KTH Electrical Engineering

Exam in EG2050 / 2C1118 System Planning,
5 June 2008, 8:00–13:00, V34

Allowed aids

In this exam you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten**, **single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

PART I (MANDATORY)

Write all answers on the answer sheet provided. Motivations and calculations do not have to be presented.

Part I can yield 40 points in total. The examinee is guaranteed to pass if the score is at least 33 points. If the result in part I is at least 31 points, then there will be a possibility to complement for passing the exam with the grade E.

Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

a) (1 p) Which of the following players have the responsibility to continuously maintain the physical balance between generation and consumption?

1. Each producer and consumer.
2. The balance responsible players.
3. The system operator.

b) (1 p) We use the notion "ahead trading" to describe all the trading which occurs before the hour of delivery (or any other trading period). Which of the following contracts can be traded in an ahead market?

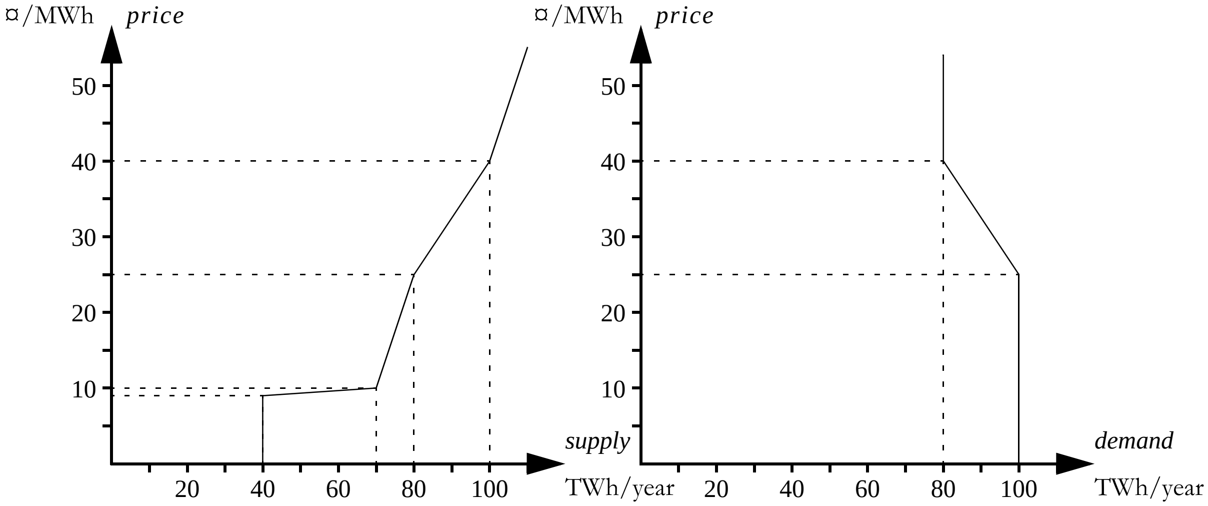
1. Balance power, i.e., when a balance responsible player is selling any surplus in their balance to the system operator, or when a balance responsible player is buying from the system operator to cover for any deficit in their balance.
2. Firm power, i.e., the customer buys the same amount of energy in each trading period as long as the contract is valid.
3. Regulation power, i.e., when a player at request from the system operator is supplying more power to the system (up-regulation) or when a player at request from the system operator is supplying less power to the system (down-regulation).

c) (2 p) Consider a balance responsible company which during one hour has generated 876 MWh, sold 400 MWh to the power exchange, sold 458 MWh to consumers with take-and-pay contracts and sold 20 MWh regulation power to the system operator. How large is the imbalance of this company for this hour?

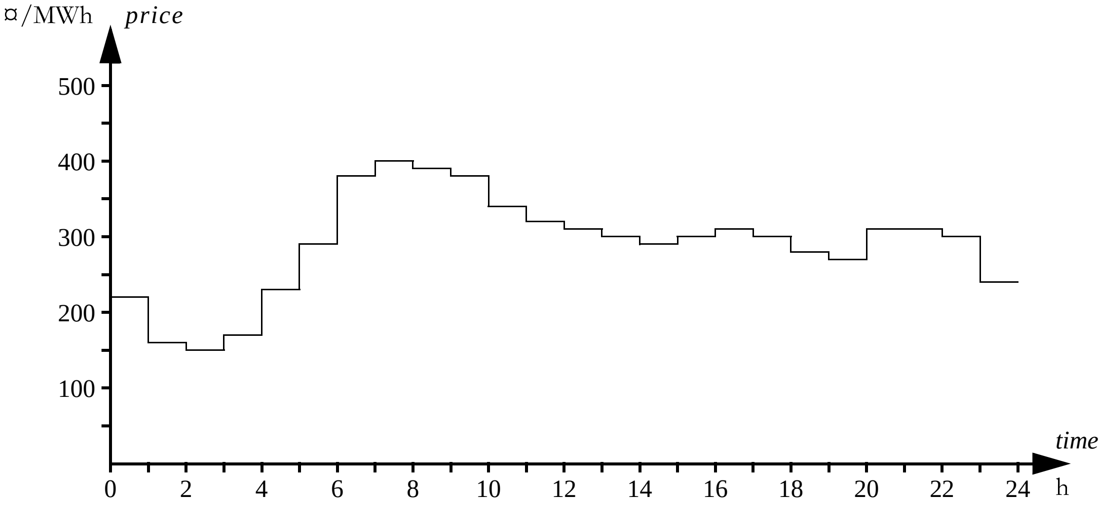
1. 0 MWh.
2. 2 MWh.
3. 18 MWh.
4. 22 MWh.
5. None of the alternatives above is correct.

Problem 2 (6 p)

a) (3 p) The figures below shows the supply and demand curves of a certain electricity market. What will the electricity price become in this electricity market if we assume perfect competition, perfect information and that there are neither transmission, reservoir nor capacity limitations?



b) (3 p) The figure below shows the electricity price in a certain electricity market during one day. Assume that there is perfect competition in this electricity market, that all players have access to perfect information, and that there are neither transmission nor reservoir limitations. How much will be generated during this day in a power plant with the variable operation cost 200 ¤/MWh and the installed capacity 100 MW?



Problem 3 (6 p)

Consider a power system divided in three areas, which are connected by AC lines. Each transmission line is equipped with a protection system which after a short time delay disconnects the line if the power flow exceed the maximal capacity. Data for the interconnections are given in table 1.

The frequency of the system is 50.02 Hz at 8:00, and the total gain in the system is 4 000 MW/Hz and is available in the interval 50±1 Hz. Shortly afterwards, a nuclear power plant (which is not participating in the primary control) is stopped in area C, which results in a loss of 1 000 MW generation. The primary control responds to this outage by increasing the generation by 250 MW in area A, 500 MW ni area B and 250 MW in area C.

Table 1 Data for the interconnections in problem 3.

Interconnection	Capacity [MW]	Transmission at 8:00
A ↔ B	1 000	300 MW from A to B
B ↔ C	2 000	100 MW from B to C

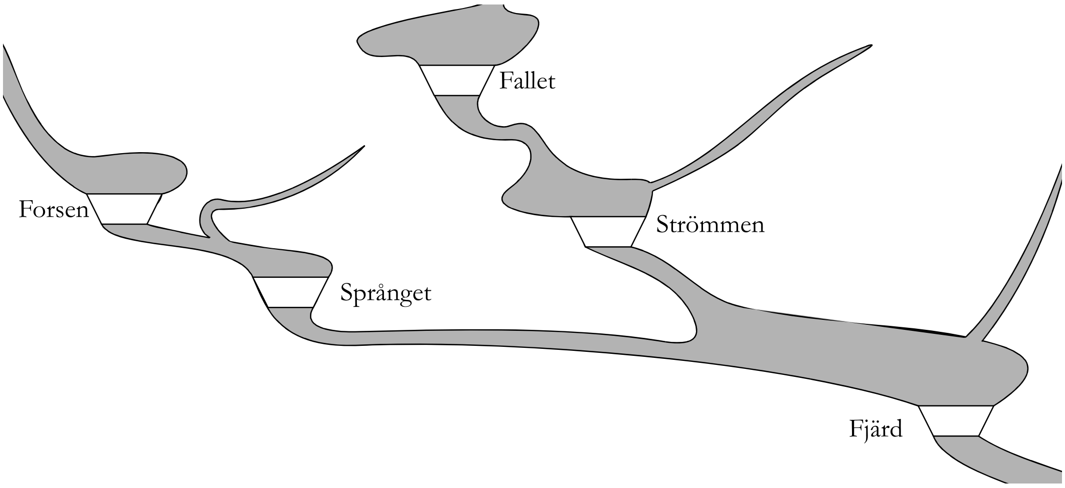
a) (1 p) How large is the gain in area A?

b) (1 p) How large is the gain in area B?

c) (1 p) How large is the gain in area C?

d) (3 p) How large is the transmission from area B to C when the primary control has restored the balance between generation and demand. (Answer 0 MW if the interconnection is disconnected due to overloading.).

Problem 4 (12 p)



a) (4 p) Formulate the hydrological constraint of Fjärd, hour *t*. The water delay time between the power plants can be neglected. Use the following symbols:

Indices for the power plants: Forsen 1, Språnget 2, Fallet 3, Strömmen 4, Fjärd 5.

$M_{i,0}$ = contents of reservoir i at the beginning of the planning period, $i = 1, \ldots, 5$,

$M_{i,t}$ = contents of reservoir i at the end of hour t, $i = 1, \ldots, 5$, $t = 1, \ldots, 24$,

$Q_{i,j,t}$ = discharge in power plant i, segment j, during hour t, $i = 1, \ldots, 5$, $j = 1, 2$, $t = 1, \ldots, 24$.

$S_{i,t}$ = spillage from reservoir i during hour t, $i = 1, \ldots, 5$, $t = 1, \ldots, 24$,

$V_{i,t}$ = local inflow to reservoir i during hour t, $i = 1, \ldots, 5$, $t = 1, \ldots, 24$.

b) (3 p) Which of the symbols in part a denote optimisation variables and parameters respectively?

c) (1 p) The best efficiency of the power plant Strömmen is achieved at the discharge 100 m^3/s. The power plant is then generating 37 MW. How large is the maximal production equivalent of the power plant?

d) (2 p) In the following cases it is necessary to use integer variables to model the electricity generation in a thermal power plant: I) When the power plant has a start-up cost stated in SEK/start, II) When the power plant has a stop cost which is stated in SEK/stop, III) When the power plant has a minimum generation level, $\underline{G}$, when committed.

1. None of the statements is true.
2. I and II are true but not III.
3. I and III are true but not II.
4. II and III are true but not I.
5. All the statements are true.

e) (2 p) Assume that it has been decided that a thermal power plant should not be committed for a shorter time than four hours, i.e., if the power plant is started 12:00 then it may not be shut down again before 16:00. Introduce the following symbols:

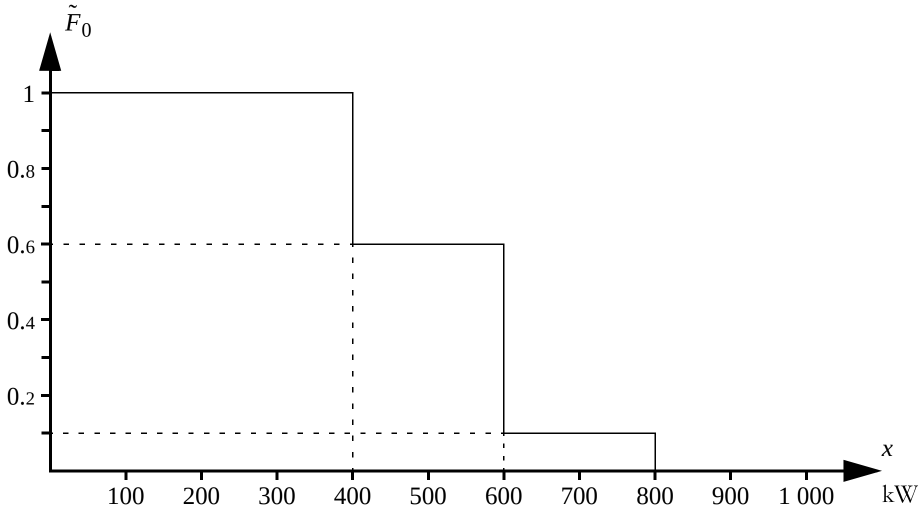
s_t^+ = start-up variable for hour *t* (1 if the power plant is starting generation at the beginning of hour *t*, otherwise 0),
s_t^- = stop variable for hour *t* (1 if the power plant is stopping generation at the beginning of hour *t*, otherwise 0).

How should a linear constraint be formulated in order to describe the relation between s_t^+, s_{t+1}^-, s_{t+2}^- and s_{t+3}^-?

1. $s_t^+ - s_{t+1}^- - s_{t+2}^- - s_{t+3}^- = 0.$
2. $s_t^+ - s_{t+1}^- - s_{t+2}^- - s_{t+3}^- \geq 0.$
3. $s_t^+ - s_{t+1}^- - s_{t+2}^- - s_{t+3}^- = 1.$
4. $s_t^+ + s_{t+1}^- + s_{t+2}^- + s_{t+3}^- = 1.$
5. $s_t^+ + s_{t+1}^- + s_{t+2}^- + s_{t+3}^- \leq 1.$

Problem 5 (12 p)

Akabuga is town in East Africa. The town is not connected to a national grid, but a group of local businessmen are considering to start a local power company, Akabuga Electricity Company Ltd. (AECL). One of the options that are studied for AECL is to supply the system with a hydro power plant and a diesel generator set. The hydro power plant would be a run-of-the-river plant (i.e., there would not be a reservoir) having an installed capacity of 600 kW. The hydro power plant can be assumed to be 100% available and the operation cost is negligible. The diesel generator set would have the capacity 150 kW, availability 90% and operation cost 0.5 ¤/kWh.



a) (1 p) The figure above shows a simplified load duration curve for Akabuga. Calculate the equivalent load duration curve for including outages in both the hydro power plant and the diesel generator set, $\tilde{F}_2(x)$, for the interval $750 \le x < 800$.

b) (1 p) Calculate the equivalent load duration curve for including outages in both the hydro power plant and the diesel generator set, $\tilde{F}_2(x)$, for the interval $800 \le x < 850$.

c) (1 p) Calculate the equivalent load duration curve for including outages in both the hydro power plant and the diesel generator set, $\tilde{F}_2(x)$, for the interval $850 \le x < 900$.

d) (3 p) Use probabilistic production cost simulation to calculate the expected electricity generation per hour in the diesel generator set.

e) (3 p) Use the inverse transform method to randomise a value of the available generation capacity of the diesel generator set, $\overline{G}$. Start with the random number 0.04 from a $U(0,1)$-distribution. What is the corresponding complementary random number, $\overline{G}*$.

f) (3 p) Assume that the power system in Akabuga is simulated using stratified sampling. Five strata have been defined, and the stratum weights are displayed in table 2. Load shedding does never occur in the strata 1, 2 or 3. In stratum 4, load shedding occurs in 40% of the studied scenarios. In stratum 5, all scenarios result in load shedding. Which estimate of *LOLP* is obtained for the system?

Table 2 Strata in the simulation of Akabuga.

Stratum, h	1	2	3	4	5
Stratum weight, ω_h	0.7	0.03	0.2	0.02	0.05

PART II (FOR HIGHER GRADES)

All introduced symbols must be defined. Solutions should include sufficient detail that the argument and calculations can be easily followed.

The answer to each problem must begin on a new sheet, but answers to different parts of the same problem (a, b, c, etc.) can be written on the same sheet. The fields *Namn* (Name), *Blad nr* (Sheet number) and *Uppgift nr* (Problem number) must be filled out on every sheet.

Part II gives a total of 60 points, but this part will only be marked if the candidate has obtained at least 33 points in part I. Then the results of parts I and II and the bonus points will be added together to determine the examination grade (A, B, C, D, E).

Problem 6 (10 p)

The electricity market in Republiken is dominated by three large companies. Moreover, there are several municipal electricity boards. Data for the power producers are given in table 3 below. The variable operation costs are assumed to be linear within the intervals, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level. The electricity consumption is not price sensitive and amounts to 100 TWh/year.

Assume perfect information and that there are neither transmission, reservoir nor capacity limitations. Which of the three large players in the electricity market of Republiken would profit if they reduce the generation in one of their own nuclear power plants by 1 TWh/year?

Table 3 Data for the producers in Republiken.

Power source	Production capability [TWh/year]				Variable costs [¤/MWh]
	Allmänna elbolaget	Byarnas energi AB	Centrala Republikens elbolag	Various municipal electricity boards	
Wind	2	1	1	1	0
Hydro	25	15	5		0
Nuclear	20	10	10		100
Industrial backpressure				5	80–120
Fossil fuels	5	5	10	5	180–580

Problem 7 (10 p)

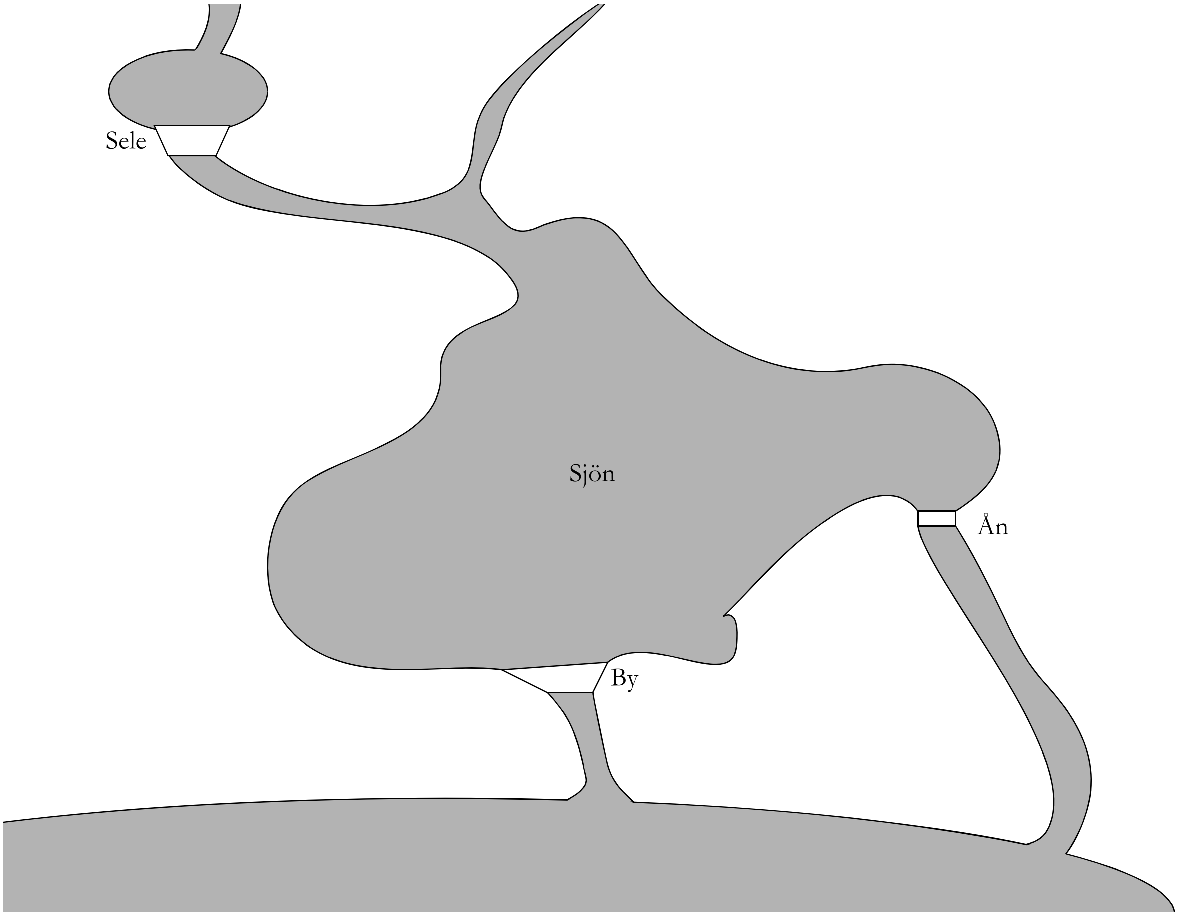
The neighbouring countries Rike and Maa have a common electricity market, and the two system operators, Riksnät and Maa Grid, are jointly responsible for the frequency control of this electricity market. The system operators have agreed that Riksnät should provide a gain of 2 500 MW/Hz in Rike and Maa Grid should ensure that the gain in Maa is 1 500 MW/Hz. The gain should be available in the interval 49.5 to 50.5 Hz.

Rike and Maa are interconnected via a number of parallel AC transmission lines, having a total maximal capacity of 2 000 MW as well as an HVDC lie with maximal capacity of 600 MW. The transmission line are equipped with a protection system which after a short time delay disconnects the line if the power flow exceed the maximal capacity. The power flow on the HVDC line is not affected by the frequency of the system, but can be controlled manually from the control room of Riksnät.

The result of the electricity trading at a certain occasion is a planned generation (i.e., the generation obtained if the frequency is exactly 50 Hz) is 19 000 MW in Rike and 8 000 MW in Maa. The planned consumption during the same hour is expected to be 17 500 MW in Rike and 9 500 MW in Maa. Which is the least transmission from Rike to Maa that can be set for the HVDC line if it should be possible to manage an outage of 1 200 MW in either country without overloading the AC transmission lines. You can assume that the frequency in the system is 50 Hz when the failure occurs (i.e., the transmission between Rike and Maa will be equal to the planned transmission).

Hint: The flow on the HVDC line must be chosen so that there is enough margins on the AC transmission lines to accommodate the power flow changes between the two countries that are caused by the changes in the generation of the power plants participating in the primary control.

Problem 8 (20 p)



AB Vattenkraft owns the two hydro power plants Sele and By located as in the figure above. Sjön is used as reservoir for the power plant By. Water from Sjön can also be released through the regulation dam at Ån. The Environment Court has judged that AB Vattenkraft always must maintain a flow of at least s 1 m^3/s through Ån. Data for the power plants are given in table 4. The company is planning to sell the generated electricity to the local power exchange, ElKräng. It is assumed that they can sell as much as they want to the prices stated in table 5. Stored water is assumed to be used for electricity generation at average production equivalent (i.e., installed capacity divided by maximal discharge) and future generation is valued 400 SEK/MWh. The water delay time between the power plants can be neglected.

a) (10 p) Formulate the planning problem of AB Vattenkraft as an LP problem. Use the notation in table 6 for the parameters (it is however permitted to add further symbols if you consider it necessary).

NOTICE! The following is required to get full score for this problem:

- The symbols for the optimisation variables must be clearly defined.
- The optimisation problem should be formulated so that it is easy to determine what the objective function is, which constraints there are and which limits there are.
- The possible values for all indices should be clearly stated for each equation.

b) (1 p) The discharge in the power plant By is governed by a simple automatic control system. If the contents of Sjön exceed 10 HE in the beginning of an hour then 3 m^3/s will be discharged through the turbine in By during that hour. Nothing is generated in By if the contents of Sjön is less than 10 HE. Which future production equivalent should be used for water stored in Sjön if the control system is considered? Do not forget to motivate your answer!

Table 4 Data for the power plants of AB Vattenkraft.

Power plant	Installed capacity [MW]	Start contents of reservoir [HE]	Maximal contents of reservoir [HE]	Marginal production equivalents [MWh/HE]		Maximal discharge [HE]		Local inflow [HE]
				Segment 1	Segment 2	Segment 1	Segment 2	
Sele	4.9	35	50	0.72	0.65	5	2	2
By	0.4	15	20	0.11	0.07	3	1	0.2

Table 5 Expected prices at ElKräng.

Hour	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Price at ElKräng [SEK/MWh]	355	355	335	320	355	385	430	475	515	515	460	430
Hour	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24
Price at ElKräng [SEK/MWh]	395	375	370	365	365	365	375	380	380	375	370	355

Table 6 Notation for the planning problem of AB Vattenkraft.

Symbol	Explanation	Value
$\overline{H}_i$	Installed capacity in power plant *i*	See table 4
$M_{i,0}$	Start contents of reservoir *i*	See table 4
$\overline{M}_i$	Maximal contents of reservoir *i*	See table 4
$\mu_{i,j}$	Marginal production equivalent in power plant *i*, segment *j*	See table 4
$\overline{Q}_{i,j}$	Maximal discharge in power plant *i*, segment *j*	See table 4
V_i	Local inflow to reservoir *i*	See table 4
λ_t	Expected price at ElKräng hour *t*	See table 5
λ_f	Expected future electricity price	400

c) (9 p) How must the planning problem from part a be reformulated to consider the control system in By? Do not forget to define all new variables and parameters that you introduce.

Hint: Introduce an integer variable which states if By is producing or not.

Problem 9 (20 p)

The general public in Land is concerned with global warming, since a government report has concluded that the social cost for emission of CO_2 amounts to 50 ¤/ton. The national energy authority has therefore been commissioned by the politicians to investigate if it would be beneficial to the society to introduce electricity disclosure of electricity in Land. The idea is that aware consumers, who are willing to pay a little bit extra for environmentally benign electricity, will force the power companies of Land to increase the electricity generation in power plants which do not emit CO_2.

In the study, the energy authority assumes that the introduction of electricity disclosure would give the consumers the choice between two different kinds of electricity: grey electricity and carbon-free electricity. The system would be organised in such a manner that if the consumers during one hour demand 5 000 MWh carbon-free electricity, then at least 5 000 MWh must be generated in nuclear power plants and bio-fuelled power plants. If the demand for carbon-free electricity would exceed the generation capacity of these power plants then some consumers who actually would prefer carbon-free electricity will have to buy grey electricity instead. However, the energy authority does not think that such a situation is likely, as the expect that the share of consumers opting for carbon-free electricity will not exceed 50%.

The energy authority uses a simplified model of the electricity market in Land for the study. It is assumed that all power plants always are available and that the consumers are not price sensitive. The figure on the next page shows a duration curve for the electricity demand and data for the power plants in Land are listed in table 7. As a measure of the benefits to the society, the energy authority uses the total surplus, which is defined as

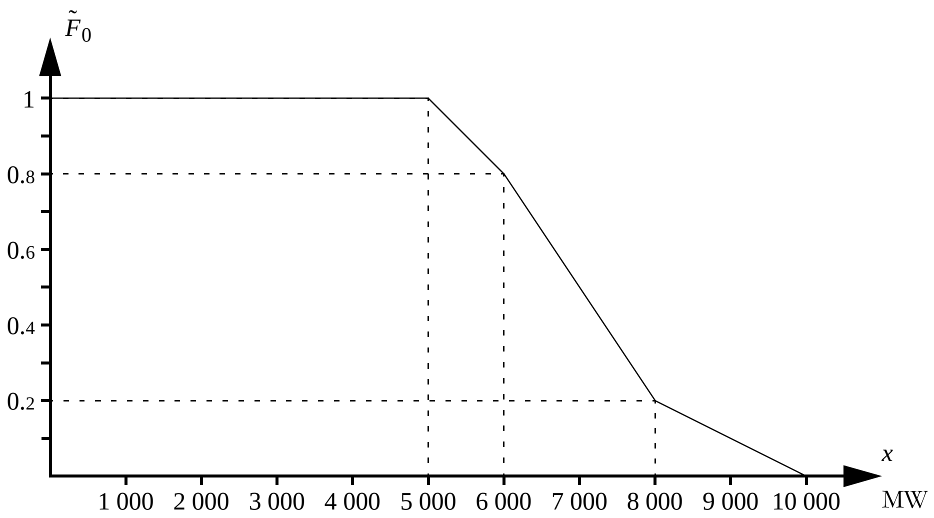
$$ETS = ETVC - ETOC - EEC,$$

where

ETS = expected total surplus,
$ETVC$ = expected value of electricity consumption,
$ETOC$ = expected total operation cost,
EEC = expected emission cost.

Table 7 Data for the power plants in Land.

Power source	Installed capacity [MW]	Variable costs [¤/MWh]	Emission of CO_2 [ton/MWh]
Nuclear			
Low operation cost	1 000	8	0
High operation cost	1 000	9	
Biomass			
Low operation cost	1 000	22	0
Medium operation cost	1 000	32	
High operation cost	1 000	42	
Fossil gas			
Low operation cost	1 000	20	0.4
Medium operation cost	1 000	30	
High operation cost	1 000	40	
Coal condensing			
Low operation cost	1 000	15	1
Medium operation cost	1 000	25	
High operation cost	1 000	35	



a) (1 p) The value of the electricity consumption is a constant, because the consumption is not price sensitive and the risk of failures in the power plants is neglected. How large is the *ETVC* if the value of consuming one MWh electricity is estimated to 100 ¤?

b) (5 p) Consider the electricity market in Land if electricity disclosure is *not* introduced. The producers will then in the first place use the power plants with the lower variable costs, i.e., they will not consider the emissions of a particular power plant. Calculate the expected generation of nuclear, biomass, fossil gas and coal condensing respectively.

c) (4 p) Calculate *ETS* for the electricity market in Land if electricity disclosure is *not* introduced.

d) (10 p) A Monte Carlo model is used to study the electricity market in Land after the introduction of electricity disclosure. Let the index g denote power plants with a certain variable cost (sorted in ascending order). A scenario can then be analysed according to the following procedure:

Scenario parameters:

$D_{CO2\text{-}free}$ = demand for carbon-free electricity,
D_{grey} = demand for grey electricity.

Model constants:

$\hat{G}_g$ = installed capacity in power plant g, $g = 1, \ldots, 11$ (see table 7),
β_g = variable cost of power plant g, $g = 1, \ldots, 11$ (see table 7).

Result variables:

$G_{CO2\text{-}free,\, g}$ = generation sold as carbon-free electricity, $g = 1, 2, 5, 8, 11$,
$G_{grey,\, g}$ = generation sold as grey electricity, $g = 1, \ldots, 11$,
TOC = total operation cost,
EC = emission cost.

The first step is to calculate how much generation is required to cover the demand for carbon-free electricity, which is done by solving the following optimisation problem:

$$\begin{aligned} \text{minimise} \quad & \beta_1 G_{CO2\text{-}free,\,1} + \beta_2 G_{CO2\text{-}free,\,2} + \beta_5 G_{CO2\text{-}free,\,5} + \beta_8 G_{CO2\text{-}free,\,8} + \beta_{11} G_{CO2\text{-}free,\,11} \\ \text{subject to} \quad & G_{CO2\text{-}free,\,1} + G_{CO2\text{-}free,\,2} + G_{CO2\text{-}free,\,5} + G_{CO2\text{-}free,\,8} + G_{CO2\text{-}free,\,11} = D_{CO2\text{-}free}, \\ & 0 \le G_{CO2\text{-}free,\,g} \le \hat{G}_g, \quad g = 1, 2, 5, 8, 11. \end{aligned}$$

From the solution to the above problem, it is possible to determine how much generation is needed to cover the demand for grey electricity by solving the following optimisation problem:

$$\text{minimise} \quad \sum_{g=1}^{11} \beta_g G_{grey,\,g}$$

$$\text{subject to} \quad \sum_{g=1}^{11} G_{grey,\,g} = D_{grey},$$

$$0 \le G_{grey,\,g} \le \hat{G}_g - G_{CO2\text{-}free,\,g},\; g = 1, 2, 3, 5, 8,$$

$$0 \le G_{grey,\,g} \le \hat{G}_g,\; g = 4, 6, 7, 9, 10, 11.$$

From the solutions to the problems above, the total operation cost is given by

$$TOC = \sum_{g\,=\,1,\,2,\,5,\,8,\,11} \beta_g G_{C02\text{-}free,\,g} + \sum_{g=1}^{11} \beta_g G_{grey,\,g},$$

and the emission cost is

$$EC = 50 \cdot 0.4 \cdot (G_4 + G_7 + G_{10}) + 50 \cdot 1 \cdot (G_6 + G_9 + G_{11}).$$

Table 8 shows four scenarios of a Monte Carlo simulation of the electricity market in Land. Which estimate of *ETS* is obtained from these scenarios if the control variate method is used?

Table 8 Some scenarios for a Monte Carlo simulation of the electricity market in Land.

Scenario	1	2	3	4
Demand for grey electricity [MW]	3 600	5 200	4 300	3 600
Demand for carbon-free electricity [MW]	1 600	4 300	2 700	3 400

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Answer sheet for part I

Name: ....

Personal number: ....

Problem 1

a) Alternative .... is correct.

b) Alternative .... is correct.

c) Alternative .... is correct.

Problem 2

a) .... ¤/MWh **b)** .... MWh

Problem 3

a) .... MW/Hz **b)** .... MW/Hz

c) .... MW/Hz

d) .... MW from area B to area C.

Problem 4

a) ....

b) Parameters: ....

Optimisation variables: ....

c) .... MWh/HE

d) Alternative .... is correct.

e) Alternative .... is correct.

Problem 5

a) .... **b)** ....

c) .... **d)** .... kWh/h

e) $\overline{G}$: .... kW $\overline{G}^*$: .... kW

f) .... %

Problem 1

a) 3, **b)** 2, **c)** 2.

Problem 2

b) The electricity price is determined by the intersection of the supply and demand curves. The intersection can be found graphically by drawing both curves in the same figure. An alternative method of solution is to assume an electricity price, λ, between 25 and 40 ¤/MWh. The supply at these price levels is given by

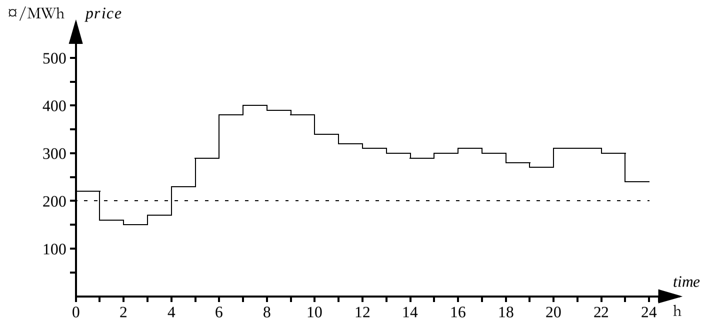
$$80 + \frac{\lambda - 25}{40 - 25}20,$$

whereas the demand is

$$80 + \frac{40 - \lambda}{40 - 25}20.$$

These two expressions should be equal, which results in the electricity price $\lambda = 32.5$ ¤/MWh.

b) The power plant will generate its installed capacity during those hours when the electricity price is higher than 200 ¤/MWh. Drawing a line at the level 200 ¤/MWh shows that the power plant will generate 100 MW during 21 hours, which results in a total generation of 2 100 MWh.



Problem 3

a) As $\Delta f = \Delta G_{tot}/R_{tot}$ and $\Delta G_A = R_A \Delta f$ we get that $R_A/R_{tot} = \Delta G_A/\Delta G_{tot}$ (i.e., the share of area A of the total gain is proportional to the share of area A of the total generation increase due to the failure), which means that $R_A = 250/1\,000 \cdot 4\,000 = 1\,000$ MW/Hz.

b) The same reasoning as in part a yields $R_B = 500/1\,000 \cdot 4\,000 = 2\,000$ MW/Hz.

c) The same reasoning as in part a yields $R_C = 250/1\,000 \cdot 4\,000 = 1\,000$ MW/Hz.

d) Area C has lost 1 000 MW generation, out of which 250 MW has been replaced by the primary control in area C. The remaining 750 MW must be covered by increased import from area B. Hence, the new flow from area B to area C is 100 + 750 = 850 MW, which is less than the maximal capacity of the line.

Problem 4

a) $M_{5,t} = M_{5,t-1} + V_{5,t} + Q_{2,1,t} + Q_{2,2,t} + S_{2,t} + Q_{4,1,t} + Q_{4,2,t} + S_{4,t} - Q_{5,1,t} - Q_{5,2,t} - S_{5,t}$

b) Parameters: $M_{i,0}$ and $V_{i,t}$. Optimisation variables: $M_{i,t}$, $S_{i,t}$ and $Q_{i,j,t}$

c) The maximal production equivalent is obtained at the discharge where we have the best efficiency. The definition of production equivalent then gives us $\gamma_{max} = H/Q = 0.37$ MWh/HE.

d) 5.

e) 5.

Problem 5

a) $\tilde{F}_1(x) = \tilde{F}_0(x)$, $\tilde{F}_2(x) = 0.9\tilde{F}_1(x) + 0.1\tilde{F}_1(x - 150) \Rightarrow \tilde{F}_2(x) = 0.9 \cdot 0.1 + 0.1 \cdot 0.1 = 0.1$ for the interval $750 \le x < 800$.

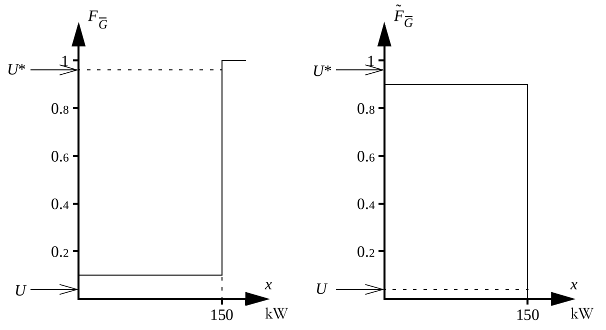
b) $\tilde{F}_1(x) = \tilde{F}_0(x)$, $\tilde{F}_2(x) = 0.9\tilde{F}_1(x) + 0.1\tilde{F}_1(x - 150) \Rightarrow \tilde{F}_2(x) = 0.9 \cdot 0 + 0.1 \cdot 0.1 = 0.01$ for the interval $800 \le x < 850$.

c) $\tilde{F}_1(x) = \tilde{F}_0(x)$, $\tilde{F}_2(x) = 0.9\tilde{F}_1(x) + 0.1\tilde{F}_1(x - 150) \Rightarrow \tilde{F}_2(x) = 0.9 \cdot 0 + 0.1 \cdot 0.1 = 0.01$ for the interval $850 \le x < 900$.

d) The expected generation in the diesel generator set is calculated as

$$EG_2 = EENS_1 - EENS_2 = \int_{600}^{\infty} \tilde{F}_1(x)dx - \int_{750}^{\infty} \tilde{F}_2(x)dx = 0.1 \cdot 200 - (0.1 \cdot 50 + 0.01 \cdot 150) =$$
$$= 13.5 \text{ kW/h/h}.$$

e) If we draw the distribution function or the duration curve of $\overline{G}$ it is easy to see that $U = 0.04$ and $U^* = 1 - U$ are transformed to 0 and 150 respectively, i.e., the result is either $\overline{G} = 0$ and $\overline{G}^* = 150$, or $\overline{G} = 150$ and $\overline{G}^* = 0$.



f) According to the problem text, we have the following estimates for the strata: $m_{LOLO1} = m_{LOLO2} = m_{LOLO3} = 0$, $m_{LOLO4} = 0.4$ and $m_{LOLO5} = 1$. Hence, we get

$$m_{LOLO} = \sum_{h=1}^{5} \omega_h m_{LOLOh} = 0 + 0 + 0 + 0.02 \cdot 0.4 + 0.05 \cdot 1 = 5.8\%.$$

Problem 6

We start by calculating the electricity price if none of the players is executing market power, i.e., we assume perfect competition. The generation at the electricity price 180 ¤/MWh will be 5 TWh wind power, 45 TWh hydro power, 40 TWh nuclear and 5 TWh industrial backpressure. This gives in total 95 TWh, which is not sufficient. Therefore, assume an electricity price in the interval 180 to 580 ¤/MWh. The total generation is then

$$95 + \frac{\lambda - 180}{580 - 180} 25.$$

Setting this expression equal to 100 and solving for λ yields the electricity price $\lambda = 260$ ¤/MWh

Now, assume that a player withholds 1 TWh generation capacity in a nuclear power plant. The lost terawatt-hour will be replaced by increased usage of fossil fuels, which will require an increase of the electricity price to 276 ¤/MWh.

If one of the dominating companies is choosing to reduce their generation in a nuclear power plant, the income from this power plant will be reduced by 1 TWh · 260 ¤/MWh. Meanwhile, the variable costs decrease by 1 TWh · 100 ¤/MWh. Hence, the lowering results in a lost profit of 160 M¤. However, at the same time the profits of the remaining generation of the company is increasing by 16 ¤/MWh. If the profit increase due to the price increase is to be larger than the lost profit then the remaining generation of the company must be at least 160 M¤/16 ¤/MWh = 10 TWh. This condition is fulfilled by all three of the large players.

Problem 7

At nominal frequency there is a surplus of 1 500 MW in Rike and this surplus has to be exported to Maa. When the frequency control is compensating an outage of generation, the generation in the primary control power plants will change in both Rike and Maa, which results in a changed power flow between the two countries. As the power flow of the HVDC line is not affected, all changes have to take place on the AC lines; thus, there must be sufficient reserves on these lines. It is these reserves that determines how much power can be transmitted on the HVDC line.

If there is an upper limit to how much the AC lines can transfer then the remaining power flow must be on the HVDC line, which means that there must be a lower limit to the flow on this line. The maximal limit for the AC transmission depends on how much the transmission from Rike to Maa increases due to an outage. The worst case is losing 1 200 MW in Maa, because 62.5% (corresponding to the share of Rike of the total gain) of this outage will be replaced by the primary control in Rike and must be transported to Maa via the AC lines. The planned transmission may therefore not exceed 2 000 – 0.625 · 1 200 = 1 250 MW, which means that out of the planned transmission of 1 500 MW, at least 1 500 – 1 250 = 250 MW must be transmitted on the HVDC line.

Problem 8

a) The problem we want to solve is

maximise	*income of sold electricity* + *value of stored water*,
subject to	*hydrologiscal balance for the hydro power plants*,
	limitations in generation capacity, etc.

Indices for the power plants

Sele 1, By/Ån 2.

Parameters

Most of the parameters are defined in table 6 of the problem text. The average production equivalent is calculated according to the instruction, i.e.,

$$\gamma_i = \text{average production equivalent in power plant } i = \frac{\overline{H}_i}{\overline{Q}_{i,1} + \overline{Q}_{i,2}} = \begin{cases} 0.7 & i = 1, \\ 0.1 & i = 2. \end{cases}$$

Optimisation variables

$Q_{i,t}$ = discharge in power plant i, segment j, during hour t, $i = 1, 2$, $t = 1, \ldots, 24$,
$S_{i,t}$ = spillage from reservoir i during hour t, $i = 1, 2$, $t = 1, \ldots, 24$,
$M_{i,t}$ = contents of reservoir i at the end of hour t, $i = 1, 2$, $t = 1, \ldots, 24$.

Objective function

$$\text{maximise} \quad \sum_{t=1}^{24} \lambda_t \left(\sum_{i=1}^{2} \sum_{j=1}^{2} \mu_{i,j} Q_{i,j,t} \right) + \lambda_f (\gamma_1 + \gamma_2) M_{1,24} + \lambda_f \gamma_2 M_{2,24}.$$

Constraints

Hydrological balance for Sele:

$$M_{1,t} - M_{1,t-1} + Q_{1,1,t} + Q_{1,2,t} + S_{1,t} = V_1, \qquad t = 1, \ldots, 24.$$

Hydrological balance for By/Ån:

$$M_{2,t} - M_{2,t-1} + Q_{2,1,t} + Q_{2,2,t} + S_{2,t} - Q_{1,1,t} - Q_{1,2,t} - S_{1,t} = V_2, \qquad t = 1, \ldots, 24.$$

Variable limits

$0 \le Q_{i,j,t} \le \overline{Q}_{i,j}$, $\quad i = 1, 2$, $t = 1, \ldots, 24$,
$0 \le S_{1,t}$ $\quad t = 1, \ldots, 24$,
$1 \le S_{2,t}$ $\quad t = 1, \ldots, 24$,
$0 \le M_{i,t} \le \overline{M}_3$, $\quad i = 1, 2$, $t = 1, \ldots, 24$.

b) As the discharge of the power plant always will be 3 HE when it is committed, it is logical to use the production equivalent for this discharge, i.e., $\gamma_2 = 0.11$ MWh/HE.

c) Introduce the following new optimisation variables:

u_t = unit commitment of By (1 if water is discharged at By, 0 otherwise).

The discharge in By is a direct function of the unit commitment variable; hence, we can substitute the variables $Q_{2,j}$ for $3u_t$. The resulting objective function is then

$$\text{maximise} \quad \sum_{t=1}^{24} \lambda_t \left(\sum_{j=1}^{2} \mu_{1,j} Q_{1,j,t} + \gamma_2 \cdot 3u_t \right) + \lambda_f(\gamma_1 + \gamma_2) M_{1,24} + \lambda_f \gamma_2 M_{2,24}.$$

The hydrological balance for By/Ån is changed to

$$M_{2,t} - M_{2,t-1} + 3u_t + S_{2,t} - Q_{1,1,t} - Q_{1,2,t} - S_{1,t} = V_2, \qquad t = 1, \ldots, 24.$$

There is also a new for new constraints that model the relation between the contents of Sjön and the discharge in By. If u_t is equal to zero then $M_{2,t}$ may not be larger than 10, which can be achieved using the following constraint:

$$M_{2,t} \le 10 + (\overline{M}_2 - 10)u_t, \qquad t = 1, \ldots, 24.$$

If u_t is equal to one, then $M_{2,t}$ must be at least equal to 10:

$$M_{2,t} \ge 10u_t, \qquad t = 1, \ldots, 24.$$

Finally, the variable limits must be updated. The limits of $Q_{2,j}$ can be removed, as these variables are no longer included in the problem. It is also possible to remove the limits of $M_{2,t}$, because the limits of this variable will be set by the new constraints. On the other hand, we have to add a requirement that u_t is a binary variable:

$$u_t \in \{0, 1\}, \qquad t = 1, \ldots, 24.$$

Problem 9

a) The expected load is given by

$$E[D] = EENS_0 = \int_0^{\infty} \tilde{F}_0(x)dx =$$

$$= 5\,000 \cdot 1 + 1\,000 \cdot (1 + 0.8)/2 + 2\,000 \cdot (0.8 + 0.2)/2 + 2\,000 \cdot 0.2/2 = 7\,100 \text{ MWh/h}.$$

The expected value of the electricity consumption is then $ETVC = 100E[D] = 710\,000$ ¤/h.

b) The expected generation in each power plant can be calculated using probabilistic production cost simulation. The power plants are added in ascending cost order. Since all power plants have 100% availability, we get $\tilde{F}_g(x) = \tilde{F}_{g-1}(x)$ every time a new power plant is added. Using the formula

$$EG_g = T \cdot p_g \int_{\hat{G}_{g-1}^{tot}}^{\hat{G}_g^{tot}} \tilde{F}_{g-1}(x)dx.$$

we can now calculate the expected generation in each power plant:

$$EG_1 = \int_0^{1\,000} \tilde{F}_0(x)dx = 1\,000 \text{ MWh/h}, \; EG_2 = \int_{1\,000}^{2\,000} \tilde{F}_1(x)dx = 1\,000 \text{ MWh/h},$$

$$EG_3 = \int_{2\,000}^{3\,000} \tilde{F}_2(x)dx = 1\,000 \text{ MWh/h}, \; EG_4 = \int_{3\,000}^{4\,000} \tilde{F}_3(x)dx = 1\,000 \text{ MWh/h},$$

$$EG_5 = \int_{4\,000}^{5\,000} \tilde{F}_4(x)dx = 1\,000 \text{ MWh/h}, \; EG_6 = \int_{5\,000}^{6\,000} \tilde{F}_5(x)dx = 900 \text{ MWh/h},$$

$$EG_7 = \int_{6\,000}^{7\,000} \tilde{F}_6(x)dx = 650 \text{ MWh/h}, \; EG_8 = \int_{7\,000}^{8\,000} \tilde{F}_7(x)dx = 350 \text{ MWh/h},$$

$$EG_9 = \int_{8\,000}^{9\,000} \tilde{F}_8(x)dx = 150 \text{ MWh/h}, \; EG_{10} = \int_{9\,000}^{10\,000} \tilde{F}_9(x)dx = 50 \text{ MWh/h},$$

$$EG_{11} = \int_{10\,000}^{11\,000} \tilde{F}_{10}(x)dx = 0 \text{ MWh/h}.$$

The nuclear power is represented by the two least expensive units, i.e., $EG_{nuclear} = EG_1 + EG_2 =$ 2 000 MWh/h. The biomass power plants are added as number 5, 8 and 11, which results in the expected generation $EG_{bio} = EG_5 + EG_8 + EG_{11} = 1\,350$ MWh/h. The fossil gas constitutes power plants 4, 7 and 10, which yields $EG_{fossilgas} = EG_4 + EG_7 + EG_{10} = 1\,700$ MWh/h. Finally, the expected coal condensing generation is $EG_{coal} = EG_3 + EG_6 + EG_9 = 2\,050$ MWh/h.

c) The expected total operation cost is calculated as

$$ETOC = \sum_{g=1}^{11} \beta_g EG_g = \ldots = 134\,450 \text{ ¤/h}.$$

The fossil gas power plants emit 0.4 ton/MWh and the coal condensing 1 ton/MWh. With a cost of 50 ¤/ton we get the following total emission cost

$$EEC = 50 \cdot 0.4 \cdot EG_{fossilgas} + 50 \cdot 1 \cdot EG_{coal} = 136\,500 \text{ ¤/h}.$$

The expected total surplus is thus $ETS = 710\,000 - 134\,450 - 136\,500 = 439\,050$ ¤/h.

d) The control variates are taken from a simplified model, where no difference is made between grey and carbon-free electricity:

$$\text{minimise} \quad \sum_{g=1}^{11} \beta_g \tilde{G}_g$$

$$\text{subject to} \quad \sum_{g=1}^{11} \tilde{G}_g = D_{grey} + \ddot{D}_{CO2\text{-}fre},$$

$$0 \le \tilde{G}_g \le \hat{G}_g, \; g = 1, \ldots, 11.$$

The control variates can be calculated from the solution to the above optimisation problem:

$$\tilde{TOC} = \sum_{g=1}^{11} \beta_g \tilde{G}_g$$

and

$$\tilde{EC} = 50 \cdot 0.4 \cdot (\tilde{G}_6 + \tilde{G}_9 + \tilde{G}_{11}) + 50 \cdot 1 \cdot (\tilde{G}_4 + \tilde{G}_7 + \tilde{G}_{10}).$$

The given scenarios results in the following values of the control variates and result variables:

Scenario, i	1	2	3	4
toc_i [¤/h}	79 000	216 600	129 000	129 800
$\tilde{toc}_i$ [¤/h]	79 000	216 000	129 000	129 000
ec_i [¤/h}	80 000	194 000	140 000	132 000
$\tilde{ec}_i$ [¤/h]	80 000	200 000	140 000	140 000

The expected operation cost is estimated by

$$m_{TOC} = \mu_{\tilde{TOC}} + \frac{1}{4}\sum_{i=1}^{4}(toc_i - \tilde{toc}_i) = 134\,450 + (0 + 600 + 0 + 800)/4 = 134\,800 \text{ ¤/h},$$

where $\mu_{\tilde{TOC}}$ is taken from the solution to part c. Similarly, we get the estimate

$$m_{EC} = \mu_{\tilde{EC}} + \frac{1}{4}\sum_{i=1}^{4}(ec_i - \tilde{ec}_i) = 136\,500 + (0 - 6\,000 + 0 - 8\,000)/4 = 133\,000 \text{ ¤/h}.$$

The expected total surplus is in this case $ETS = 710\,000 - 134\,800 - 133\,000 = 442\,200$ ¤/h.