KTH Electrical Engineering

Exam in EG2050 / 2C1118 System Planning, 13 March 2008, 14:00–19:00, Q13, Q15

Allowed aids

In this exam in you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten**, **single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

PART I (MANDATORY)

Write all answers on the answer sheet provided. Motivations and calculations do not have to be presented.

Part I can yield 40 points in total. The examinee is guaranteed to pass if the score is at least 33 points. If the result in part I is at least 31 points, then there will be a possibility to complement for passing the exam with the grade E.

Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

a) (2 p) The consumers in a bilateral electricity market has the following choices: I) They can choose which system operator they want, II) They can choose which retailer they want, III) The can choose which player should manage their balance responsibility.

1. None of the statements is true.
2. Only I is true.
3. Only III is true.
4. I and II are true but not III.
5. II and III are true but not I.

b) (1 p) We use the notion "real-time trading" to describe all the trading which occurs during the hour of delivery (or any other trading period). Which of the following contracts can be traded in a real-time market?

1. Balance power, i.e., when a balance responsible player is selling any surplus in their balance to the system operator, or when a balance responsible player is buying from the system operator to cover for any deficit in their balance.
2. Firm power, i.e., the customer buys the same amount of energy in each trading period as long as the contract is valid.
3. Regulation power, i.e., when a player at request from the system operator is supplying more power to the system (up-regulation) or when a player at request from the system operator is supplying less power to the system (down-regulation).

c) (1 p) Some power exchanges allow so-called block bids. What does that mean?

1. A block bid is a sell or purchase bid which is valid for one single trading period.
2. A block bid is a sell or purchase bid which is valid for several trading periods, and which can only be accepted as a whole.
3. A block bid is a sell or purchase bid which is valid for one single trading period, but which can only be accepted if the same player during that trading period is allowed to sell regulation power to the system operator.

Problem 2 (6 p)

Consider a simplified model of the electricity market in Land. The maximal daily production and the variable costs are shown in table 1 below. The variable operation costs are assumed to be linear within the intervals, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

Table 1 Data for the electricity producers in Land.

Power source	Production capability [TWh/year]	Variable costs [¤/MWh]
Coal condensing	10	300–500
Nuclear power	70	100
Hydro power	70	5
Wind power	1	5

a) (2 p) Assume that the electricity market in Land has perfect competition, perfect information and that there are neither capacity, transmission nor reservoir limitations. How large is the electricity consumption if the electricity price is 380 ¤/MWh?

b) (1 p) Assume that the electricity price is 380 ¤/MWh. AB Elbolaget owns hydro power with a production capability of 20 TWh/year and nuclear power with a production capability of 25 TWh/year. How large is the gross income of AB Elbolaget for sold electricity?

c) (1 p) How much does the total variable costs of AB Elbolaget decrease if the production capability of the nuclear power plants is decreased to 24 TWh/year due to maintenance works. The demand is the same as in the previous problems.

d) (2 p) How large is the gross income of AB Elbolaget if the production capability of the nuclear power plants is decreased to 24 TWh/year due to maintenance works. The demand is the same as in the previous problems.

Problem 3 (6 p)

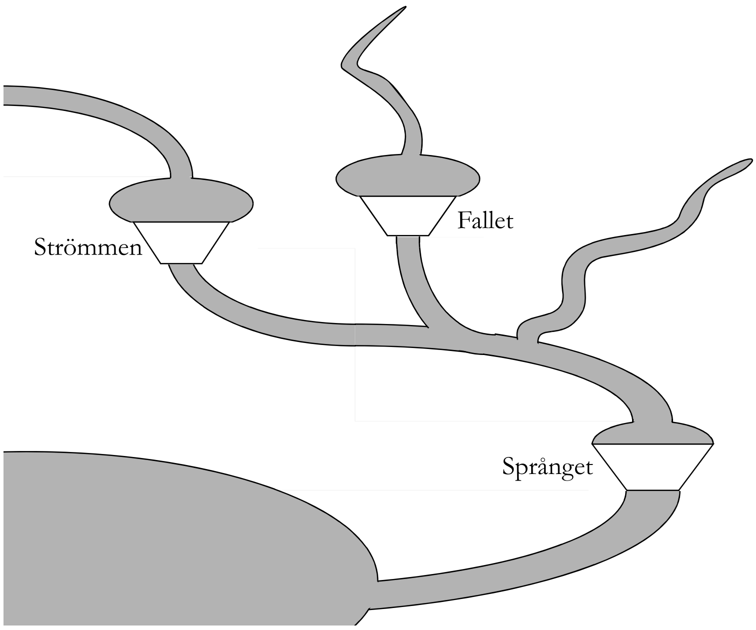
The hydro power plant Forsen has an installed capacity of 50 MW. To avoid damage on the turbines of the power plant, it is never allowed to generate less than 22 MW when it is committed. The power plant is part of the primary control of Land and has a gain of 80 MW/Hz. Forsen is producing 36 MW at 2 pm. The other power plants in Land that participate in the primary control has a total gain of 3 920 MW/Hz. The gain of the other power plants is available in the frequency range 50±0.4 Hz.

The frequency of the system is 50.075 Hz at 2 pm. Shortly thereafter a nuclear power plant in Land is started again after an earlier outage, which means that the power system is supplied another 896 MW.

a) (3 p) How much will Forsen produce when the primary control has restored the balance between generation and consumption in the system?

b) (3 p) What will the frequency be in Land? Answer with three decimals!

Problem 4 (12 p)



a) (4 p) The best efficiency in the hydro power plant Strömmen is obtained for the discharge 160 m^3/s and the electricity generation is then 40 MW. The maximal discharge in Strömmen is 200 m^3/s and the relative efficiency is then 96%. Assume that we need a piecewise linear model of electricity generation as function of the discharge in Strömmen. The model should have two segments and the breakpoint between them should be located at the best efficiency. Calculate the following parameters:

μ_j = marginal production equivalent in Strömmen, segment j,
$\overline{Q}_j$ = maximal discharge in Strömmen, segment j.

b) (4 p) AB Vattenkraft owns the three hydro power plants Strömmen, Fallet and Språnget (see the figure above). AB Vattenkraft sells power to customers with firm power contracts, but the company also has the possibility to trade at the local power exchange ElKräng. The following symbols have been introduced in a short-term planning problem for these hydro power plants:

Indices for the power plants: Strömmen 1, Fallet 2, Språnget 3.

γ_i = expected future production equivalent for water stored in reservoir i, i = 1, 2, 3,
λ_t = expected electricity price at ElKräng hour t, t = 1, …, 24,
λ_{25} = expected electricity price at ElKräng after the end of the planning period,
$M_{i,0}$ = contents of reservoir i at the beginning of the planning period, i = 1, 2, 3,
$M_{i,t}$ = contents of reservoir i at the end of hour t, i = 1, 2, 3, t = 1, …, 24,
$\mu_{i,j}$ = marginal production equivalent in power plant i, segment j, i = 1, 2, 3, j = 1, 2,
p_t = purchase from ElKräng hour t, t = 1, …, 24,
r_t = sales to ElKräng hour t, t = 1, …, 24,
$Q_{i,j,t}$ = discharge in power plant i, segment j, during hour t, i = 1, 2, 3, j = 1, 2, t = 1, …, 24.

Formulate the objective function if the aim of the planning problem is to maximise the income of sold electricity at ElKräng plus the value of stored water minus the costs of purchasing electricity from ElKräng. Use the symbols defined above.

c) (2 p) The following symbols in the planning problem of AB Vattenkraft denote optimisation variables: I) $M_{i,0}$, II) p_t, III) $Q_{i,j,t}$.

1. None of the statements is true.
2. Only I is true.
3. Only III is true.
4. II and III are true but not I.
5. All the statements are true.

d) (2 p) The following variables and parameters have been introduced in a short-term planning problem for a thermal power plant:

C^+ = start-up cost of the power plant,
C^- = stop cost of the power plant,
$G_{g,t}$ = generation in the power plant during hour t,
s_t^+ = start-up variable for hour t (1 if the power plants starts generating at the beginning of hour t, otherwise 0),
s_t^- = stop variable for hour t (1 if the power plant stops generating at the beginning of hour t, otherwise 0).
u_t = unit commitment during hour t (1 if the power plant is committed, otherwise 0),
β = variable generation cost.

The following objective function is used in the planning problem:

$$\text{minimise} \quad \sum_{t \in \mathcal{T}} (\beta G_t + C^+ s_t^+ + C^- s_t^-).$$

Which of the following constraints can be used to control the relation between start-up, stop and unit commitment?

I) $u_t - u_{t-1} - s_t^+ + s_t^- = 0.$

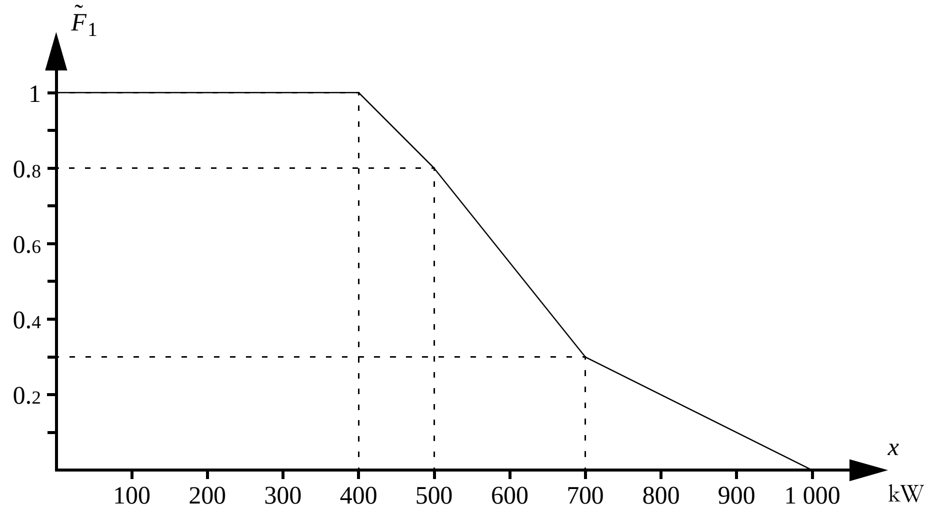
II) $u_t - u_{t-1} - s_t^+ \leq 0.$

III) $u_t - u_{t-1} + s_t^+ - s_t^- = 0.$

1. None of the alternatives is correct.
2. Only alternative I is correct.
3. Only alternative II is correct.
4. Only alternative III is correct.
5. It is possible to choose between using alternative I and alternative II.

Problem 5 (12 p)

Ekibuga is a town in East Africa. The town is not connected to a national grid, but has a local system of its own. The local grid is supplied by a hydro power plant and a diesel generator set. The hydro power plant does not have a reservoir, but the water flow is always sufficient to generate the installed capacity (800 kW) and the risk for outages in the power plant is negligible. The diesel generator set has a capacity of 200 kW, the availability is 85% and the operation cost is 2 ¤/kWh.



a) (2 p) The figure above shows the equivalent load duration curve including outages in the hydro power plant. How large is the expected energy not served per hour if only the hydro power plant is considered?

b) (2 p) The expected energy not served when considering both the hydro power plant and the diesel generator is 3 kWh/h. Calculate the expected total operation cost per hour for the system.

c) (2 p) Calculate the risk of power deficit in the system.

d) (3 p) There is some correlation between the load and the risk of outages in the diesel generator set as well as the electricity grid in Ekibuga. A Monte Carlo simulation of the system in Ekibuga has been performed in order to assess the impact of these correlations. The results of the twelve first scenarios are shown in table 2. As can be seen in the table, both stratified sampling and control variates has been used in the simulation. Calculate for each stratum the expected difference in risk of power deficit between the two models.

e) (3 p) The simplified model that has been used in the simulation of Ekibuga corresponds to the model used in probabilistic production cost simulation. Which estimate of the *LOLP* for the power system of Ekibuga is obtained from the results in table 2?

Table 2 Results from a Monte Carlo simulation of the power system in Ekibuga.

Stratum, h	Stratum weight, ω_h	Observations of *LOLO* according to the detailed model, $x_{h,i}$	Observations of *LOLO* according to the simplified model, $z_{h,i}$
1	0.82	0, 0, 0, 0, 0	0, 0, 0, 0, 0
2	0.15	0, 1, 0, 0, 0	0, 0, 0, 0, 0
3	0.03	1, 1	1, 1

PART II (FOR HIGHER GRADES)

All introduced symbols must be defined. Solutions should include sufficient detail that the argument and calculations can be easily followed.

The answer to each problem must begin on a new sheet, but answers to different parts of the same problem (a, b, c, etc.) can be written on the same sheet. The fields *Namn* (Name), *Blad nr* (Sheet number) and *Uppgift nr* (Problem number) must be filled out on every sheet.

Part II gives a total of 60 points, but this part will only be marked if the candidate has obtained at least 33 points in part I. Then the results of parts I and II and the bonus points will be added together to determine the examination grade (A, B, C, D, E).

Problem 6 (10 p)

The electricity market in Rike is dominated by two large power companies, Rikskraft and Södra Rikes allmänna elbolag (SRAE). There is also a number of smaller municipal power companies.

Table 3 shows the forecasts of Rikskraft and SRAE respectively concerning the total available generation capability (i.e., the sum of the production capability of the company itself and the production capability of the competitors) for the coming year. The table also shows the variable costs, which are assumed to be linear within the intervals, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

Table 3 Forecasts of the available production capability of the electricity market in Rike.

	Rikskraft	SRAE	Variable cost [¤/MWh]
Hydro power	70	65	5
Nuclear power	51	52	100
Biomass	20	20	150–350
Coal condensing	20	20	300–500

a) (3 p) Rikskraft estimates the so-called water value as the average electricity price for the coming year. In this estimation, it is assumed that there is perfect competition on the electricity market in Rike and that there are neither capacity, transmission nor reservoir limitations. The company is using the forecasts stated in table 3, and a consumption forecast which states that the electricity demand for the next year will be 135 TWh. Which water value will Rikskraft be using?

b) (3 p) SRAE is estimating the water value using the same method as Rikskraft, but they use their own forecasts for available production capability and their own consumption forecast, which suggests that the load will be 136 TWh/year. Which water value will SRAE be using?

c) (4 p) Table 4 shows the available capacity for the power plants of different companies during hour *t*. Assume that all electricity trading in Rike is done at the power exchange, and that the companies submits bids of thermal power according to variable costs, and that the hydro power plant is submitted according to the water value (as calculated in problem a and b respectively). Which electricity price will there be in the power exchange if the price is determined by a price cross, and if there are no grid limitations during hour *t*? The demand is 15 000 MW during hour *t* and the consumers are not price sensitive.

Table 4 Available production capacity for a certain hour on the electricity market in Rike.

	Available hydro power capacity [MWh/h]	Available nuclear power capacity [MWh/h]	Available biomass capacity [MWh/h]	Available coal condensing capacity [MWh/h]
Rikskraft	5 000	4 000	250	–
SRAE	3 000	3 000	500	500
Other power companies	–	–	1 750	2 000

Problem 7 (10 p)

The power system in Rike is divided in two parts. There are large amounts of hydro power in the northern part of the system, but the main consumption centres are in the southern part. There are several parallel AC transmission lines between the two parts. The maximal flow on these lines is 7 000 MW – if this limit is exceeded the power system becomes unstable and there is a risk for extensive blackouts in the entire or parts of the system.

The requirements for the frequency control in Rike is that the system should be able to manage a dimensioning fault (i.e., an outage of the largest power plant) without the frequency dropping below 49.0 Hz and without exceeding the maximal power flow between the two areas. Part of the transmission capability must therefore be reserved for frequency control, which means that the maximal transmission at nominal frequency (50 Hz) must be less than 7 000 MW.

Table 5 The frequency control in Rike.

	Northern Rike	Southern Rike
Gain, 49.9–50.1 Hz [MW/Hz]	2 000	1 000
Gain, 49.0–49.9 Hz [MW/Hz]	900	300
Dimensioning fault [MW]	750	1 200

a) (5 p) How large is the maximal transmission from north to south at nominal frequency?

b) (5 p) How large is the maximal transmission from south to north at nominal frequency?

Problem 8 (20 p)

The electricity market in Land is subject to electricity disclosure, and the consumers can choose between several electricity products. Stads energi AB offers their customers two electricity products: grey electricity and bio electricity. The grey electricity can be produced in any power plant, but the bio electricity must be produced in biomass-fuelled power plants. The electricity disclosure is accounted per hour, which means that if the customers buying bio electricity are consuming 100 MWh during a certain hour, then Stads energi AB must produce at least 100 MWh bio electricity that hour.

Stads energi AB owns two power plants: the coal condensing power plant Sotinge and the biomass-fuelled power plant Flisinge. Each power plant has two blocks. The data of the different blocks are shown in table 6.

a) (16 p) On Thursday afternoon, the company has received notice about how much they will sell of each electricity product during the Friday. Formulate the planning problem of Stads energi AB as a MILP problem. Use the notation in table 9 for the parameters (it is however permitted to add further symbols if you consider it necessary).

NOTICE! The following is required to get full score for this problem:

- The symbols for the optimisation variables must be clearly defined.
- The optimisation problem should be formulated so that it is easy to determine what the objective function is, which constraints there are and which limits there are.
- The possible values for all indices should be clearly stated for each equation.

b) (4 p) Assume that a solution to the planning problem from part a is available, and that this solution includes both the primal optimisation variables as well as the dual variables. Do you have enough information to determine how much Stads energi AB would earn if the electricity disclosure was accounted per day instead of per hour? If that is the case, show how the calculations are performed. Otherwise, explain how to set about calculating the saving.

Hint: If the electricity disclosure is accounted per day, then the company every hour has to produce as much as the total sales of the two electricity products during that hour, i.e., if the customers consume 50 MWh grey electricity and 50 MWh bio electricity, then the company must generate 100 MWh during this hour. However, it is acceptable to for example generate 75 MWh in Sotinge and 25 MWh in Flisinge, as long as the generation and demand of bio electricity is in balance during the day; if the total sales of bio electricity is 2 400 MWh one day, then the company must generate 2 400 MWh ni Flisinge, but these 2 400 MWh can be generated during any time of the day.

Table 6 Data for the power plants of Stads energi AB.

	Sotinge block I	Sotinge block II	Flisinge block I	Flisinge block II
Installed capacity [MW]	150	120	90	80
Minimal generation when committed [MW]	40	30	20	15
Variable generation cost [SEK/MWh]	280	350	250	300
Start-up cost [SEK]				
After 1 hour down-time	10 000	7 500	5 000	4 500
After 2 hours down-time	18 000	13 500	9 000	8 500
After 3 hours down-time or more	30 000	22 500	15 000	14 000

Table 7 Generation schedule for Thursday evening.

Time	Generation [MWh]			
	Sotinge block I	Sotinge block II	Flisinge block I	Flisinge block II
21–22	85	0	90	15
22–23	80	0	90	0
23–24	70	0	90	0

Table 8 Contracted load during Friday.

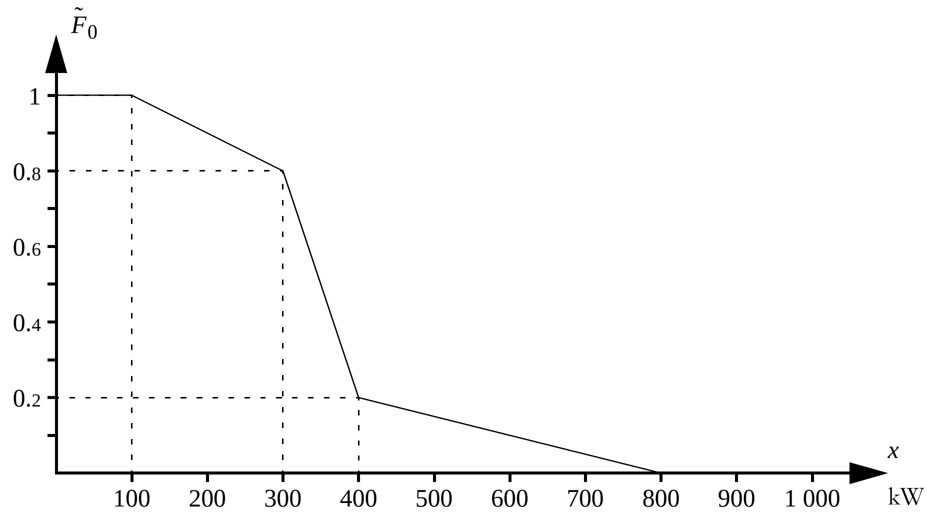
Hour	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Grey electricity [MWh]	90	90	90	90	90	100	110	120	120	120	125	120
Bio electricity [MWh]	65	60	55	50	60	65	70	80	85	80	80	80
Hour	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24
Grey electricity [MWh]	120	125	125	125	115	100	90	90	90	90	90	90
Bio electricity [MWh]	85	80	80	85	85	95	105	105	100	95	85	75

Table 9 Notation for the planning problem of Stads energi AB.

Symbol	Explanation	Value
$\overline{G}_g$	Installed capacity in power plant g	See table 6
$\underline{G}_g$	Minimal generation when power plant g is committed	See table 6
β_{Gg}	Variable generation cost in power plant g	See table 6
C_g^*	Start-up cost in power plant g after one hour down-time.	See table 6
C_g^{**}	Start-up cost in power plant g after two hours down-time.	See table 6
C_g^{***}	Start-up cost in power plant g after at least three hours down-time.	See table 6
$u_{g,t}$	Unit commitment in power plant g before the start of the planning problem, $t = -2, -1, 0$	See table 7
$D_{1,t}$	Sales of grey electricity hour t	See table 8
$D_{2,t}$	Sales of bio electricity hour t	See table 8

Problem 9 (20 p)

Large parts of the rural areas of Eggwanga have no access to electricity. The Eggwangan authorities have started a rural electrification programme, and as part of this programme, the Eggwanga National Electricity Supply Company Ltd. (ENESCO) has been given the task of electrifying the village Ekyaro and its surroundings. ENESCO is now investigating how much it would cost to connect Ekyaro to the national grid.



a) (5 p) The figure above show the load duration curve of Ekyaro. The line between Ekyaro and the national grid will have a capacity of 500 kW. As the electricity demand in the national grid is large compared to the generation capacity, rotating load curtailment is common practice. ENESCO estimates that Ekyaro will be disconnected in average 438 hours per year. Assume that the average generation cost of electricity fed into the national grid is 5 ¤/kWh and that the losses on the line between Ekyaro and the national grid can be neglected. Use probabilistic production cost simulation to calculate the expected annual operation cost of the power system in Ekyaro.

b) (5 p) To increase the reliability of supply in Ekyaro, ENESCO is considering installing a number of local diesel generator sets, which can be used when the line from the national grid is disconnected, or when the load in Ekyaro is higher than the capacity of the line. Each diesel generator set will have a capacity of 150 kW and an availability 90%. How many diesel generator sets are needed to keep the risk of load shedding in Ekyaro below 10%?

c) (6 p) To obtain a more accurate calculation of the expected operation cost of Ekyaro, ENESCO would like to simulate the power system in Ekyaro using a more detailed model. The objective of this model is to include that the marginal generation cost in the national grid is different at different times, and that the losses on the line between the national grid and Ekyaro amounts to 4%. ENESCO is using the following frequency function to model the marginal generation cost:

$$f(x) = \begin{cases} 0.45 & x = 2, \\ 0.35 & x = 6, \\ 0.2 & x = 10, \\ 0 & \text{all other } x. \end{cases}$$

Consider the system where Ekyaro is only supplied by the line from the national grid (i.e., there are no diesel generator sets). Define an electricity market model which can be used to analyse the scenarios in a Monte Carlo simulation of the power system in Ekyaro, where the objective of the simulation is to estimate the expected operation cost and the risk of load shedding in the system. State the scenario parameters, model constants and result variables of your model, and describe how the result variables are calculated.

d) (3 p) Use the random number 0.3 from a $U(0, 1)$-distribution to generate a random value of the marginal generation cost of the national grid. Moreover, state the complementary random number of this value.

e) (1 p) When using complementary random numbers in a Monte Carlo simulation, a negative correlation is created between the input values of the electricity market model (the scenario parameters) in different scenarios. How must this negative correlation affect the correlation between the outputs (the result variables) in different scenarios if the method should reduce the variance of the estimate, $Var[m_X]$?

1. The result variables must be negatively correlated.
2. The result variables must be independent.
3. The result variables must be positively correlated.

KTH Electrical Engineering

Answer sheet for part I

Name: ....

Personal number: ....

Problem 1

a) Alternative .... is correct.

b) Alternative .... is correct.

c) Alternative .... is correct.

Problem 2

a) .... TWh/year **b)** .... M¤/year

c) .... M¤/year **d)** .... M¤/year

Problem 3

a) .... MW **b)** .... Hz

Problem 4

a) μ_1 .... MWh/HE μ_2 .... MWh/HE

$\bar{Q}_1$ HE .... $\bar{Q}_2$ .... HE

b) ....

c) Alternative .... is correct.

d) Alternative .... is correct.

Problem 5

a) .... kWh/h **b)** .... ¤/h

c) .... %

d) Stratum 1: .... % Stratum 2: .... %

Stratum 3: .... %

e) .... %

Suggested solution for exam in i EG2050/2C1118 System Planning, 13 March 2008.

Problem 1

a) 5, **b)** 3, **c)** 2.

Problem 2

a) At the electricity price 380 ¤/MWh, we get maximal generation from hydro power, wind power and nuclear power, i.e., in total 141 TWh. Besides, 40% of the price interval for the coal condensing is used, which gives another 0.4·10 = 4 TWh. The consumption is thus 145 TWh/year.

b) AB Elbolaget sells in total 45 TWh/year for the price 380 ¤/MWh, which results in an income of 45 000 000 MWh · 380 ¤/MWh = 17 100 M¤/year.

c) The variable operation cost of 1 TWh nuclear power is 1 000 000 MWh · 100 ¤/MWh. The total variable costs of AB Elbolaget would therefore decrease by 100 M¤/year.

d) The 1 TWh decrease in nuclear power must be replaced by the same amount of coal condensing, which means that another 10% of the coal condensing capacity will be used. This results in a price increase corresponding to 10% of the price interval for coal condensing, i.e., 20 ¤/MWh. The total income of the company is then 44 TWh · 400 ¤/MWh = 17 600 M¤/year.

Problem 3

a) The power plants that are part of the primary control compensates the start of the nuclear power plant by decreasing the generation by in total 896 MW. The contribution of each individual power plant is determined by its share of the total gain. Forsen corresponds to 2% of the gain in the system; hence, it should reduce the generation by 17.92 MW. However, the power plant only has the possibility to reduce by 14 MW. Consequently, the power plant will generate 22 MW (least possible power when the unit is committed).

b) The other power plants in the system must reduce the generation by 882 MW, which requires a that the frequency is increasing by 882 MW/3 920 MW/Hz = 0.225 Hz. The new frequency in the system is then 50.075 + 0.225 = 50.3 Hz.

Problem 4

a) The following data are given in the problem text:

$\overline{Q}$ = maximal discharge in Strömmen = 200,
$\hat{Q}$ = discharge in Strömmen at best efficiency = 160,
$\hat{H}$ = generation in Strömmen at best efficiency = 40,
$\eta(\overline{Q})$ = relative efficiency at maximal discharge in Strömmen = 0.96.

To calculate the marginal production equivalents, we need the generation at maximal discharge, which can be calculated using the formula $H = \gamma_{max} \cdot \eta(Q) \cdot Q$. First however, we must calculate the maximal production equivalent, which is obtained at best efficiency:

γ_{max} = maximal production equivalent in Strömmen = 40/160 = 0.25 MWh/HE.

The generation we need is now given by

$\overline{H}_i$ = maximal generation in Strömmen = 0.25 · 0.96 · 200 = 48 MW.

The marginal production equivalents can now be calculated according to

$$\mu_1 = \frac{\hat{H}}{\hat{Q}}$$

and

$$\mu_2 = \frac{\overline{H} - \hat{H}}{\overline{Q} - \hat{Q}},$$

which results in the following linear models of the power plant:

μ_j = marginal production equivalent in Strömmen, segment j =

$$= \begin{cases} 0.25 & j = 1, \\ 0.20 & j = 2, \end{cases}$$

$$\overline{Q}_j = \text{maximal discharge in Strömmen, segment } j = \begin{cases} 160 & j = 1, \\ 40 & j = 2. \end{cases}$$

b) maximise $\sum_{t=1}^{24} \lambda_t(r_t - p_t) + \lambda_f((\gamma_1 + \gamma_3)M_{1,24} + (\gamma_2 + \gamma_3)M_{2,24} + \gamma_3 M_{3,24})$.

c) 4.

d) 2.

Problem 5

a) The unserved energy during an hour is given by

$$EENS_1 = 1 \cdot \int_{800}^{\infty} \tilde{F}_1(x)dx = 0.2 \cdot 200/2 = 20 \text{ kWh/h}.$$

b) The expected generation in the diesel generator set is $EG_2 = EENS_1 - EENS_2 = 17$ kWh/h. Hence, the expected operation cost is f $ETOC = 2EG_2 = 34$ ¤/h.

c) The risk of power deficit is given by $\tilde{F}_2(1\,000) = 0.85\tilde{F}_1(1\,000) + 0.15\tilde{F}_1(800) = 0 + 0.15 \cdot 0.2 = 0.03$. The risk of power deficit is thus 3%.

d) The expected difference between the two models in one stratum is given by

$$m_{(X-Z)h} = \frac{1}{n_h} \sum_{i=1}^{n_h} (x_{h,i} - z_{h,i}) = \begin{cases} 0 & h = 1, \\ 0.20 & h = 2, \\ 0 & h = 3. \end{cases}$$

e) The final estimate of *LOLP* is obtained by weighting together the expected difference per stratum and then adding the expectation value of the simplified model (i.e., the *LOLP* calculated in part c):

$$LOLP \approx \sum_{h=1}^{3} \omega_h m_{(X-Z)h} + \mu_Z = 0 + 0.15 \cdot 0.2 + 0 + 0.03 = 6\%.$$

Problem 6

a) According to the forecast of Rikskraft, the hydro power and nuclear power can supply 121 TWh, which means that the other power plants have to produce 14 TWh. Assume that the electricity price is λ. For λ in the interval 150–300 ¤/MWh the biomass power plants will generate

$$\frac{\lambda - 150}{350 - 150}20.$$

Setting this expression equal to 14 and solving for λ yields $\lambda = 290$ ¤/MWh, which is within the assumed interval. Thus, according to the forecast of Rikskraft, there will be balance between production and consumption if the electricity price is 290 ¤/MWh.

b) According to the forecast of SRAE, the hydro power and nuclear power can supply 117 TWh, which means that the other power plants have to produce 19 TWh. Assume that the electricity price is λ. For λ in the interval 300–350 ¤/MWh the total contribution from biomass and coal condensing is

$$\frac{\lambda - 150}{350 - 150}20 + \frac{\lambda - 300}{500 - 300}20.$$

Setting this expression equal to 19 and solving for λ yields $\lambda = 320$ ¤/MWh, which is within the assumed interval. Thus, according to the forecast of SRAE, there will be balance between production and consumption if the electricity price is 320 ¤/MWh.

c) Assume that the electricity price is equal to the water value of SRAE, i.e., 320 ¤/MWh. At this price all nuclear power plants will be utilised (because the sell price in the bids of the nuclear power plants is 100 ¤/MWh) as well as all the hydro power of Rikskraft (because the sell price in this bid is 290 ¤/MWh). Moreover, 85% of the bids from biomass-fuelled power plants will be accepted (because the electricity price 320 ¤/MWh corresponds to 85% of the price interval 150–300 ¤/MWh) and 10% of the coal condensing (because 320 ¤/MWh corresponds to 10% of the price interval 300–500 ¤/MWh). This gives in total $7\,000 + 5\,000 + 0.85 \cdot 2\,500 + 0.1 \cdot 2\,500 = 14\,375$ MWh. The remaining 625 MW can be covered by the hydro power of SRAE, which means that there will be balance between production and consumption at the electricity price 320 ¤/MWh.

Problem 7

a) If a fault occurs in northern Rike then part of the generation in northern Rike will be replaced by import from the southern part of the country. A dimensioning fault in northern Rike will therefore not constitute a limitation to the maximal transmission from north to south. If a dimensioning fault is occurring in southern Rike then the power flow from north to south is going to increase, because part of the outage in southern Rike will be replaced by generation in the primary control of northern Rike. The transmission between the two areas may not so large that there are no margins for this transmission increase.

The total gain in Rike is 3 000 MW/Hz in the interval 49.9–50.1 Hz. If the frequency drops to 49.9 Hz the total generation increase will be 300 MW, which is not sufficient to replace an outage of 1 200 MW in southern Rike. The frequency must therefore drop even more, so that the power plants which still contribute to the gain when the frequency is less than 49.9 Hz will increase the generation by another 900 MW. As the total gain now is reduced to 1 200 MW/Hz, the frequency change will be $\Delta f = 900/1\,200$ Hz compared to the frequency $f = 49.9$, i.e., the final frequency will be $49.9 - 0.75 = 49.15$ Hz. At this frequency the power plants in northern Rike have increased the generation by $0.1 \cdot 2\,000 + 0.75 \cdot 900 = 875$ MW. The maximal transmission from north to south may therefore not exceed $7\,000 - 875 = 6\,125$ MW at nominal frequency.

b) If a dimensioning fault is occurring in northern Rike then the power flow from south to north is going to increase, because part of the outage in northern Rike will be replaced by generation in the primary control of southern Rike. Similar reasoning as in part a yields that the frequency in Rike drops to $49.9 - 450/1\,200 = 49.525$ Hz. At this frequency the power plants in southern Rike have increased the generation by $0.1 \cdot 1\,000 + 0.375 \cdot 300 = 212.5$ MW. The maximal transmission from south to north may therefore not exceed $7\,000 - 212{,}5 = 6\,787.5$ MW at nominal frequency.

Problem 8

a) The problem we want to solve is

minimise	*generation costs + start-up costs*
subject to	*limitations in generation capacity,*
	relation between unit commitment and start-up of power plants,
	load balance.

The differentiation between two different electricity products can be managed in different ways. One way would be to have a load balance constraint for the total load (i.e., *total generation = total contracted load*)and another constraint for the bio electricity (i.e., *total generation in Flisinge ≥ sold bio electricity*). In this solution, we have opted for a division of the generation in each power plant; one part which is sold as grey electricity and one part which is sold as bio electricity.

Indices for the power plants

Sotinge I - 1, Sotinge II - 2, Flisinge I - 3, Flisinge II - 4.

Indices for electricity products

Grey electricity - 1, bio electricity - 2.

Parameters

The parameters are defined in table 9 in the problem text.

Optimisation variables

$G_{g,e,t}$ = generation in power plant g, hour t, sold as electricity product e, $g = 1, \ldots, 4$, $e = 1, 2$, $t = 1, \ldots, 24$,

$u_{g,t}$ = unit commitment in power plant g during hour t, $g = 1, \ldots, 4$, $t = 1, \ldots, 24$,

$s^{*}_{g,t}$ = start-up of power plant g, hour t, after one hour down-time, $g = 1, \ldots, 4$, $t = 1, \ldots, 24$,

$s^{**}_{g,t}$ = start-up of power plant g, hour t, after two hours down-time, $g = 1, \ldots, 4$, $t = 1, \ldots, 24$,

$s^{***}_{g,t}$ = start-up of power plant g, hour t, after at least three hours down-time, $g = 1, \ldots, 4$, $t = 1, \ldots, 24$.

Objective function

$$\text{minimise} \sum_{t=1}^{24} \sum_{g=1}^{4} (\beta_{Gg}(G_{g,1,t} + G_{g,2,t}) + C_g{}^{*} s_{g,t}{}^{*} + C_g{}^{**} s_{g,t}{}^{**} + C_g{}^{***} s_{g,t}{}^{***}).$$

Constraints

Maximal generation in the power plants:

$$G_{g,1,t} + G_{g,2,t} \le u_{g,t}\overline{G}_g, \qquad g = 1, \ldots, 4, t = 1, \ldots, 24.$$

Minimal generation in the power plants:

$$G_{g,1,t} + G_{g,2,t} \ge u_{g,t}\underline{G}_g, \qquad g = 1, \ldots, 4, t = 1, \ldots, 24.$$

Relation between unit commitment and start-up after a certain down-time:

$$u_{g,t} - u_{g,t-1} - u_{g,t-2} - u_{g,t-3} - s_{g,t}^{***} \le 0, \qquad g = 1, \ldots, 4, t = 1, \ldots, 24,$$

$$u_{g,t} - u_{g,t-1} - u_{g,t-2} - s_{g,t}^{***} - s_{g,t}^{**} \le 0, \qquad g = 1, \ldots, 4, t = 1, \ldots, 24,$$

$$u_{g,t} - u_{g,t-1} - s_{g,t}^{***} - s_{g,t}^{**} - s_{g,t}^{*} \le 0, \qquad g = 1, \ldots, 4, t = 1, \ldots, 24.$$

Load balance for grey electricity:

$$\sum_{g=1}^{4} G_{g,1,t} = D_{1,t}, \qquad t = 1, \ldots, 24.$$

Load balance for bio electricity:

$$\sum_{g=3}^{4} G_{g,2,t} = D_{2,t}, \qquad t = 1, \ldots, 24.$$

Variable limits

$$0 \le G_{g,e,t}, \qquad g = 1, \ldots, 4, e = 1, 2, t = 1, \ldots, 24,$$

$$u_{g,t} \in \{0, 1\}, \qquad g = 1, \ldots, 4, t = 1, \ldots, 24,$$

$$s_{g,t}^{*} \in \{0, 1\}, \qquad g = 1, \ldots, 4, t = 1, \ldots, 24,$$

$$s_{g,t}^{**} \in \{0, 1\}, \qquad g = 1, \ldots, 4, t = 1, \ldots, 24,$$

$$s_{g,t}^{***} \in \{0, 1\}, \qquad g = 1, \ldots, 4, t = 1, \ldots, 24.$$

b) It does not help to have access to the dual variables, because the dual variables state how much the optimal value of the objective function will change if there is a small change in the right-hand-side of a constraint. To investigate how the objective function is changing when the electricity disclosure is accounted on a daily basis instead of per hour, it will be necessary to reformulate the load balance constraints, so that there is a requirement that the generation equals the total sales per hour, i.e.,

$$\sum_{g=1}^{4} \sum_{e=1}^{2} G_{g,e,t} = \sum_{e=1}^{2} D_{e,t}, \qquad t = 1, \ldots, 24,$$

and a requirement that the total sales per day of an electricity product is in balance with the total generation during that day, i.e.,

$$\sum_{t=1}^{24} \sum_{g=1}^{4} G_{g,1,t} = \sum_{t=1}^{24} D_{1,t},$$

$$\sum_{t=1}^{24} \sum_{g=3}^{4} G_{g,2,t} = \sum_{t=1}^{24} D_{2,t}.$$

Problem 9

a) The line can be expected to be disconnected during 438 of the 8 760 hours in a year, which corresponds to5% of the time. The line can therefore be modelled as a power plant with a capacity of 500 kW, generation cost 5 ¤/kWh and availability 95%. The expected annual generation in such a power plant is calculated according to

$$EG_1 = 8\,760 \cdot 0{,}95 \int_0^{500} \tilde{F}_0(x)dx = 8\,760 \cdot 0.95(1 \cdot 100 + (1 + 0.8)/2 \cdot 200 + (0.8 + 0.2)/2 \cdot 100$$
$$+ (0.2 + 0.15)/2 \cdot 100) \approx 2.89 \text{ GWh/year}.$$

The expected operation cost in a year is then $ETOC = 5EG_1 \approx 14.46$ M¤/year.

b) The risk of load shedding in Ekyaro when there is only the line from the national grid is given by

$$LOLP_1 = \tilde{F}_1(500) = 0.95\tilde{F}_0(500) + 0.05\tilde{F}_0(500 - 500) = 0.95 \cdot 0.15 + 0.05 \cdot 1 = 19.25\%.$$

With one diesel generator set, we get

$$LOLP_2 = \tilde{F}_2(650) = 0.9\tilde{F}_1(650) + 0.1\tilde{F}_1(500) =$$

$$= 0.9(0.95\tilde{F}_0(650) + 0.05\tilde{F}_0(650 - 500)) + 0.1 \cdot 0.1925 =$$
$$= 0.9(0.95 \cdot 0.075 + 0.05 \cdot 0.95) + 0.1 \cdot 0.1925 \approx 12{,}61\%.$$

With two diesel generator sets, we get

$$LOLP_3 = \tilde{F}_3(800) = 0.9\tilde{F}_2(800) + 0.1\tilde{F}_2(650) =$$

$$= 0.9(0.9\tilde{F}_1(800) + 0.1\tilde{F}_1(800 - 150)) + 0{,}1 \cdot 0{,}126125 =$$

$$= 0.9(0.9(0.95\tilde{F}_0(800) + 0.05\tilde{F}_0(800 - 500)) + 0.1\tilde{F}_1(650)) + 0.1 \cdot 0.126125 =$$
$$= 0.9(0.9 \cdot (0.95 \cdot 0 + 0.05 \cdot 0.8) + 0.1 \cdot (0.95 \cdot 0.075 + 0.05 \cdot 0.95)) + 0.1 \cdot 0.126125 = 5{,}57\%.$$

In conclusion, two diesel generator sets are needed to keep the *LOLP* ni Ekyaro below 10%.

c) Introduce the following scenario parameters:

D = load in Ekyaro,
β = marginal production cost in the national grid,
$\overline{P}$ = available transmission capacity from the national grid.

Introduce the following model constant:

β_L = loss coefficient for transmission from the national grid.

The import from the national grid, i.e., the power that is injected on the line, is denoted P and is a result variable. It is calculated according to

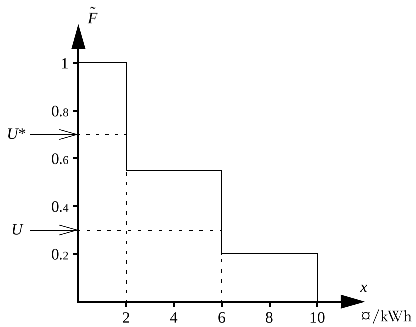
$$P = \begin{cases} 0 & \text{if } \bar{P} = 0, \\ \dfrac{D}{1-\beta_L} & \text{if } \bar{P} = 500 \text{ and } D \le (1-\beta_L)\bar{P} = 480, \\ 500 & \text{if } \bar{P} = 500 \text{ and } D > (1-\beta_L)\bar{P} = 480. \end{cases}$$

Finally, we have the two result variables which are needed to estimate *ETOC* and *LOLP*: the operation cost is given by $TOC = \beta \cdot P$, and *LOLO* which is equal to 1 if $\bar{P} = 0$ (i.e., if the line is disconnected) or if $D > 480$ (i.e., if power that reaches Ekyaro if 500 kW is injected to the line), and which otherwise is equal to 0.

d) According to the definition we have

$$\tilde{F}(x) = \sum_{t > x} f(t) = \begin{cases} 1 & x < 2, \\ 0.55 & 2 \le x < 6, \\ 0.2 & 6 \le x < 10, \\ 0 & 10 \le x. \end{cases}$$

By drawing the duration curve, it is easy to see that $U = 0.3$ is transformed to 6 ¤/kWh and $U^* = 1 - U = 0.7$ is transformed to 2 ¤/kWh.



e) 1.