1,t} + Q_{i,2,t} + S_{i,t} = V_i, \qquad i = 1, 2, t = 1, \ldots, 24.$$

Hydrological balance for Ede:

$$M_{3,t} - M_{3,t-1} + Q_{3,1,t} + Q_{3,2,t} + S_{3,t} - \sum_{i=1}^{2} \left(\sum_{j=1}^{2} Q_{i,j,t} + S_{i,t} \right) = V_3, t = 1, \ldots, 24.$$

Variable limits

$0 \le Q_{i,j,t} \le \overline{Q}_{i,j}$, $\quad i = 1, 2, 3, j = 1, 2, t = 1, \ldots, 24$,

$0 \le S_{i,t}$ $\quad i = 1, \ldots, 4, t = 1, \ldots, 24$,

$0 \le M_{i,t} \le \overline{M}_i$, $\quad i = 1, \ldots, 4, t = 1, \ldots, 24$.

b) In the new planning problem we are planning the discharge at base generation, i.e., the electricity generation at 50 Hz. The base generation must be chosen so that there are sufficient primary control reserves in each power plant.

Introduce the following new parameters:

R_{tot} = contracted total gain = 320,

$R_{i,k}$ = gain set in power plant i, state k =

$$= \begin{cases} 0 & i = 1, 2, 3, \ k = 1, \\ 40 & i = 1, 2, \ k = 2, \\ 80 & i = 1, 2, \ k = 3, \\ 80 & i = 3, \ k = 2, \\ 160 & i = 1, 2, \ k = 4, \\ 160 & i = 3, \ k = 3, \\ 320 & i = 3, \ k = 4, \end{cases}$$

Δf = frequency change at maximal up or down regulation = 0.1.

Introduce the following new optimisation variable:

$r_{i,k,t}$ = gain activated in hydro power plant i, state k, during hour t (1 if the gain in the power plant is set to the value corresponding to this state, otherwise 0), $i = 1, 2, 3, k = 1, \ldots, 4, t = 1, \ldots, 24$.

Then we must introduce new constraints which makes sure that the company supplies enough gain and that sufficient reserves are available. Let us start by securing that the contracted total gain is supplied:

$$\sum_{i=1}^{3}\sum_{k=1}^{4} R_{i,k} r_{i,k,t} \geq R_{tot}, \qquad t = 1, \ldots, 24.$$

In each power plant we must choose between four possible states of the gain. Hence, in a given power plant during a given hour, we must have one $r_{i,k,t}$ which is equal to one and the others must be zero. The following constraint guarantees this:

$$\sum_{k=1}^{4} r_{i,k,t} = 1, \qquad i = 1, 2, 3, t = 1, \ldots, 24.$$

We must also make sure that there are sufficient reserves in the power plants. In case of up-regulation we must require that the base generation is not larger than installed capacity minus the largest possible up-regulation, i.e.,

$$\sum_{j=1}^{2} \mu_{i,j} Q_{i,j,t} \leq \overline{H}_i - \Delta f \sum_{k=1}^{4} R_{i,k} r_{i,k,t}, \qquad i = 1, 2, 3, t = 1, \ldots, 24.$$

In the case of down-regulation, the requirement is that the base generation must not be less than the largest possible down-regulation, i.e.,

$$\sum_{j=1}^{2} \mu_{i,j} Q_{i,j,t} \geq \Delta f \sum_{k=1}^{4} R_{i,k} r_{i,k,t}, \qquad i = 1, 2, 3, t = 1, \ldots, 24.$$

Finally we state the variable limits of the new optimisation variables:

$$r_{i,k,t} \in \{0, 1\}, \qquad i = 1, 2, 3, k = 1, \ldots, 4, t = 1, \ldots, 24.$$

Problem 9

a) We must neglect the lower limit on generation in the diesel generator when it is committed.

b) The risk of power deficit is given by

$$LOLP = \tilde{F}_2(300) = 30.32\%.$$

c) The increase of the costs of the power company is equal to the cost of unserved load multiplied by the expected energy not served during a year, i.e.,

$$\Delta ETOC = 2 \cdot 8760 \int_{300}^{\infty} \tilde{F}_2(x)dx = 552\,055.2 \text{ ¤/year}.$$

d) The intervals should be chosen so that we can separate scenarios where only the wave power plant is used ($TOC = 0$), scenarios where the diesel generator set is operating at its minimal capacity ($TOC = 50$), scenarios where the diesel generator set is operating at its maximal capacity ($TOC = 200$) and scenarios where the diesel generator set is generating between 50 and 100 kW ($50 < TOC < 200$). Moreover, we should differentiate between load shedding scenarios and scenarios where there is sufficient generation capacity. The table below shows intervals fulfilling these requirements:

Node	Interval	*TOC*	*LOLO*	Node weight	Stratum weight
i	$D \leq 50$	0	0	0.0013	$0.1 \cdot 0.0013 \approx 0.0001$
ii	$50 < D$	0	1	0.9987	$0.1 \cdot 0.9987 \approx 0.0999$
iii	$D \leq 100$	0	0	0.0228	$0.15 \cdot 0.0228 \approx 0.0034$
iv	$100 < D$	0	1	0.9772	$0.15 \cdot 0.9772 \approx 0.1466$
v	$D \leq 50$	0	0	0.0013	$0.3 \cdot 0.0013 \approx 0.0004$
vi	$50 < D \leq 100$	50	0	0.0214	$0.3 \cdot 0.0214 \approx 0.0064$
vii	$100 < D \leq 250$	$50 < D \leq 200$	0	0.8186	$0.3 \cdot 0.8186 \approx 0.2456$
viii	$250 < D$	200	1	0.1587	$0.3 \cdot 0.1587 \approx 0.0476$
ix	$D \leq 100$	0	0	0.0228	$0.45 \cdot 0.0228 \approx 0.0102$
x	$100 < D \leq 150$	50	0	0.1359	$0.45 \cdot 0.1359 \approx 0.0612$
xi	$150 < D \leq 300$	$50 < D \leq 200$	0	0.8186	$0.45 \cdot 0.8186 \approx 0.3684$
xii	$300 < D$	200	1	0.0228	$0.45 \cdot 0.0228 \approx 0.0102$

To calculate the node weights we need the probability distribution of the total load:

$$\mu_{Dtot} = \mu_{D1} + \mu_{D1} = 110 + 90 = 200,$$

$$\sigma_{Dtot} = \sqrt{\sigma_{D1}^2 + \sigma_{D2}^2} = \sqrt{40^2 + 30^2} = 50.$$

The node weights can now be calculated using the distribution function of the standardised normal distribution. Consider for example node vi, the node weight of which is given by

$$P(50 < D \leq 100) = P(D \leq 100) - P(D \leq 50) = \Phi\left(\frac{100 - 150}{50}\right) - \Phi\left(\frac{50 - 150}{50}\right) =$$

$$= \{\text{read in table 10}\} = 0.0228 - 0.0013 = 0.0214.$$

Finally the stratum weights are obtained by multiplying the node weights along each branch of the strata tree.

e) From part d we see that there are only two strata where *TOC* is varying, namely the strata including node vii and node xi respectively. The first two of the given scenarios belong to node vii and the others to node xi. Hence, we get the following estimates of *ETOC* and *LOLP*:

$$m_{TOC} = \sum_{h=1}^{12} \omega_h m_{TOCh} = 0 + 0 + 0 + 0 + 0 + 0.0064 \cdot 50 + 0.2456 \cdot \frac{160 + 192}{2} +$$

$$+ 0.0476 \cdot 200 + 0 + 0.0612 \cdot 50 + 0.3684 \cdot \frac{83 + 71}{2} + 0.0102 \cdot 200 \approx 86.53 \text{ ¤/h},$$

$$m_{LOLO} = 0 + 1 \cdot 0.0999 + 0 + 1 \cdot 0.1466 + 0 + 0 + 0 + 1 \cdot 0.0476 + 0 + 0 + 0 + 1 \cdot 0.0102 \approx 30.43\%.$$