KTH Electrical Engineering

Exam in 2C1118 System Planning,
8 June 2007, 8:00–13:00, V34

Allowed aids

In this exam you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten**, **single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the exam.

PART I (MANDATORY)

Write all answers on the answer sheet provided. Motivations and calculations do not have to be presented.

Part I can yield 40 points in total. The examinee is guaranteed to pass if the score is at least 35 points. If the result in part I is at least 32 points, then there will be a possibility to complement for passing the exam with the grade 3.

Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

a) (2 p) We use the notion "ahead trading" to describe all the trading which occurs before the hour of delivery (or any other trading period). In the ahead market it is possible to trade with the following contracts: I) Take-and-pay contracts, i.e., when the customer subscribes to a specific maximal power, and during the duration of the contract is allowed to buy any amount of energy per trading period as long as the maximal power is not exceeded, II) Firm power, i.e., when the seller is committed to deliver a specific amount of energy per trading period during the duration of the contract, III) Regulation power, i.e., when a player at request from the system operator is supplying more power to the system (up-regulation) or when a player at request from the system operator is supplying less power to the system (down-regulation).

1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. I and II are true but not III.

b) (1 p) Consider a producer which between 10 and 11 is producing according to plan, except that at 10:45 the producer carries out a down regulation bid at the request of the system operator. Which real-time trading has this company been involved in during this hour?

1. The company has purchased 50 MWh regulating power from the system operator.
2. The company has sold 50 MWh regulating power to the system operator.
3. The company has sold more than 50 MWh regulating power to the system operator.

c) (1 p) Consider a balance responsible player which during one hour has generated 876 MWh, sold 400 MWh to the power exchange, sold 458 MWh to consumers with take-and-pay contracts and sold 20 MWh regulation power to the system operator. What obligation does this player have in the post trading for this hour?

1. The player must buy balance power from the system operator.
2. The player must sell balance power to the system operator.
3. None—it is voluntary for balance responsible players to participate in the post trading.

Problem 2 (6 p)

Assume that the electricity market in Land has perfect competition, all players have perfect information, and there are neither transmission nor capacity limitations. The hydro reservoirs of Land can store at most 60 TWh. On 1 January the reservoirs holds in total 20 TWh and according to the long-term forecast for the electricity market (which as already mentioned is assumed to be faultless), the reservoirs should hold 25 TWh on 31 December. The inflow is 150 TWh between 1 January and 30 June, and 55 TWh between 1 July and 31 December. The variable operation cost of the hydro power is negligible.

In addition to the hydro power there is also coal condensing units in Land. The coal condensing can produce 50 TWh between 1 January and 30 June, and the same amount between 1 July and 31 December. The relation between electricity price and coal condensing generation is assumed to be linear; if the electricity price is 400 ¤/MWh or less, the coal condensing units will not generate anything, and if the price is 500 ¤/MWh or higher, the generation will be as large as possible.

The electricity consumption in Land is not price sensitive and amounts to 130 TWh between 1 January and 30 June, and 120 TWh between 1 July and 31 December.

a) (2 p) What would the price be in the electricity market of Land if there was *no* reservoir limitation?

b) (2 p) How much would the reservoirs hold at midnight between 30 June and 1 July if there was *no* reservoir limitation?

c) (1 p) Which electricity price will there be between 1 January and 30 June if the reservoir limitation is considered?

d) (1 p) Which electricity price will there be between 1 July and 31 December if the reservoir limitation is considered?

Problem 3 (6 p)

a) (2 p) Consider a power system at a time when the frequency is stable at 49.97 Hz. At this moment there is an error in a transformer station and a power plant generating 500 MW is disconnected from the grid. Which of the following alternatives describes what happens in the system?

1. There is a surplus of energy, which results in a voltage increase in the grid. The control systems of the power plants participating in the primary control responds to the voltage increase by reducing the electricity generation.
2. There is a surplus of energy, which results in a voltage decrease in the grid. The control systems of the power plants participating in the primary control responds to the voltage decrease by reducing the electricity generation.
3. There is a deficit of energy, which results in a voltage increase in the grid. The control systems of the power plants participating in the primary control responds to the voltage increase by increasing the electricity generation.
4. There is a deficit of energy, which results in a frequency decrease in the system. The control systems of the power plants participating in the primary control responds to the frequency decrease by increasing the electricity generation.
5. There is a deficit of energy, which results in a frequency increase in the system. The control systems of the power plants participating in the primary control responds to the frequency increase by increasing the electricity generation.

b) (2 p) Consider a power system divided in two areas, A and B. There is only one interconnection between these two areas. This interconnection is composed of a 220 kV AC line with them maximal capacity 600 MW. The line is equipped with a protection system which after a short time delay disconnects the line if the power flow exceeds the maximal capacity of the line.

The transmission at a certain occasion is 500 MW from area A to B. At this occasion the load in area B is increasing by 150 MW. The gain in area A is 2 000 MW/Hz and the gain in area B is equally large. How large is the transmission from area A to area B when the primary control has restored the balance between generation and consumption? (Answer 0 MW if the line is disconnected due to overloading.)

c) (2 p) A certain power plant has a gain of 100 MW/Hz. The base generation (i.e., the generation when the frequency is exactly 50 Hz) is 80 MW and the installed capacity of the power plant is equal to 90 MW. How much will the power plant generate when the frequency of the system is 49.88 Hz?

Problem 4 (12 p)

Stads energi AB owns a thermal power plant with three blocks. Moreover, the company owns a wind farm. Assume that the company has formulated their short-term planning problem as a MILP problem and that the following symbols have been introduced:

Indices for the power plants: Block I - 1, Block II - 2, Block III - 3.

C_g^+ = start-up cost in power plant g, $g = 1, 2, 3$,
D_t = contracted load during hour t, $t = 1, \ldots, 24$,
$G_{g,t}$ = generation in power plant g, hour t, $g = 1, 2, 3$, $t = 1, \ldots, 24$,
$\overline{G}_g$ = installed capacity in power plant g, $g = 1, 2, 3$,
$\underline{G}_g$ = minimal generation when power plant g is committed, $g = 1, 2, 3$,
$s_{g,t}^+$ = start-up variable for power plant g, hour t, $g = 1, 2, 3$, $t = 1, \ldots, 24$,
$u_{g,0}$ = unit commitment of power plant g at the beginning of the planning period, $g = 1, 2, 3$,
$u_{g,t}$ = unit commitment of power plant g, hour t, $g = 1, 2, 3$, $t = 1, \ldots, 24$,
W_t = expected wind power generation in hour t, $t = 1, \ldots, 24$,
β_{Gg} = variable operation cost in power plant g.

a) (3 p) Which of the symbols above represent optimisation variables and parameters respectively?

b) (4 p) Assume that the expected wind power generation always is less than the contracted load. Then, how should the load balance constraint of Stads energi AB be formulated for hour t. Use the symbols defined above.

c) (3 p) At installed capacity the hydro power plant Fallet generates 100 MW and the production equivalent is then 0.5 MWh/HE. The reservoir can hold 12 960 000 m^3. Assume that the power plants up-stream neither discharge or spill any water and that the local inflow is negligible. If we start with a full reservoir how many hours can Fallet generate its installed capacity before the reservoir is empty?

d) (2 p) Assume that a short-term planning problem for three hydro power plants has been formulated as an LP problem, where the objective is to maximise the value of stored water. The optimisation problem is then solved using commercial software (for example GAMS). Let $v_{i,t}$ denote the dual variable of the hydrological constraint for power plant i, hour t. In the following situations we can be certain that there is something wrong with the solution we have obtained: I) If any $v_{i,t} < 0$, II) If any $v_{i,t} = 0$, III) If power plant j is located directly downstream power plant i and $v_{j,t} > v_{i,t}$.

1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. All the statements are true.

Problem 5 (12 p)

a) (2 p) Consider an electricity market with three power plants. The following values of the unserved energy has been obtained from a probabilistic production cost simulation:

$EENS_0$ = 420 MWh/h, $EENS_1$ = 195 MWh/h,
$EENS_2$ = 34.8 MWh/h, $EENS_3$ = 5.01 MWh/h.

What is the *ETOC* of the system if all power plants have the operation cost 10 ¤/MWh?

b) (2 p) The following duration curve has been calculated for the equivalent load including outages in the three power plants:

$$\tilde{F}_3(x) = \begin{cases} 1 & x < 400, \\ 0.4168 & 400 \le x < 500, \\ 0.2710 & 500 \le x < 650, \\ 0.0766 & 650 \le x < 750, \\ 0.0280 & 750 \le x < 900, \\ 0.0064 & 900 \le x < 1\,000, \\ 0.0010 & 1\,000 \le x < 1\,150, \\ 0.0002 & 1\,150 \le x < 1\,250, \\ 0 & 1\,250 \le x, \end{cases}$$

What is the *LOLP* of the system if each power plant has an installed capacity of 250 MW?

c) (2 p) Assume that there is another power plant in the system and that this power plant has the installed capacity 200 MW, availability 80% and the operation cost 12 ¤/MWh. In which interval is $\tilde{F}_4(x)$ equal to 0.30016?

d) (6 p) The same electricity market has also been simulated using Monte Carlo methods. The model used in the Monte Carlo simulation is slightly more advanced than the model used in probabilistic production cost simulation; for example, it considers that some consumers are price sensitive. To obtain an accurate result from the Monte Carlo simulation, it has been decided to se stratified sampling.

The results of the fifteen first scenarios of the Monte Carlo-simulation are compiled in table 1. Which estimates of *ETOC* and *LOLP* are obtained from these results?

Table 1 Results from a Monte Carlo simulation of an electricity market.

Stratum	Stratum weight	Observations of *TOC* [¤/h]	Observations of *LOLO*
1	0.9	3 500, 5 000, 4 500, 3 000, 4 000	0, 0, 0, 0, 0
2	0.09	5 250, 5 500, 7 500, 5 500, 6 250	0, 0, 1, 0, 0
3	0.01	7 500, 7 500, 7 500, 5 000, 7 500	1, 1, 1, 1, 1

PART II (FOR HIGHER GRADES)

All introduced symbols must be defined. Solutions should include sufficient detail that the argument and calculations can be easily followed.

The answer to each problem must begin on a new sheet, but answers to different parts of the same problem (a, b, c, etc.) can be written on the same sheet. The fields *Namn* (Name), *Blad nr* (Sheet number) and *Uppgift nr* (Problem number) must be filled out on every sheet.

Part II gives a total of 60 points, but this part will only be marked if the candidate has obtained at least 35 points in part I. Then the results of parts I and II and the bonus points will be added together to determine the examination grade (3, 4, 5).

Problem 6 (10 p)

Unionen has four member states: Aland, Beland, Celand and Deland. Data for the common electricity market of Unionen are given in table 2. The variable operation costs are assumed to be linear within the intervals, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

Table 2 Data of the common electricity market in Unionen.

Power source	Production capability [TWh/year]				Variable costs [¤/MWh]	CO_2 emissions [ton/MWh]
	Aland	Beland	Celand	Deland		
Hydro	60	115	15	–	6–7	0
Wind	4	4	2	10	10	0
Nuclear	80	–	20	–	80–120	0
Biomass	10	–	15	5	100–250	0
Fossil gas	–	–	10	30	200–360	0,4
Coal condensing	5	–	30	10	220–400	1,0
Consumption	150	120	90	40		

Assume perfect competition, perfect information, and that there are neither capacity, transmission nor reservoir limitations in the common electricity market. Moreover, assume that Unionen has introduced a system for tradable emission rights, which means that the producers must buy an emission right for each ton of CO_2 that they have emitted during the year. The emission rights are traded in a separate market, which means that there will be a market price on emission rights. This market price depends on how many emission rights that are demanded by the electricity producers, as shown in table 3. Estimate the electricity price in the common electricity market of Unionen under these circumstances.

Table 3 The price of emission rights.

The demand of the electricity market [million emission rights]	Price [¤/emission right]
0–30	100
30–45	130
45–55	160
55–65	200

Problem 7 (10 p)

The power system in Rike is divided in two parts. There is a lot of hydro power in the northern part of the system, but most of the load is located in the southern part. There are several parallel AC transmission lines between the two areas. The maximal flow from north to south is 7 000 MW—if this limit is exceeded, the power system becomes unstable and there is a risk that there will be extensive blackouts in the entire or parts of the power system. Therefore, in order to avoid this, the maximal flow is set to 6 000 MW at nominal frequency. The unused transmission capacity is reserved for frequency control.

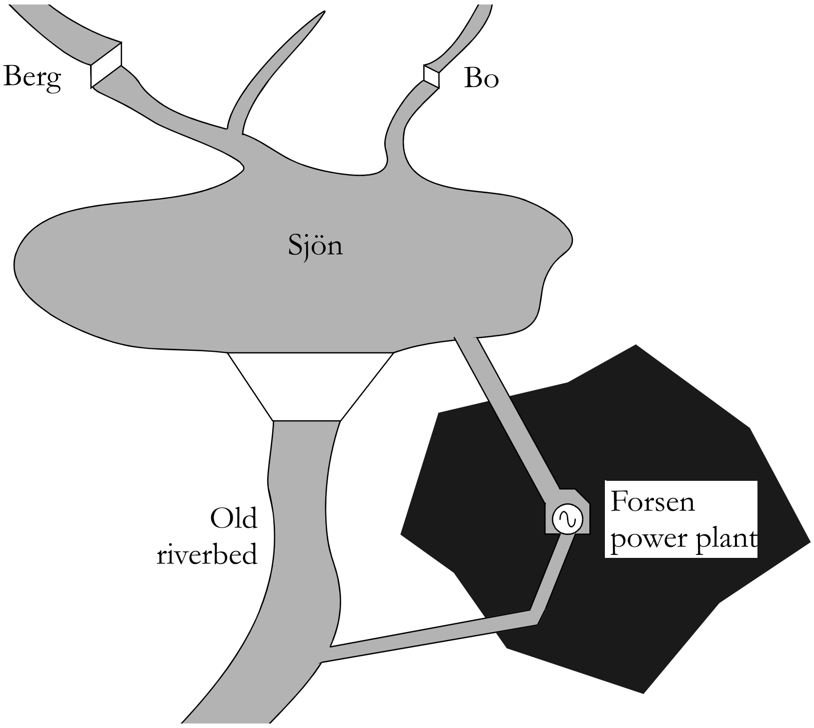
Riksnät is the system operator in Rike and has the responsibility for the frequency control. The primary control that Riksnät is purchasing is divided in two parts: the ordinary primary control capacity and the momentary disturbance reserve. The ordinary primary control capacity should be available in the frequency interval 49.9 to 50.1 Hz and has a total gain of 3 000 MW/Hz, where 2 000 MW/Hz is located in power plants in the northern part of Rike. The momentary disturbance reserve should be available in the frequency interval 49.5 to 49.9 Hz and must be large enough to make the system capable of managing an outage in the largest power plant during normal operation (i.e., when the frequency varies between 49.9 and 50.1 Hz) without overloading the interconnections between the northern and the southern part of the country.

a) (5 p) The largest power plant in Rike is a nuclear power plant, located in the southern part of Rike, with the installed capacity 1 400 MW. How large gain must the momentary disturbance reserve have in order to guarantee that a failure in this power plant during normal operation does not cause the frequency to drop below 49.5 Hz?

b) (5 p) How much of the momentary disturbance reserve must be located in the southern part of Rike?

Hint: Keep in mind that a part of the transmission capability from north to south may be used be the ordinary primary control!

Problem 8 (20 p)



AB Vattenkraft owns three hydro power plants located as in the figure above. Berg and Bo are small run-of-the-river power plants. The power plant Forsen is built underground and the water is led to a from the turbine in tunnels. Sjön serves as reservoir for Forsen and the old riverbed is used for spillage. As the power plant is located in an recreational area, which is popular among trekkers and fishers, the Environment Court has ordered AB Vattenkraft to maintain a flow of at least 10 m^3/s in the old riverbed. More data for the power plants are given in table 4.

AB Vattenkraft is planning to sell their electricity generation at the local power pool, ElKräng. It is assumed that the company can sell as much as they want to the prices listed in table 5. The table also shows the expected local inflow. Stored water is assumed to be used for electricity generation at the best marginal production equivalent in each power plant and future electricity generation is valued 210 SEK/MWh. The water delay time between the power plants can be neglected.

a) (12 p) Formulate the planning problem of AB Vattenkraft as an LP problem. Use the notation in table 6 for the parameters (it is however permitted to add further symbols if you consider it necessary).

NOTICE! The following is required to get full score for this problem:

- The symbols for the optimisation variables must be clearly defined.
- The optimisation problem should be formulated so that it is easy to determine what the objective function is, which constraints there are and which limits there are.
- The possible values for all indices should be clearly stated for each equation.

b) (8 p) Assume that it is desirable to use a better model of the electricity generation as a function of the discharge in Forsen. Any discharge below 50 HE results in poor efficiency and can also cause vibrations that harm the turbine. Therefore, it is preferable to use a model which allows the power plant to be stopped (i.e., no water is discharged through the turbine), but which otherwise does not allow the discharge to be less than 50 HE. The discharge range above 50 HE should be divided in three linear segments. The chosen breakpoints between the linear segments are stated in table 7.

How must the planning problem from parta be reformulated in order to consider the improved model of Forsen. Do not forget to define all new variables and parameters that you introduce!

Table 4 Data for the hydro power plants of AB Vattenkraft.

Power plant	Installed capacity [MW]	Production equivalent [MWh/HE]	Maximal discharge [HE]	Maximal contents of reservoir [HE]	Start contents of reservoir [HE]
Berg	5.5	0.10	55	–	–
Bo	3.2	0.08	40	–	–
Forsen	50	0.5	100	4 000	2 500

Table 5 Forecast for the planning period.

Hour	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Price at ElKräng [SEK/MWh]	193	189	180	175	175	192	202	222	233	233	232	229
Local inflow [HE]												
Berg	49	49	51	55	59	57	52	48	46	45	45	45
Bo	28	30	33	37	35	32	27	25	25	25	25	25
Forsen	11	11	12	12	14	12	11	10	10	10	10	10
Hour	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24
Price at ElKräng [SEK/MWh]	222	212	209	206	202	200	200	203	203	206	207	196
Local inflow [HE]												
Berg	45	45	47	47	48	47	46	44	42	41	40	40
Bo	25	25	25	25	29	32	28	24	23	23	23	23
Forsen	10	10	10	10	11	11	11	9	9	9	9	9

Table 6 Notation for the planning problem of AB Vattenkraft.

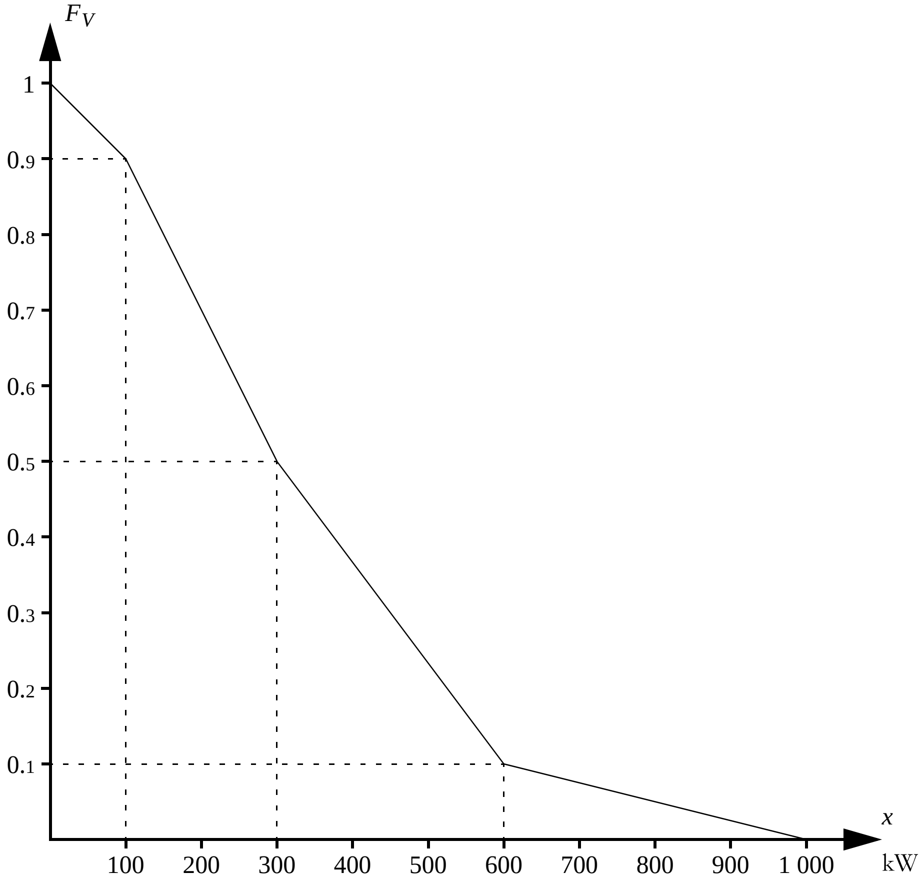
Symbol	Explanation	Value
$\overline{H}_i$	Installed capacity in power plant *i*	See table 4
γ_i	Production equivalent in power plant *i*	See table 4
$\overline{Q}_i$	Maximal discharge in power plant *i*	See table 4
$M_{i,0}$	Start contents of reservoir *i*	See table 4
$\overline{M}_i$	Maximal contents of reservoir *i*	See table 4
λ_t	Expected price at ElKräng hour *t*	See table 5
$V_{i,t}$	Local inflow to power plant *i*, hour *t*	See table 5
S_{river}	Minimum water flow in the old riverbed	10
λ_f	Expected future electricity price	210

Table 7 Electricity generation in Forsen.

Breakpoint	Discharge, *Q* [TE]	Generation, *H* [MWh/TE]	Comment
1	50	23.7	Minimum discharge when Forsen is committed.
2	65	30.9	Local best efficiency
3	80	39.6	Best efficiency
4	100	50.0	Maximal discharge.

Problem 9 (20 p)

Isla Desierta is located in the Atlantic Ocean. The electricity consumption of the island can be considered normally distributed with the mean 600 kW and the standard deviation 100 kW. THe power supply is managed by a local power company, Electricidad de Isla Desierta (EdID). The company can import electricity from the mainland via an HVDC transmission line, which is 100% available. This line is one-directional, i.e., it is only possible to transfer power from the mainland to Isla Desierta and not the other way around. The line has a capacity of 1 000 kW and EdID pays 1 ¤/kWh for imported electricity. The company also has a hydro power plant of their own on the island. This hydro power plant has a capacity of 500 kW, the availability is 100% and the operation cost is negligible. The hydro power plant has no reservoir; hence, the maximal generation is limited by the water flow, which depends on the precipitation and how much water is flowing from the volcanic wells of the island. A duration curve for the available water flow, V (converted to kW), is shown in the figure below. The water flow can be assumed to be independent of the load in the system.



a) (5 p) Formulate a discrete model of the available generation capacity in the hydro power plant. The model should have three states: no generation capacity available, installed capacity available, and a state in between. Do not forget to motivate your calculations!

b) (5 p) Use the model from part a and probabilistic production cost simulation to calculate the expected operation cost of EdID. To simplify the calculations you may use the following approximation of the load duration curve:

$$\tilde{F}_0(x) = \begin{cases} 1 & x < 500, \\ 0.4 & 500 \le x < 750, \\ 0 & 750 \le x. \end{cases}$$

If you have not solved the problem in part a, then you may use the following model of the hydro power plant: 50% probability that installed capacity is available, 40% probability that half the installed capacity is available and 10% probability that no generation capacity is available.

c) (3 p) Assume that the EdID system is to be simulated using Monte Carlo methods. How is it then appropriate to model the available generation capacity in the hydro power plant? Do not forget to motivate your answer!

d) (1 p) How many random numbers are necessary to generate a scenario for a Monte Carlo simulation of the EdID system?

e) (6 p) Use an appropriate set of random numbers from table 8 to generate a scenario for a Monte Carlo simulation of the EdID system. Moreover, state the complementary scenarios that can be created from the same random numbers.

Hint: For symmetrical probability distributions, it holds that if $X = \mu + \delta$, where $\mu = E[X]$ and δ is an arbitrary number, then the complementary random number is $X^* = \mu - \delta$.

Table 8 Some random numbers for a Monte Carlo simulation of the EdID system.

Probability distribution	$U(0, 1)$	$N(600, 100)$
Random number	0.95 0.25 0.60	595 725 675

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Answer sheet for part I

Name: ....

Personal number: ....

Problem 1

a) Alternative .... is correct.

b) Alternative .... is correct.

c) Alternative .... is correct.

Problem 2

a) .... ¤/MWh

b) .... TWh

c) .... ¤/MWh **d)** .... ¤/MWh

Problem 3

a) Alternative .... is correct.

b) .... MW **c)** .... MW

Problem 4

a) Parameters: ....

Optimisation variables: ....

b) ....

c) .... hours.

d) Alternative .... is correct.

Problem 5

a) .... ¤/h

b) .... %

c) .... $\leq x <$ ....

d) *ETOC* .... ¤/h *LOLP* .... %

Problem 1

a) 5, **b)** 1, **c)** 2.

Problem 2

a) The total load in Land during the year is 250 TWh. The inflow and the start contents in the reservoirs together amount to 225 TWh, but as 25 TWh are to be saved for next year, the hydro power can only generate 200 TWh. The remaining 50 TWh must originate from the coal condensing, which means that half the coal condensing capacity is utilised. The price must then be 450 ¤/MWh.

b) During the first half of the year the coal condensing is generating 25 TWh, which means that the hydro power generates 105 TWh, while the inflow is 150 TWh. Hence, the reservoirs are filled by 45 TWh during this period. As the reservoirs held 20 TWh at the beginning, they must then hold 65 TWh at the end of the period.

c) From part b we can conclude that the reservoirs must be filled on 30 June. To avoid spilling water, the hydro power must generate in total 110 TWh between 1 January and 30 June, which means that the coal condensing has to generate 20 TWh during the same period. This means that 40% of the coal condensing capacity is utilised and therefore the price must be 440 ¤/MWh.

d) During the period 1 July to 31 December, the hydro power has access to 60 TWh (stored water from the first half of the year) + 55 TWh (inflow) – 25 TWh (water stored for next year) = 90 TWh, which means that the coal condensing must generate 30 TWh during the same period. This means that 60% of the coal condensing capacity is utilised and therefore the price must be 460 ¤/MWh.

Problem 3

a) 4.

b) Since the gain is the same in both areas, the generation increase will also be the same. In area B the load has increased by 150 MW, but the generation has only increased by 75 MW, which means that the remaining 75 MW must be due to import from A. The transmission from A to B must therefore increase to 575 MW, which is within the limit.

c) The relation between frequency and generation yields that the power plant should generate

$$G = G_0 - R(f - f_0) = 80 - 100(49.88 - 50) = 92 \text{ MW},$$

which is not possible. The power plant will then generate as much as possible, i.e., 90 MW.

Problem 4

a) Parameters: C_g^+, D_t, $\overline{G}_g$, $\underline{G}_g$, $u_{g,0}$, W_t and β_{Gg}. Optimisation variables: $G_{g,t}$, $s_{g,t}^+$, and $u_{g,t}$.

b) $\sum_{g=1}^{3} G_{g,t} + W_t = D_t$.

c) The discharge at installed capacity is given by the relation $Q = H/\gamma(Q) = 100/0{,}5 = 200$ HE. As the reservoir can store 12 960 000/3 600 = 3 600 H, a full reservoir will be enough for 18 hours maximal generation.

d) 2.

Problem 5

a) The total generation in the three power plants is given by $(EENS_0 - EENS_1) + (EENS_1 - EENS_2) + (EENS_2 - EENS_3) = EENS_0 - EENS_3 = 420 - 5.01 = 414.99$ MWh/h. The expected operation cost is then $ETOC = 10 \cdot 414.99 = 4\,149.9$ ¤/h.

b) The risk of power deficit is given by $LOLP = \tilde{F}_3(750) = 2.8\%$.

c) As $\tilde{F}_4(x) \geq \tilde{F}_3(x)\ \forall x$, the lower limit for the interval in question must be at least 500 MW. We can calculate $\tilde{F}_4(x)$ for $x \geq 500$ using the convolution formula until we find the requested interval:

$$\tilde{F}_4(x) = 0.8\tilde{F}_3(x) + 0.2\tilde{F}_3(x - 200) =$$

$$= \begin{cases} \dots & \\ 0.8 \cdot 0.271 + 0.2 \cdot 1 = 0.271 & 500 \leq x < 600, \\ 0.8 \cdot 0.271 + 0.2 \cdot 0.4168 = 0.30016 & 600 \leq x < 650, \\ \dots & \end{cases}$$

d) The following estimates are obtained of the expectation value in each stratum:

$m_{TOC1} = 20\,000/5 = 4\,000$ $\quad m_{LOLO1} = 0$
$m_{TOC2} = 30\,000/5 = 6\,000$ $\quad m_{LOLO2} = 1/5 = 0.2$
$m_{TOC3} = 35\,000/5 = 7\,000$ $\quad m_{LOLO3} = 5/5 = 1$

Thus, we get

$$m_{TOC} = \sum_{h=1}^{3} \omega_h m_{TOCh} = 0.9 \cdot 4\,000 + 0.09 \cdot 6\,000 + 0.01 \cdot 7\,000 = 4\,210 \text{ ¤/h},$$

$$m_{LOLO} = \sum_{h=1}^{3} \omega_h m_{LOLOh} = 0 + 0.09 \cdot 0.2 + 0.01 \cdot 1 = 2.8\%.$$

Problem 6

Let us start by studying the electricity market without considering the price of emission rights, in order to get an idea about the size of the emission from the electricity producers of Unionen. The total demand is 400 TWh/year. Wind power, hydro power, nuclear power and biomass can in total contribute with 340 TWh/year, which is not sufficient. Thus, we will also have to use fossil gas and coal condensing. Assume that the electricity price is λ. For λ in the interval 220–360 ¤/MWh the fossil gas and coal condensing are generating

$$\frac{\lambda - 200}{360 - 200}40 + \frac{\lambda - 250}{400 - 220}45.$$

This contribution should be equal to 60, because the other power sources generate 340 TWh/year and the demand is 400 TWh/year. Solving this equation yields the electricity price $\lambda = 330$, which results in a total emission of 40.5 Mton CO_2.

Based on the results above we can assume that the price of an emission right will be 130 ¤. The consequence for fossil gas producers is that the variable operation cost is increasing by 0.4 · 130 = 52 ¤/MWh and for coal condensing the increase is 130 ¤/MWh. The contribution form these two power sources should still be 60 TWh, which gives the following equation:

$$\frac{\lambda - 252}{412 - 252}40 + \frac{\lambda - 350}{530 - 350}45 = 60.$$

Solving this equation yields $\lambda = 421$ ¤/MWh, but at this price all fossil gas should be utilised (40 TWh). Hence, we know that the coal condensing should generate 20 TWh, i.e., we need to use 4/9 of the coal condensing capacity. Consequently, we use 4/9 of the price interval for coal condensing, resulting in the electricity price 430 ¤/MWh.

Now we just have to validate our assumption on the price of emission rights. The fossil gas is emitting 40 TWh · 0.4 ton/MWh = 16 Mton CO_2 and the coal condensing is emitting 20 Mton. The producers will thus need to purchase 36 million emission rights, which according to table 3 results in the assumed price 130 ¤/emission right.

Problem 7

a) Assume that the nuclear power plant is disconnected in a situation when the resources of the ordinary primary control are exhausted, i.e., when the frequency is 49.9 Hz. The momentary disturbance reserve must then supply 1 400 MW without allowing the frequency to become lower than 49.5 Hz, which requires a gain of 1 400/0.4 = 3 500 MW/Hz.

b) Firsts we must calculate how large part of the reserves on the interconnection between north and south which is used by the ordinary primary control. At the frequency 49.9 Hz, the ordinary primary control in the northern part is increasing its generation by 2 000 · 0.1 = 200 MW. The worst case scenario is that the entire increase must be transferred to the southern part of Rike. In that case there will be a reserve of 800 MW transmission capacity left. Apparently, if the momentary then should supply 1 400 MW then at least 600 MW must be located in the southern part. In other words, the disturbance reserve in the southern part must have a gain of 600/0.4 = 1 500 MW/Hz.

Problem 8

a) The problem we want to solve is

maximise *income of sold electricity + value of stored water*,
subject to *hydrological balance of hydro power plant*,
limittions in generation capacity, etc.

Indices of the power plants

Berg 1, Bo 2, Forsen 3.

Parameters

The parameters are defined in table 6 in the problem text.

Optimisation variables

$Q_{i,t}$ = discharge in power plant i during hour t, $i = 1, 2, 3$, $t = 1, \ldots, 24$,
$S_{i,t}$ = spillage from reservoir i during hour t, $i = 1, 2, 3$, $t = 1, \ldots, 24$,
$M_{i,t}$ = contents of reservoir i at the end of hour t, $i = 1, 2, 3$, $t = 1, \ldots, 24$.

Objective function

$$\text{maximise} \quad \sum_{t=1}^{24} \lambda_t \left(\sum_{i=1}^{3} \gamma_i Q_{i,t} \right) + \lambda_f \gamma_3 M_{3,24}.$$

Constraints

Hydrological balance for Berg and Bo:

$$Q_{i,t} + S_{i,t} = V_{i,t}, \qquad i = 1, 2, t = 1, \ldots, 24.$$

Hydrological balance for Forsen:

$$M_{3,t} - M_{3,t-1} + Q_{3,t} + S_{3,t} - \sum_{i=1}^{2} (Q_{i,t} + S_{i,t}) = V_{3,t}, \quad t = 1, \ldots, 24.$$

Variable limits

$0 \le Q_{i,t} \le \bar{Q}_{i,j}$, $\quad i = 1, 2, 3, t = 1, \ldots, 24,$
$0 \le S_{i,t}$ $\quad i = 1, 2, t = 1, \ldots, 24,$
$S_{river} \le S_{3,t}$ $\quad t = 1, \ldots, 24,$
$0 \le M_{3,t} \le \bar{M}_3,$ $\quad t = 1, \ldots, 24.$

b) Let $Q_{3,b}$ and $H_{3,b}$ denote discharge and electricity generation respectively at the breakpoint b as defined in table 7. The maximal discharge in each segment is then given by

$$\bar{Q}_{3,j} = Q_{3,j+1} - Q_{3,j} = \begin{cases} 15 & j = 1, \\ 15 & j = 2, \\ 20 & j = 3. \end{cases}$$

and the marginal production equivalents by

$$\mu_{3,j} = \frac{H_{3,j+1} - H_{3,j}}{\bar{Q}_{3,j}} = \begin{cases} 0{,}48 & j = 1, \\ 0{,}58 & j = 2, \\ 0{,}52 & j = 3. \end{cases}$$

We can now observe that the model of Forsen will require two integer variables: one to manage the forbidden interval for the discharge and another one to prevent discharge in segments 2 and 3 before segment 1 is fully utilised. Hence, we need to introduce the following new optimisation variables:

$u_{3,t}$ = unit commitment for Forsen during hour t (1 if the power plant is committed, 0 otherwise),
$z_{3,t}$ = permission to discharge in segments 2 and 3 (1 if discharge is possible in seg-

ments 2 and 3, 0 otherwise),

$Q_{3,j,t}$ = discharge in segment j, hour t.

Besides, we need two more parameters:

$\underline{H}_3$ = minimal generation when Forsen is committed = 23.7,
$\underline{Q}_3$ = minimal discharge when Forsen is committed = 50,

The modified model of the electricity generation of Forsen must then be included in the objective function:

$$\text{maximise} \quad \sum_{t=1}^{24} \lambda_t \left(\sum_{i=1}^{2} \gamma_i Q_{i,t} + \underline{H}_3 u_{3,t} + \sum_{j=1}^{3} \mu_{3,j} Q_{3,j,t} \right) + \lambda_f \gamma_3 M_{3,24}.$$

As we now have four optimisation variables representing the discharge in Forsen, we must also modify the hydrological constraint of this power plant:

$$M_{3,t} - M_{3,t-1} + \underline{Q}_3 u_{3,t} + \sum_{j=1}^{3} Q_{3,j,t} + S_{3,t} - \sum_{i=1}^{2} (Q_{i,t} + S_{i,t}) = V_3, t = 1, \ldots, 24.$$

It is only possible to discharge in segment 1 when Forsen is committed; therefore, we introduce the following constraint:

$$Q_{3,1,t} \le \overline{Q}_{3,1} u_{3,t}, \qquad t = 1, \ldots, 24.$$

Discharge in segments 2 and 3 is only allowed then the discharge in segment 1 equals its maximal value. The following constraints guarantees that the discharge in segment 1 is maximal whenever the binary variable allowing discharge in segments 2 and 3 is equal to one:

$$\overline{Q}_{3,1} z_{3,t} \le Q_{3,1,t}, \qquad t = 1, \ldots, 24.$$

Finally, we have the following constraint which ensures that nothing is discharged in segments 2 and 3 unless $z_{3,t} = 1$:

$$Q_{3,j,t} \le \overline{Q}_{3,j} z_{3,t}, \qquad j = 2, 3, t = 1, \ldots, 24.$$

The upper limits of the discharge variables are set by the constraints above, which leaves us with the following variable limits:

$$u_{3,t} \in \{0, 1\}, \qquad t = 1, \ldots, 24,$$

$$z_{3,t} \in \{0, 1\}, \qquad t = 1, \ldots, 24,$$

$$0 \le Q_{3,j,t}, \qquad j = 1, 2, 3, t = 1, \ldots, 24.$$

Problem 9

a) In reality the available generation capacity can assume any value between 0 and 500 kW, but in our model we only consider three possible states: 0, 250 and 500 kW respectively. The question is which probability should be assigned to each of these states. In other words, we need a discrete approximation of the continuous duration curve for the water flow. A simple and good solution is to represent flows between 0 and 125 kW in the continuous model by the discrete state $\overline{H} = 0$ kW, flows between 125 kW and 375 kW in the continuous model corresponds to the discrete state $\overline{H} = 250$ kW, and finally all flows larger than 375 kW are represented by the discrete state $\overline{H} = 500$ kW. Consequently, we get

$$P(\overline{H} = 0) = P(V < 125) = 1 - \tilde{F}_V(125) = \{\text{see figure}\} = 0.15,$$

$$P(\overline{H} = 250) = P(V > 375) - P(V < 125) = \tilde{F}_V(125) - \tilde{F}_V(375) = \{\text{see figure}\} = 0.45,$$

$$P(\overline{H} = 500) = P(V > 375) = \tilde{F}_V(375) = \{\text{see figure}\} = 0.4.$$

By that, we have identified three states for the hydro power plant and calculated the probability of each state, which is what is needed for a power plant model in a probabilistic production cost simulation.

b) *ETOC* depends on the cost to import electricity from the mainland. Therefore, we need to calculate *EENS* with and without the import. As the hydro power plant has the least operation cost, we add this unit first:

$$\tilde{F}_1(x) = 0.4\tilde{F}_0(x) + 0.45\tilde{F}_0(x - 250) + 0.15\tilde{F}_0(x - 500) = \\ = \begin{cases} 0.4 \cdot 1 + 0.45 \cdot 1 + 0.15 \cdot 1 = 1 & x < 500, \\ 0.4 \cdot 0.4 + 0.45 \cdot 1 + 0.15 \cdot 1 = 0.76 & 500 \le x < 750, \\ 0.4 \cdot 0 + 0.45 \cdot 0.4 + 0.15 \cdot 1 = 0.33 & 750 \le x < 1\,000, \\ 0.4 \cdot 0 + 0.45 \cdot 0 + 0.15 \cdot 0.4 = 0.06 & 1\,000 \le x < 1\,250, \\ 0 & 1\,250 \le x. \end{cases}$$

The electricity import can be considered as a 100% reliable power plant with a capacity of 1 000 kW, i.e., $\tilde{F}_2(x) = \tilde{F}_1(x)$. Thus, we get

$$EENS_1 = \int_{500}^{\infty} \tilde{F}_1(x)dx = 250 \cdot 0.76 + 250 \cdot 0.33 + 250 \cdot 0.06 = 287.5 \text{ kWh/h},$$

$$EENS_2 = \int_{1\,500}^{\infty} \tilde{F}_2(x)dx = 0 \text{ kWh/h}.$$

The expected import is $EENS_1 - EENS_2 = 237.5$ kWh/h, which results in $ETOC = 287.5$ ¤/h.

c) We do not need a discrete approximation of the hydro power plant in the Monte Carlo simulation. To calculate the available generation capacity in a specific scenario, we just randomise a water flow from the given duration curve.

d) In each scenario there are two scenario parameters (i.e., values that vary in a random manner from scenario to scenario): water flow and electricity consumption. Hence, we need two random numbers to create a scenario.

e) Assume that the first random number from the$U(0, 1)$-distribution is used to generate the water flow and that the first random number from the $N(600, 100)$-distribution is used to generate the total load.

The water flow is calculated using the inverse transform method: $V = F_V^{-1}(0.95) = \{\text{see figure}\} = 50$ kW. The complementary random number is then $V^* = F_V^{-1}(1 - 0{,}95) = \{\text{see figure}\} = 800$ kW. Hence, we get the available generation capacities $\overline{H} = 50$ kW and $\overline{H}^* = 500$ kW respectively. (In the second case we cannot exceed the installed capacity of the power plant, even though the water flow is large enough to generate more than 500 kW).

The load is given directly by the first random number form the normal distribution: $D = 595$ kW. As the normal distribution is symmetrical, we then have $D^* = 605$ kW.

These random numbers and complementary random numbers can be combined into four scenarios:

- **Scenario 1.** Available hydro power capacity: 50 kW. Load: 595 kW.
- **Scenario 2.** Available hydro power capacity: 50 kW. Load: 605 kW.
- **Scenario 3.** Available hydro power capacity: 500 kW. Load: 595 kW.
- **Scenario 4.** Available hydro power capacity: 500 kW. Load: 605 kW.