KTH Electrical Engineering

Exam in EG2050 / 2C1118 System Planning, 14 March 2009, 8:00–13:00, Q13, Q15, Q17

Allowed aids

In this exam you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten**, **single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

PART I (MANDATORY)

Write all answers on the answer sheet provided. Motivations and calculations do not have to be presented.

Part I can yield 40 points in total. The examinee is guaranteed to pass if the score is at least 33 points. If the result in part I is at least 31 points, then there will be a possibility to complement for passing the exam with the grade E.

Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

a) (2 p) A balance responsible player is economically responsible that the system during each trading period (for example one hour) is supplied a much energy as consumed by the customers of the player. In practice, this responsibility is managed by I) The balance responsible player is obliged to buy financial instruments in the ahead market, II) The balance responsible player is obliged to buy and sell regulating power in the real-time market, III) The balance responsible player is obliged to buy and sell balance power in the post market.

1. Only I is true.
2. Only II is true.
3. Only III is true.
4. I and II are true but not III.
5. II and III are true but not I.

b) (2 p) The following applies to a bilateral electricity market: I) Producers are free to sell to any other producer, retailer or consumer, II) All electricity trading has to be performed at a power pool, III) The consumers are free to buy from any producer or retailer.

1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. I and III are true but not II.

Problem 2 (6 p)

The electricity market in Land has perfect competition, all players have perfect information and there are neither capacity, transmission nor reservoir limitations in the system. Data for the electricity producers of Land are given in table 1. The variable costs of the coal condensing are assumed to be linear in the given interval, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

Table 1 Data for the electricity producers in Land.

Power source	Production capability [TWh/year]	Variable costs [¤/MWh]
Hydro	60	5
Nuclear	60	90–100
Biofuel	20	100–300
Fossil fuel	10	200–400

a) (1 p) How much is generated in the bio-fuelled power plants during a year if the electricity price is 250 ¤/MWh?

b) (3 p) What will the electricity price be in Land if the electricity consumption is not price sensitive and amounts to 142 TWh/year?

c) (2 p) Assume that the electricity price in Land is 320 ¤/MWh. The power company Strålinge owns a nuclear power plant with a production capability of 10 TWh per year. The variable costs are 100 ¤/MWh and the fixed costs of the power plant are 2 000 M¤/year. How large is the profit of the company?

Problem 3 (6 p)

a) (2 p) A certain power system can be divided in five areas. The transmission interconnections between these areas are listed in table 2. The following applies to this system: I) There are two synchronous grids in the system, II) Area A and area E belong to the same synchronous grid, III) Area B and area E belong to the same synchronous grid.

Table 2 Transmission interconnections in problem a.

Interconnection	Capacity [MW]	Type
A ↔ B	600	High voltage direct current (HVDC)
A ↔ C	1 000	Alternating current
A ↔ D	1 200	Alternating current
B ↔ E	2 000	Alternating current
C ↔ D	1 500	Alternating current
D ↔ E	400	High voltage direct current (HVDC)

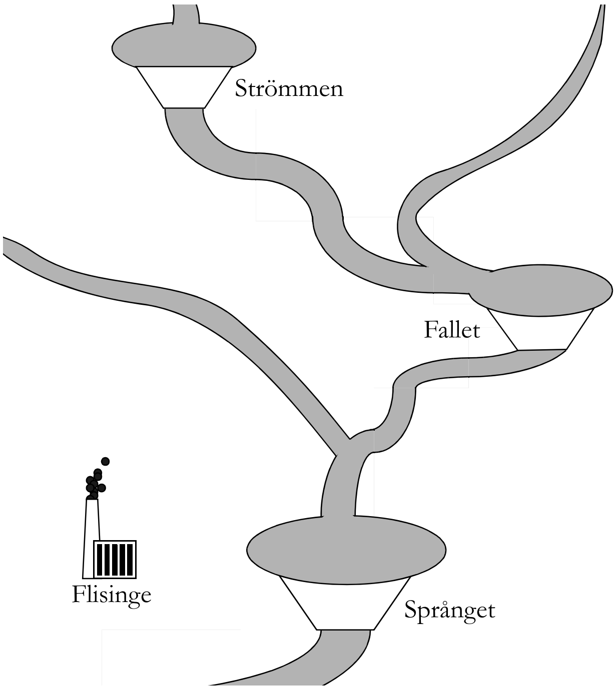
1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. I and III are true but not II.
5. II and III are true but not I.

b) (2 p) The hydro power plant Sele has a gain of 100 MW/Hz. The base generation (i.e., the generation when the frequency is exactly 50 Hz) is 50 MW and the installed capacity of the power plant is equal to 90 MW. To avoid damaging the turbines it is not allowed to generate less than 40 MW. How much will the power plant generate when the frequency of the system is 50.14 Hz?

c) (2 p) Consider a power system divided in two areas, A and B. There is only one interconnection between these two areas. This interconnection is composed of a 400 kV AC line with them maximal capacity 900 MW. The line is equipped with a protection system which after a short time delay disconnects the line if the power flow exceeds the maximal capacity of the line.

The transmission at a certain occasion is 500 MW from area A to B. At this occasion 200 MW generation is disconnected in area B due to a failure in a transformer. The gain in area A is 2 000 MW/Hz and the gain in area B is 3 000 MW/Hz. How large is the transmission from area A to area B when the primary control has restored the balance between generation and consumption? (Answer 0 MW if the line is disconnected due to overloading.)

Problem 4 (12 p)



AB Elkraft owns three hydro power plant located as in the figure above. Besides, the company also owns the biomass-fuelled power plant Flisinge. The following symbols have been introduced in a short-term planning problem for the power plants of the company:

Indices for the hydro power plants: Strömmen 1, Fallet 2, Språnget 3.

- C^+ = start-up cost in Flisinge,
- γ_i = expected future production equivalent for water stored in reservoir i, $i = 1, 2, 3$,
- D_t = contracted load during hour t, $t = 1, \ldots, 24$,
- G_t = electricity generation in Flisinge, hour t, $t = 1, \ldots, 24$,
- λ_{25} = expected electricity price at ElKräng after the end of the planning period,
- $\mu_{i,j}$ = marginal production equivalent in power plant i, segment j, $i = 1, 2, 3, j = 1, 2, 3$,
- $Q_{i,j,t}$ = discharge in hydro power plant i, segment j, hour t, $i = 1, 2, 3, j = 1, 2, 3, t = 1, \ldots, 24$.

a) (3 p) Which of the symbols above represent optimisation variables and parameters respectively?

b) (4 p) Formulate the load balance constraint of AB Elkraft for hour t. Use the symbols defined above.

c) (3 p) At installed capacity the hydro power plant Fallet generates 100 MW and the production equivalent is then 0.8 MWh/HE. The reservoir can hold 5 400 000 m^3. Assume that the power plants up-stream neither discharge or spill any water and that the local inflow is negligible. If we start with a full reservoir how many hours can Fallet generate its installed capacity before the reservoir is empty?

d) (2 p) AB Kraftverket owns a thermal power plant. Assume that the company wants to formulate a short-term planning problem for this power plant and that the objective is to maximise the income of sold electricity minus the generation costs during 24 hours. The following objective function and constraints have been suggested:

$$\text{maximise} \quad \sum_{t=1}^{24} ((\lambda_t - \beta_t)G_t - C^+ s_t^+)$$

$$\text{subject to} \quad G_t \geq u_t \underline{G}, \qquad t = 1, \ldots, 24,$$

$$G_t \leq u_t \overline{G}, \qquad t = 1, \ldots, 24.$$

In addition to the constraints above, a unit commitment constraint will also be necessary. Which of the following constraints can be used?

I) $u_t - u_{t-1} = s_t^+$,

II) $u_t - u_{t-1} \leq s_t^+$,

III) $u_t - u_{t-1} = s_t^+ - s_t^-$.

1. It is only possible to use constraint I.
2. It is only possible to use constraint II.
3. It is only possible to use constraint III.
4. It is possible to use either constraint I or III.
5. It is possible to use either constraint II or III.

The symbols that are used in the problem above are defined as follows:

β = variable cost in the power plant,
C^+ = start-up cost in the power plant,
G_t = generation in the power plant during hour *t*,
$\underline{G}$ = minimal generation when the power plant is committed,
$\overline{G}$ = maximal generation when the power plant is committed,
λ_t = electricity price hour *t*,
s_t^+ = start-up variable for hour *t* (1 if the power plant is started before this hour, otherwise 0),
s_t^- = stop variable for hour *t* (1 if the power plant is stopped before this hour, otherwise 0),
u_t = unit commitment during hour *t* (1 if the power plant is committed, 0 otherwise).

Problem 5 (12 p)

Consider an electricity market where the load can be assumed to be normally distributed with the expectation value 8 000 MW and the standard deviation 1 000 MW. In this electricity market there is hydro power plants with a combined installed capacity of 10 000 MW. The hydro power plants can be assumed to be 100% reliable and have negligible variable costs. Moreover, there is a thermal power plant, which has 1 000 MW installed capacity, 90% availability and the variable cost 10 ¤/MWh. Some partial results from a probabilistic production cost simulation of this electricity market are shown in table 3.

Table 3 Results from a probabilistic production cost simulation of the electricity market in problem 5a–c.

	x = 7 000	x = 8 000	x = 9 000	x = 10 000	x = 11 000	x = 12 000
$\tilde{F}_0(x)$	0.841	0.500	0.159	0.023	0.001	0.000
$\int_x^\infty \tilde{F}_0(\xi)d\xi$	1 083.3	389.9	83.3	8.5	0.4	0.0
$\int_x^\infty \tilde{F}_2(\xi)d\xi$	1 175.8	467.4	114.9	16.0	1.2	0.0

a) (1 p) Calculate $EENS_0$, i.e., the expected unserved energy if there is no power plant in the system.

b) (3 p) Use probabilistic production cost simulation to calculate the expected operation cost of the system.

c) (2 p) Use probabilistic production cost simulation to calculate the risk of power deficit in the system.

d) (2 p) Assume that a value of the load has been randomised and that the result was 7 000 MW. What is the complementary random number of this value?

e) (4 p) In order to consider losses in the transmission grid, a Monte Carlo simulation of the system is performed. Control variates have been used to achieve as good results as possible. The detailed model consider the losses. The simplified model neglects the grid, which means that the same model is used as in a probabilistic production cos simulation. The results are shown in table 4. Which estimates of *ETOC* and *LOLP* are obtained?

Table 4 Results from a Monte Carlo simulation of the electricity market in problem 5e.

Number of scenarios, n	Results from the detailed model		Results from the simplified model	
	$\sum_{i=1}^{n} toc_i$	$\sum_{i=1}^{n} lolo_i$	$\sum_{i=1}^{n} \tilde{toc}_i$	$\sum_{i=1}^{n} \tilde{lolo}_i$
10 000	975 000	41	745 000	35

PART II (FOR HIGHER GRADES)

All introduced symbols must be defined. Solutions should include sufficient detail that the argument and calculations can be easily followed.

The answer to each problem must begin on a new sheet, but answers to different parts of the same problem (a, b, c, etc.) can be written on the same sheet. The fields *Namn* (Name), *Blad nr* (Sheet number) and *Uppgift nr* (Problem number) must be filled out on every sheet.

Part II gives a total of 60 points, but this part will only be marked if the candidate has obtained at least 33 points in part I. Then the results of parts I and II and the bonus points will be added together to determine the examination grade (A, B, C, D, E).

Problem 6 (10 p)

The Rikish Energy Agency has created a future scenario for the electricity market in 2020. The model of the Energy Agency is based on the assumption that there is perfect competition in the electricity market in Rike, that all players have perfect information and that there are neither capacity, transmission nor reservoir limitations in the system. The Energy Agency also assumes that the electricity consumption in Rike will be 160 TWh/year in 2020 and that the generation capacity will consist of 66 TWh/year hydro power (variable cost 1 ¤/MWh), 60 TWh/year nuclear power (variable cost 10 ¤/MWh) as well as 40 TWh/year fossil fuels (the variable cost is assumed to be linear in the interval 30–70 ¤/MWh, i.e., the production is zero if the price is 30 ¤/MWh and the production is maximal at the price level 70 ¤/MWh).

The aim of this future scenario is to study how the electricity market would be affected by investments in new nuclear power plants and a large-scale development of wind power respectively. To support the latter alternative, it has been proposed that Rike should introduce a system with green certificates. The owner will receive a green certificate for each MWh generated in the new wind power plants. The consumers are then obliged to buy certificates corresponding to 10% of their electricity consumption, which means that a consumer with an annual consumption of 100 MWh will have to buy 10 green certificates per year. The Energy Agency believes that the price of the green certificates would be 20 ¤ per certificate.

a) (4 p) Assume that a new nuclear power plant can generate 7 TWh/year and that the variable costs are 10 ¤/MWh whereas the fixed costs are 280 M¤ per power plant and year. The more nuclear power plants that are built in Rike, the lower the electricity price will become. If too many new nuclear power plants are built then the electricity price will become so low that the income of sold electricity will not be sufficient to cover the costs in the new power plants. How many new nuclear power plants can be built in Rike without making the new nuclear power plants unprofitable?

b) (4 p) Assume that a new wind farm can generate 1 TWh/year and that the variable costs are 1 ¤/MWh whereas the fixed costs are 67 M¤ per wind farm and year. The more wind farms that are built in Rike, the lower the electricity price will become. If too many new wind farms are built then the electricity price will become so low that the income of sold electricity and sold certificates will not be sufficient to cover the costs of the new power plants. How many new wind farms can be built in Rike without making the new wind farms unprofitable?

c) (2 p) Which of the alternatives above will result in the least costs for the consumers on the electricity market in Rike?

Problem 7 (10 p)

The power system in Rike is divided in two parts. There are large amounts of hydro power in the northern part of the system, but the main consumption centres are in the southern part. There are several parallel AC transmission lines between the two parts. The maximal flow on these lines is 7 000 MW – if this limit is exceeded the power system becomes unstable and there is a risk for extensive blackouts in the entire or parts of the system. Therefore, in order to avoid this, the maximal flow is set to a value less than 7 000 MW at nominal frequency; the unused transmission capacity is reserved for power flows which are due to the primary control. Riksnät, which is the system operator in Rike, must at last 8:00 notify the players in the electricity market about how much transmission from north to south that can be allowed for each hour the next day. However, a problem for Riksnät is that they do not know exactly which gain will be available next day.

For a certain hour, Riksnät expects the gain in the northern part of Rike to be in the range from 2 000 MW/Hz to 3 000 MW/Hz and that the gain in the southern part of the country will be in the range from 1 000 MW/Hz to 1 250 MW/Hz. How much transmission capacity must be reserved for the primary control if one wants to be sure that the system can manage an outage of 1 200 MW in southern Rike without overloading the lines between northern and southern Rike?

Problem 8 (20 p)

AB Verket owns a combine heat and power plant. The power plant consists of three biomass-fuelled boilers, which generate steam. The steam is then led to three backpressure turbines which each powers an electric generator. After the turbines, the steam goes to condensers, which deliver heat to the district heating system. There is also an electric boiler in the power plant, i.e., it is possible to use electricity to generate heat for the district heating system. At maximal loading, the electric boiler consumers 25 MW. The efficiency of the electric boiler is 99%. Finally, there is also an accumulator tank, where it is possible to store thermal energy (as water at a temperature of 98 °C). The maximal storage capacity is 800 MWh heat. The heat losses in the accumulator tank are negligible in a short-term planning problem, and it can be assumed that there is no limitation in how fast the tank can be emptied and filled respectively.

a) (12 p) The ration between electricity generation and heat generation is fixed for each backpressure turbine, i.e., each MW generated electricity corresponds to a certain number of MW heat. The ration between electricity generation and heat generation as well as other data are shown in table 5. Notice that the generation cost is given MWh electricity—the heat generation is here considered as a “by-product” of the electricity generation. The electricity that is generated in the combined heat and power plant are sold at the electricity exchange ElKräng. It is assumed that the company can buy and sell as much as they want to the prices listed in table 6. The heat load depend mostly on the outdoor temperature. A forecast for the head load is given in table 6.

AB Verket wants to operate the power plants so that they maximises the income of sold electricity minus the generation costs, while each hour delivering enough heat to the district heating system. Formulate the planning problem of the company as an LP problem. The accumulator tank holds 700 MWh at the start of the planning period and should hold at least as much at the end of the planning period. Use the notation in table 7 for the parameters (it is however permitted to add further symbols if you consider it necessary).

NOTICE! The following is required to get full score for this problem:

- The symbols for the optimisation variables must be clearly defined.
- The optimisation problem should be formulated so that it is easy to determine what the objective function is, which constraints there are and which limits there are.
- The possible values for all indices should be clearly stated for each equation.

Table 5 Data for the combined heat and power plant.

	Turbine 1	Turbine 2	Turbine 3
Maximal electricity generation [MW]	30	40	25
Heat generation per MW electricity generation [MW]	2.3	2.6	2.4
Generation cost [SEK/MWh electricity generation]	390	390	390

Table 6 Forecasts for the next day.

Hour	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Price [SEK/MWh]	395	385	380	375	380	400	480	440	455	450	440	430
Heat load [MWh]	180	180	180	180	180	180	160	160	160	140	140	140
Hour	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24
Price [SEK/MWh]	425	420	415	410	410	425	450	430	405	405	395	390
Heat load [MWh]	150	160	170	180	190	200	200	200	200	200	210	210

Table 7 Symbols for the planning problem of AB Verket.

Symbol	Explanation	Value
$\overline{H}$	Maximal electricity consumption in the electric boiler	25
η_H	Efficiency in the electric boiler	0.99
$\overline{M}$	Maximal contents of the accumulator tank	800
M_0	Start contents of the accumulator tank	700
$\underline{M}_{24}$	Least allowed contents of the accumulator tank at the end of the planning period	700
$\overline{G}_g$	Maximal electricity generation in turbine g	See table 5
c_g	Obtained heat per MWh generated electricity in turbine g	See table 5
β_{Gg}	Variable generation cost in turbine g	See table 5
λ_t	Electricity price in hour t	See table 6
D_t	Heat load in hour t	See table 6

b) (8 p) Assume that it is possible to cool the steam in turbine 3 using water from the local river Ån instead of delivering heat to the district heating system. In this operation mode the maximal electricity generation in turbine 3 is increased to 30 MW. How must the planning problem from part a be reformulated in order to consider the possibility to choose between combined heat and power operation and condensing operation in turbine 3? Do not forget to define all new variables and parameters that you introduce!

Hint: Introduce an integer variable to determine the operation mode of turbine 3.

Problem 9 (20 p)

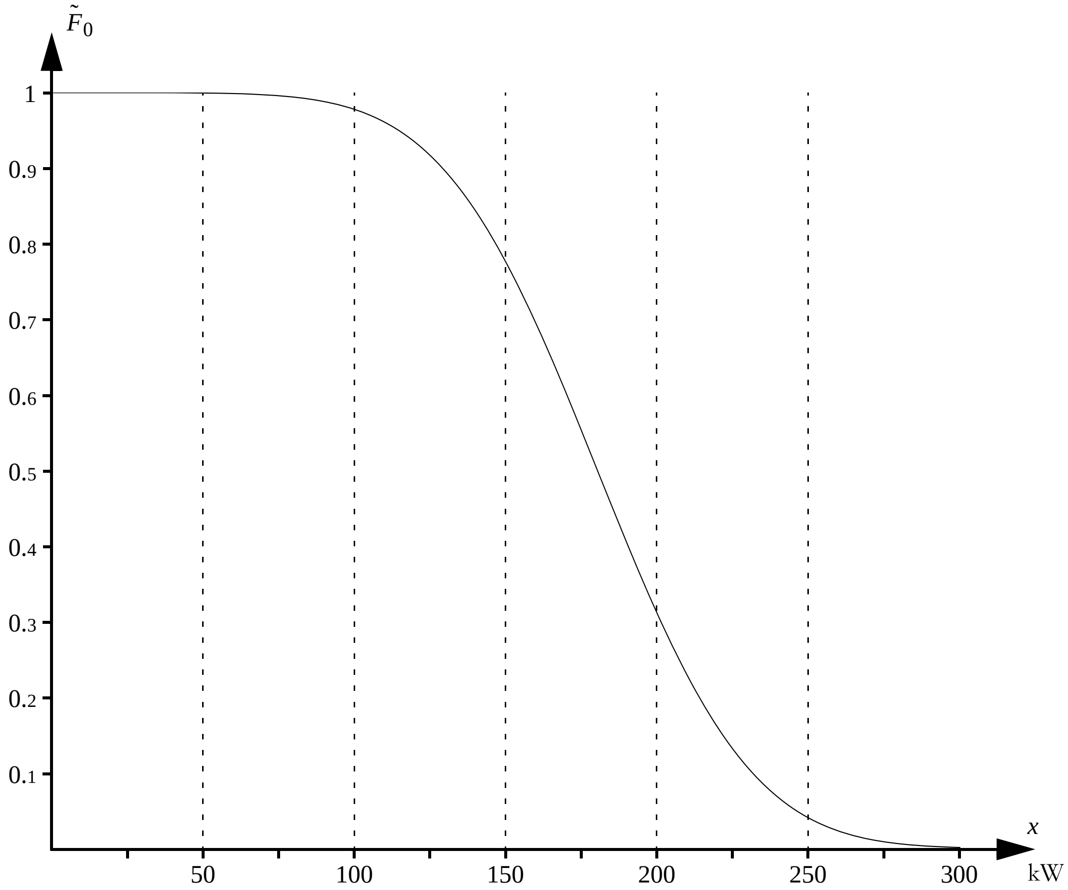
The South American village Pueblo is not connected to the national grid, but has a local grid of its own. The load in Pueblo is normally distributed with the mean 180 kW and standard deviation 40 kW. The system is supplied by three power plants: a hydro power plant and two diesel generator sets. The hydro power plant is a run-of-the-river plant (i.e., there is no reservoir) and has an installed capacity of 150 kW. The least recorded water flow in the river where the hydro power plant is located, was large enough to run the plant at installed capacity. The operation cost is negligible. The two diesel generator sets are of 100 and 50 kW respectively. The larger generator has an operation cost of 1 ¤/kWh. However, the efficiency of this unit is very poor when it is only partly loaded, and it is therefore never operated at less than 40 kW. If it is necessary a water heater can be used to consume any surplus generation. The smaller diesel generator set has an operation cost of 2 ¤/kWh. The hydro power plant has 100% availability, whereas the larger diesel generator set has the availability 90% and the availability of the smaller diesel generator set is 80%.

a) (1 p) To simulate the electricity market in Pueblo using probabilistic production cost simulation it is necessary to neglect one of the properties of the power plants described above. Explain which property that has to be neglected and why.

b) (2 p) The figure on the next page shows the load duration curve of this system. Define an approximation of this load duration curve. The approximation should consist of segments with a width of 50 kW, so that $\tilde{F}_0(x)$ is constant for the intervals $x \in [0, 50)$, $x \in [50, 100)$, etc. To get full score on this problem, your approximation may not result in serious overestimations or underestimations of *ETOC* or *LOLP* when it is used in probabilistic production cost simulation.
Hint: In practice, your task is to find a suitable "stair-case curve" to approximate $\tilde{F}_0(x)$.

c) (7 p) Use the approximated load duration curve to calculate the *ETOC* and *LOLP* of the system using probabilistic production cost simulation.

d) (10 p) Assume that Monte Carlo methods should used to simulate the system. To get as accurate results as possible, it is decided that the simulation should use control variates and stratified sampling. Suggest a strata tree for this simulation. The strata tree should be designed so that it separates those scenarios where the detailed and simplified models give the same results form those scenarios there the two model produce different results. Moreover, you should state for which strata it is possible to analytically compute the expectation values. Do not forget to motivate your answers!



$\tilde{F}_0$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
50
100
150
200
250
300
x
kW

KTH Electrical Engineering

Answer sheet for part I

Name: ....

Personal number: ....

Problem 1

a) Alternative .... is correct.

b) Alternative .... is correct.

Problem 2

a) .... TWh/year **b)** .... ¤/MWh

c) .... M¤/year

Problem 3

a) Alternative .... is correct.

b) .... MW **c)** .... MW

Problem 4

a) Parameters: ....

Optimisation variables: ....

b) ....

c) .... hours.

d) Alternative .... is correct.

Problem 5

a) .... MWh/h **b)** .... ¤/h

c) .... % **d)** .... MW

e) *ETOC* .... ¤/h *LOLP* .... %

Problem 1

a) 3, **b)** 5.

Problem 2

a) 150/200 = 75% of the biofuel capacity will be used at the electricity price 250 ¤/MWh; hence, the annual generation is 15 TWh/year.

b) Assume that the electricity price, λ, is in the range 200 to 300 ¤/MWh. Hydro power and nuclear power will generate 120 TWh; thus, the other two power sources must generate 22 TWh together. The contribution from biofuel and coal condensing can be expressed as

$$\frac{\lambda - 100}{300 - 100}20 + \frac{\lambda - 200}{400 - 200}10.$$

Setting this expression equal to 22 and solving for λ yields the electricity price $\lambda = 280$ ¤/MWh.

c) The income of Strålinge amounts to 10 TWh/year · 320 ¤/MWh = 3 200 M¤/year. From the income we subtract the total variable costs (10 TWh/year · 100 ¤/MWh = 1 000 M¤/year) and the fixed costs, which gives a profit of 200 M¤/year.

Problem 3

a) 4.

b) The relation between frequency and generation yields that the power plant should generate

$$G = G_0 - R(f - f_0) = 50 - 100(50.14 - 50) = 36 \text{ MW},$$

which is not possible. The power plant will then generate as little as possible, i.e., 40 MW.

c) Since the gain in area A is 2 000/3 000 = 40% of the total gain, 40% of the outage (i.e., 80 MW) will be replaced by increased generation in area A. This generation increase must be exported to area B. The transmission from A to B must therefore increase to 580 MW, which will not overload the transmission line.

Problem 4

a) Parameters: C^+, γ_i, D_t, λ_{25} and $\mu_{i,j}$. Optimisation variables: G_t and $Q_{i,j,t}$.

b) $\sum_{i=1}^{3} \sum_{j=1}^{3} \mu_{i,j} Q_{i,j,t} + G_t = D_t$.

c) The discharge at installed capacity is given by the relation $Q = H/\gamma(Q) = 100/0.8 = 125$ HE. As the reservoir can store 5 500 000/3 600 = 1 500 HE, a full reservoir will be enough for 12 hours maximal generation.

d) 5.

Problem 5

a) $EENS_0 = E[D] = 8\,000$ MWh/h.

b) In this case we have $\tilde{F}_0(x) = \tilde{F}_1(x)$, since the hydro power plants are 100% reliable. The expected generation in the thermal power plant are consequently given by

$$EG_2 = EENS_1 - EENS_2 = \int_{10\,000}^{\infty} \tilde{F}_1(x)dx - \int_{11\,000}^{\infty} \tilde{F}_2(x)dx = 8.5 - 1.2 = 7.3 \text{ MWh/h}.$$

Thus, the expected operation cost is $ETOC = 10EG_2 = 73$ ¤/h.

c) $LOLP = \tilde{F}_2(11\,000) = 0.9\tilde{F}_1(11\,000) + 0.1\tilde{F}_1(11\,000 - 1\,000) = 0.9 \cdot 0.001 + 0.1 \cdot 0.023 = 0.0032 =$ 0.32 %.

d) The normal distribution is symmetrical, which means that if $D = \mu_D + X$ then $D^* = \mu_D - X$. Hence, the complementary random number of $D = 7\,000$ MW must be $D^* = 9\,000$ MW.

e) $m_{TOC} = m_{(TOC - \tilde{TOC})} + \mu_{\tilde{TOC}} = \frac{1}{n}\left(\sum_{i=1}^{n} toc_i - \sum_{i=1}^{n} \tilde{toc}_i\right) + 73 =$

$= \frac{1}{10\,000}(975\,000 - 745\,000) + 73 = 96$ ¤/h.

$m_{LOLO} = m_{(LOLO - \tilde{LOLO})} + \mu_{\tilde{LOLO}} = \frac{1}{n}\left(\sum_{i=1}^{n} lolo_i - \sum_{i=1}^{n} \tilde{lolo}_i\right) + 0.0032 =$

$= \frac{1}{10\,000}(41 - 35) + 0.0032 = 0.0038 = 0.38\%$.

Problem 6

a) We can start by observing that without investments in new power plants, the electricity price in Rike will be i 64 ¤/MWh according to this model (as 34 TWh of the fossil fuel capacity would have to be utilised) and that the price will decrease by 1 ¤/MWh for each TWh generation with a lower variable cost that is supplied.

What is then the least electricity price that makes a new nuclear power plant profitable. The fixed costs plus the variable costs at maximal generation amount to 280 + 10 · 7 = 350 M¤/year. To earn at least that much, the electricity price must be 350/7 = 50 ¤/MWh. Thus, the system can be supplied 14 TWh nuclear power year—which corresponds to two units—without making the new power plants unprofitable.

b) The fixed costs plus the variable costs at maximal generation in a wind farm amount to 67 + 1 = 68 M¤/year. These costs should be covered by the income from sold electricity plus the income from the green certificates. The latter income is 20 ¤/certificate · 1 000 000 certificates/year = 20 M¤/year. Hence, the electricity price has to be at least 68 – 20 = 48 ¤/MWh if the 16 TWh wind power. Notice that we then get balance between supply and demand of green certificates, because the consumers must buy certificates corresponding to 0.1 · 160 = 16 TWh.

c) If new nuclear power plants are built, the total cost for the consumers is simply the electricity price multiplied by the total consumption, i.e., 50 · 160 = 8 000 M¤/year. If new wind farms are built then the cost of the consumers is equal to the cost of buying electricity plus the cost to buy certificates, i.e., 48 · 160 + 20 · 16 = 8 000 M¤/year. Hence, both alternatives are equivalent for the consumers (at least if only an economical comparison is made).

Problem 7

If there is an outage in southern Rike then a part of the generation in southern Rike will be replaced by import from the northern part of the country, i.e., the flow from northern to southern Rike will increase. It is the largest possible increase that determines how much transmission capacity that must be reserved for the primary control. The flow increase will become larger if the gain in northern Rike constitutes a large share of the total gain of the system. The worst case is therefore if the gain is 3 000 MW/Hz in northern Rike and 1 000 MW/Hz in southern Rike; the generation increase in the northern part of the country will then be 3 000/4 000 – 1 200 = 900 MW for an outage of 1 200 MW in southern Rike. Hence, 900 MW of the transmission capacity must be reserved for the primary control.

Problem 8

a) The problem we want to solve is

maximise *value of sold electricity – generation cost in the turbines – generation cost in the electric boiler*

subject to *heat load balance*,

energy balance of the accumulator tank,

limitations in generation capacity, etc.

The charging and discharging of the accumulator tank can be managed in different ways. One way would be to have separate variables for charging and discharging. However, in the solution here, we have chosen to model discharging as negative charging.

Parameters

The parameters are defined in table 7 in the problem text.

Optimisation variables

$G_{g,t}$ = generation in turbine g during hour t, g = 1, 2, 3, t = 1, …, 24,
H_t = electricity consumption in the electric boiler during hour t, t = 1, …, 24,
M_t = contents of the accumulator tank at the end of hour t, t = 1, …, 24,
A_t = charging of the accumulator tank during hour t, t = 1, …, 24.

Objective function

$$\text{maximise} \sum_{t=1}^{24} \left(\sum_{g=1}^{3} (\lambda_t - \beta_{Gg}) G_{g,t} - \lambda_t H_t \right).$$

Constraints

Heat load balance:

$$\sum_{g=1}^{2} c_g G_{g,t} + \eta_H H_t = D_t + A_t, \qquad t = 1, \ldots, 24.$$

Energy balance for the accumulator tank:

$$M_t = M_{t-1} + A_t, \qquad t = 1, \ldots, 24.$$

Variable limits

$$0 \le G_{g,t} \le \bar{G}_g, \qquad g = 1, 2, 3,\ t = 1, \ldots, 24,$$

$$0 \le H_t \le \bar{H}, \qquad t = 1, \ldots, 24,$$

$$0 \le M_t \le \bar{M}, \qquad t = 1, \ldots, 23,$$

$$\underline{M}_{24} \le M_{24} \le \bar{M}.$$

b) A straightforward method to model the two operation modes of turbine 3 is to introduce two different turbine models and then use a binary variable to decide which model that is used in a certain hour. Let the index g = 3 represent turbine 3 at combined heat and power operation (i.e, c_3 and $\bar{G}_3$ are the same as in part a) and let the index g = 4 correspond to turbine 3 at condensing power operation. Introduce the following new variables:

$G_{4,t}$ = generation in turbine 3 during hour t at condensing power operation, t = 1, …, 24,
$u_{3,t}$ = operation mode of turbine 3 (1 for combined heat and power operation, 0 for condensing power operation), t = 1, …, 24.

Introduce the following new parameters:

$\bar{G}_4$ = maximal generation in turbine 3 at condensing power operation = 30,
β_{G4} = variable generation cost in turbine 3 at condensing power operation = 390.

The objective function must be modified so that the generation cost in turbine 3 at condensing power operation is included:

$$\text{maximise} \sum_{t=1}^{24} \left(\sum_{g=1}^{4} (\lambda_t - \beta_{Gg}) G_{g,t} - \lambda_t H_t \right).$$

The heat load balance and the energy balance of the accumulator tank are not affected as turbine 3 does not generate any heat at condensing power operation. We will however have to introduce new constraints to manage the relation between operation mode in turbine 3 and maximal electricity generation:

$$G_{3,t} \le u_{3,t} \bar{G}_{3,t}, \qquad t = 1, \ldots, 24,$$

$$G_{4,t} \le (1 - u_{3,t}) \bar{G}_{3,t}, \qquad t = 1, \ldots, 24.$$

Finally, we have to introduce a lower limit to the electricity generation in turbine 3 at condensing power operation and state that the operation mode of turbine 3 is a binary variable:

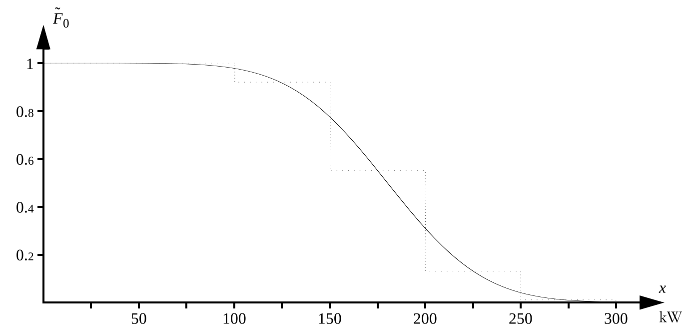
$$0 \le G_{4,t}, \qquad t = 1, \ldots, 24,$$

$$u_{3,t} \in \{0, 1\}, \qquad t = 1, \ldots, 24.$$

Problem 9

a) In probabilistic production cost simulation it is assumed that the power plants are dispatched strictly according to increasing variable operation cost, and that the lower limit of the generation in each power plant is zero. Hence, we will have to neglect the lower limit of the larger diesel generator set when using probabilistic production cost simulation.

b) What is important when approximating the load duration curve is that the surface below the curve (which corresponds to the expectation value of the load) should not change too much. A possible approximation is shown in the figure below:



Using this approximation yields the following load duration curve:

$$\tilde{F}_0(x) = \begin{cases} 1 & x < 100, \\ 0.92 & 100 \le x < 150, \\ 0.55 & 150 \le x < 200, \\ 0.13 & 200 \le x < 250, \\ 0.01 & 250 \le x < 300, \\ 0 & 300 \le x. \end{cases}$$

c) We start by adding the hydro power plant, since it has the least operation cost:

$$\tilde{F}_1(x) = 1 \cdot \tilde{F}_0(x) + 0 \cdot \tilde{F}_0(x - 150) = \tilde{F}_0(x).$$

Then we add the larger diesel generator set:

$$\tilde{F}_2(x) = 0.9\tilde{F}_1(x) + 0.1\tilde{F}_1(x - 100) =$$

$$= \begin{cases} 0.9 \cdot 1 + 0.1 \cdot 1 = 1 & x < 100, \\ 0.9 \cdot 0{,}92 + 0.1 \cdot 1 = 0.928 & 100 \le x < 150, \\ 0.9 \cdot 0{,}55 + 0.1 \cdot 1 = 0.595 & 150 \le x < 200, \\ 0.9 \cdot 0{,}13 + 0.1 \cdot 0{,}92 = 0.209 & 200 \le x < 250, \\ 0.9 \cdot 0{,}01 + 0.1 \cdot 0{,}55 = 0.064 & 250 \le x < 300, \\ 0 + 0{,}1 \cdot 0.13 = 0.013 & 300 \le x < 350, \\ 0 + 0{,}1 \cdot 0.01 = 0.001 & 350 \le x < 400, \\ 0 & 400 \le x. \end{cases}$$

The expected generation in the larger diesel generator set can now be calculated according to

$$EG_2 = EENS_1 - EENS_2 = \int_{150}^{\infty} \tilde{F}_1(x)dx - \int_{250}^{\infty} \tilde{F}_2(x)dx =$$

$$= 50(0.55 + 0.13 + 0.01) - 50(0.064 + 0.013 + 0.001) = 32.0 \text{ kWh/h}.$$

The expected generation in the smaller diesel generator set can be calculated by

$$EG_3 = 0.8 \int_{250}^{300} \tilde{F}_2(x)dx = 0.8 \cdot 50 \cdot 0.064 = 2.56 \text{ kWh/h}.$$

The expected operation cost is thus $ETOC = 1 \cdot EG_2 + 2 \cdot EG_3 = 37.12$ ¤/h. The risk of power deficit is given by

$$LOLP = \tilde{F}_3(300) = 0.8\tilde{F}_2(300) + 0.2\tilde{F}_2(300 - 50) = 0.8 \cdot 0.013 + 0.2 \cdot 0.064 = 2.32\%.$$

d) First we can observe that the two models will always give the same value of *LOLO*. Concerning *TOC* there will only be a difference between the detailed and the simplified model only in those scenarios where it is preferable to operate the smaller diesel generator set or use the water heater rather than running the larger generator on partial load. These scenarios can be identified using a strata tree with available generation capacity on the level below the root and load on the lowest level. An appropriate strata tree is shown in the table below.

Stratum, h	Available capacity in the diesel generator sets		Load, D [kW]	Difference in the results of the detailed and simplified models	
	$\overline{G}_2$ [kW]	$\overline{G}_3$ [kW]		*TOCD* [¤/h]	*LOLD*
1	0	0	> 0	0	0
2	100	0	0–150	0	0
3	100	0	150–190	> 0	0
4	100	0	> 190	0	0
5	0	50	> 0	0	0
6	100	50	0–150	0	0
7	100	50	150–190	> 0	0
8	100	50	> 190	0	0