KTH Electrical Engineering

Exam in EG2050 / 2C1118 System Planning, 10 June 2009, 8:00–13:00, V34

Allowed aids

In this exam you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten**, **single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

PART I (MANDATORY)

Write all answers on the answer sheet provided. Motivations and calculations do not have to be presented.

Part I can yield 40 points in total. The examinee is guaranteed to pass if the score is at least 33 points. If the result in part I is at least 31 points, then there will be a possibility to complement for passing the exam with the grade E.

Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

a) (2 p) The following applies to a centralised electricity market: I) Producers are free to sell to any other producer, retailer or consumer, II) All electricity trading has to be performed at a power pool, III) The consumers are free to buy from any producer or retailer.

1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. I and III are true but not II.

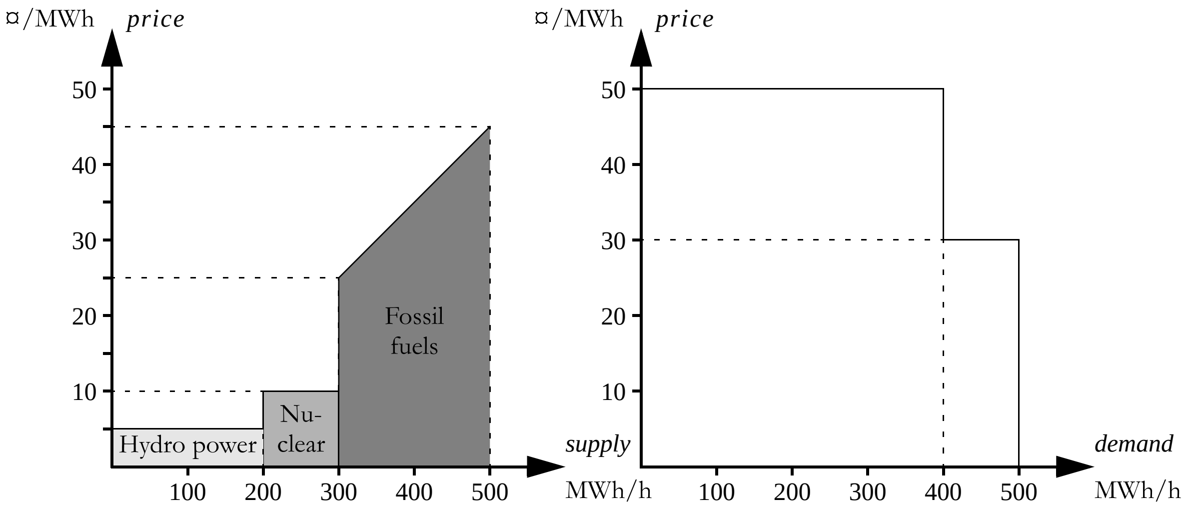
b) (1 p) What does an up-regulation bid mean?

1. A power company is selling electricity to a customer and the customer must in advance notify the power company about how much the customer will consume during each trading period.
2. A player offers to increase the generation (alternatively decrease the consumption) at the request of the system operator.
3. A player offers to decrease the generation (alternatively increase the consumption) at the request of the system operator.

c) (1 p) Consider a balance responsible player which during one hour has generated 1 031 MWh, sold 850 MWh to the power exchange, sold 158 MWh to consumers with take-and-pay contracts and purchased 20 MWh regulation power from the system operator. What obligation does this player have in the post trading for this hour?

1. The player must buy balance power from the system operator.
2. The player must sell balance power to the system operator.
3. None—it is voluntary for balance responsible players to participate in the post trading.

Problem 2 (6 p)



a) (3 p) The figures above show the supply and demand curves for of a certain hour in the electricity market of Land. What will the electricity price become in this electricity market if we assume perfect competition, perfect information and that there are neither transmission, reservoir nor capacity limitations?

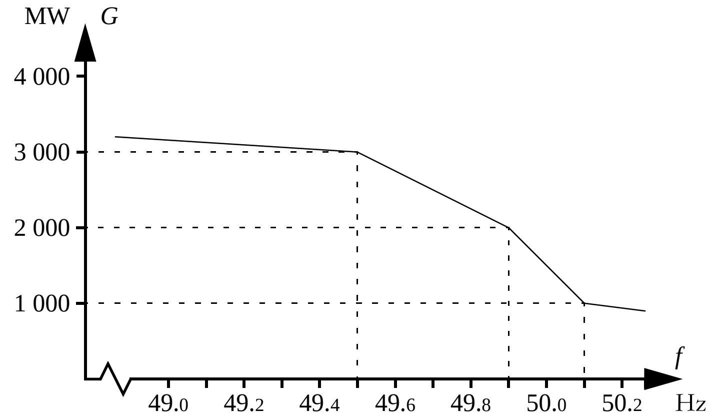
b) (2 p) What would happen to the electricity price in this electricity market if there also was 20 MWh wind power available in this hour?

c) (1 p) Consider an arbitrary electricity market and assume that the electricity price is equal to λ if there is perfect competition, perfect information and neither capacity, transmission nor reservoir limitations in the market. What will the electricity price become if the players of this electricity market do *not* have access to perfect information?

1. The electricity price must be lower than λ.
2. The electricity price must be higher than λ.
3. The electricity price can be lower than, equal to or higher than λ.

Problem 3 (6 p)

The figure below shows the total generation in the power plants participating in the primary control as a function of the frequency in a certain power system.



a) (2 p) At 11:01 there is balance between production and consumption in the system and the frequency is 50.01 Hz. At this time the thermal power plant Sotinge is increasing its generation by 100 MW. The power plant Sotinge is not participating in the primary control. What will the frequency be when the primary control has restored the balance between generation and consumption?

b) (2 p) At 11:04 there is balance between production and consumption in the system and the frequency is 50.04 Hz. At this time the load of the system is decreased by 50 MW. What will the frequency be when the primary control has restored the balance between generation and consumption?

c) (2 p) At 11:06 there is balance between production and consumption in the system and the frequency is 49.95 Hz. At this time a lightning strike in a substation causes 750 MW of generation to be lost. The concerned power plants were not part of the primary control. What will the frequency be when the primary control has restored the balance between generation and consumption?

Problem 4 (12 p)

Stads energi AB owns a thermal power plant with three blocks. Moreover, the company owns a wind farm. Assume that the company has formulated their short-term planning problem as a MILP problem and that the following symbols have been introduced:

Indices for the power plants: Block I - 1, Block II - 2, Block III - 3.

β_{Gg} = variable operation cost in power plant *g*,
C_g^+ = start-up cost in power plant *g*, *g* = 1, 2, 3,
D_t = contracted load during hour *t*, *t* = 1, …, 24,
$G_{g,t}$ = generation in power plant *g*, hour *t*, *g* = 1, 2, 3, *t* = 1, …, 24,
$\bar{G}_g$ = installed capacity in power plant *g*, *g* = 1, 2, 3,
λ_t = expected electricity price at ElKräng hour *t*, *t* = 1, …, 24,
p_t = purchase from ElKräng hour *t*, *t* = 1, …, 24,
r_t = sales to ElKräng hour *t*, *t* = 1, …, 24,
$s_{g,t}^+$ = start-up variable for power plant *g*, hour *t*, *g* = 1, 2, 3, *t* = 1, …, 24, *g* = 1, 2, 3,
$u_{g,t}$ = unit commitment of power plant *g*, hour *t*, *g* = 1, 2, 3, *t* = 1, …, 24,
W_t = expected wind power generation in hour *t*, *t* = 1, …, 24.

a) (4 p) Stads energi AB sells power to customers with firm power contracts, but the company also has the possibility to trade at the local power exchange ElKräng.. Formulate the objective function if the aim of the planning problem is to maximise the income of sold electricity at ElKräng minus the costs of purchasing electricity from ElKräng and minus the costs of the thermal power plant. Use the symbols defined above.

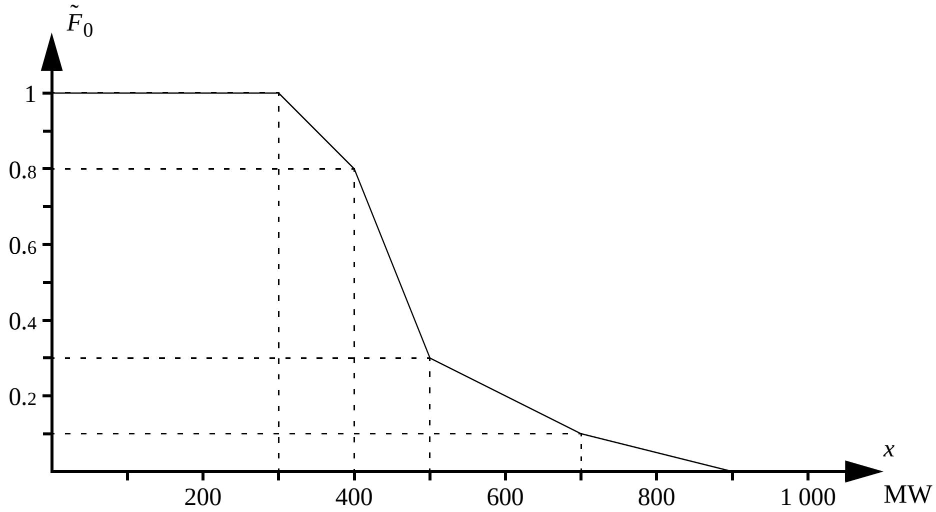
b) (4 p) Formulate the constraint that sets the relation between maximal generation and unit commitment in power plant *g*, hour *t*. Use the symbols defined above.

c) (2 p) Block III is fuelled by biomass. The fuel costs 260 ¤/m^3 and has a density of 400 kg/m^3. The heat contents of the fuel is 5 MWh/ton and the efficiency of the power plant is 40%. How large is the variable generation cost in Block III?

d) (2 p) The maximal production equivalent in Fjärd is 0.5 MWh/HE. Assume that the power plant should generate 14.7 MWh between 9:00 and 10:00. How large is the discharge in Fjärd during this hour if the relative efficiency is 98%? The answer should be given in HE.

Problem 5 (12 p)

The national grid in Nchi is supplied by three larger hydro power plants with a combined installed capacity of 750 MW and a 50 MW thermal power plant. The figure below shows the duration curve of the total load in Nchi.



a) (3 p) Assume that all power plants have 100% availability and that the variable operation cost is 10 ¤/MWh in the thermal power plant, whereas the variable operation cost in the hydro power plants is negligible. Use probabilistic production cost simulation to calculate the expected total operation cost per hour.

b) (3 p) Assume that the hydro power plants have 100% availability and that the thermal power plant has 90% availability. What is the risk of power deficit in Nchi?

c) (4 p) The Ministry of Energy in Nchi is investigating the possibility to introduce wind power. In connection to this investigation, the ministry has developed a model that includes the losses in the national grid. Ten scenarios have been generated for a Monte Carlo-simulation using this model (see table 1). Control variates are used to increase the accuracy of the simulation. Assume that *ETOC* according to the simplified model has been calculated to 20 ¤/h and that *LOLP* for the simplified model has been calculated to 3.0%. Which estimates of *ETOC* and *LOLP* are obtained for the detailed model?

Table 1 Results from a Monte Carlo simulation of the electricity market in problem 5c.

Scenario	1	2	3	4	5	6	7	8	9	10
TOC [¤/h]										
Simplified model	0	0	0	0	0	0	0	0	100	0
Detailed model	0	20	0	180	0	0	0	0	500	0
LOLO										
Simplified model	0	0	0	0	0	0	0	0	0	0
Detailed model	0	0	0	0	0	0	0	0	1	0

d) (2 p) Assume that complementary random numbers are used to improve the simulation of Nchi. What is the value of the complementary random number, D^*, if the total load of the system is randomised to D = 400 MW?

PART II (FOR HIGHER GRADES)

All introduced symbols must be defined. Solutions should include sufficient detail that the argument and calculations can be easily followed.

The answer to each problem must begin on a new sheet, but answers to different parts of the same problem (a, b, c, etc.) can be written on the same sheet. The fields *Namn* (Name), *Blad nr* (Sheet number) and *Uppgift nr* (Problem number) must be filled out on every sheet.

Part II gives a total of 60 points, but this part will only be marked if the candidate has obtained at least 33 points in part I. Then the results of parts I and II and the bonus points will be added together to determine the examination grade (A, B, C, D, E).

Problem 6 (10 p)

Unionen has four member states: Aland, Beland, Celand and Deland. The four countries have a common electricity market. Data for generation and demand are given in table 2 and data for the transmission capacity between the countries is given intable 3. The variable operation costs are assumed to be linear within the intervals, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

Table 2 Data for generation and demand in the electricity market of Unionen.

Power source	Production capability [TWh/year]				Variable costs [¤/MWh]
	Aland	Beland	Celand	Deland	
Hydro power	60	120	–	5	5
Nuclear power	80	–	–	15	80–120
Fossil fuels	20	–	60	20	300–700
Demand	133	97	32	37	

Table 3 Data for the transmission between the countries in the Unionen.

Interconnection	Capacity [TWh/year]
Aland ↔ Beland	20
Aland ↔ Celand	20
Beland ↔ Celand	20
Celand ↔ Deland	10

a) (8 p) Assume that the electricity market has perfect competition, perfect information and that there are neither capacity nor reservoir limitations. Which electricity price is obtained in each country?

b) (2 p) The consumers in Deland has taken an initiative to increase the transmission capacity between Celand and Deland to 15 TWh/year. How large may the annual investment cost be if the increased transmission capacity is to be financed only by the consumers in Deland?

Problem 7 (10 p)

Skäret is a small island in the Stockholm archipelago. The island is not connected to the national grid, but the islanders have built a small local grid, which is supplied by two diesel generator sets. Assume that the load in the system can vary between 20 and 100 kW. The frequency in the system should be kept within the interval 50 ± 1 Hz and the frequency should be exactly 50 Hz when the load is equal to 60 kW.

One of the inhabitants of the island, Inge Händig, is an engineer and has been commissioned to set the frequency control of the two diesel generator sets so that the operation cost is minimised for normal operation (i.e., then both generators are working as they should). Data for the diesel generator sets are shown in table 4. The operation cost of each diesel generator set can be divided in two terms: one part is the cost for the base generation (the electricity generation when the frequency is exactly 50 Hz) and one part for the gain, which means that the total operation cost per hour can be expressed as

$$C_{tot} = 3G_I + 3.15G_{II} + 0.30R_I + 0.32R_{II},$$

where

G_I = base generation in diesel generator set I [kW],
G_{II} = base generation in diesel generator set II [kW],
R_I = gain in diesel generator set I [kW/Hz],
R_{II} = gain in diesel generator set II [kW/Hz].

a) (2 p) Assume that Inge sets the base generation to 20 kW in diesel generator set I. Which is the highest gain R_I that he can choose, if the gain is to be available in the frequency interval 50 ± 1 Hz?

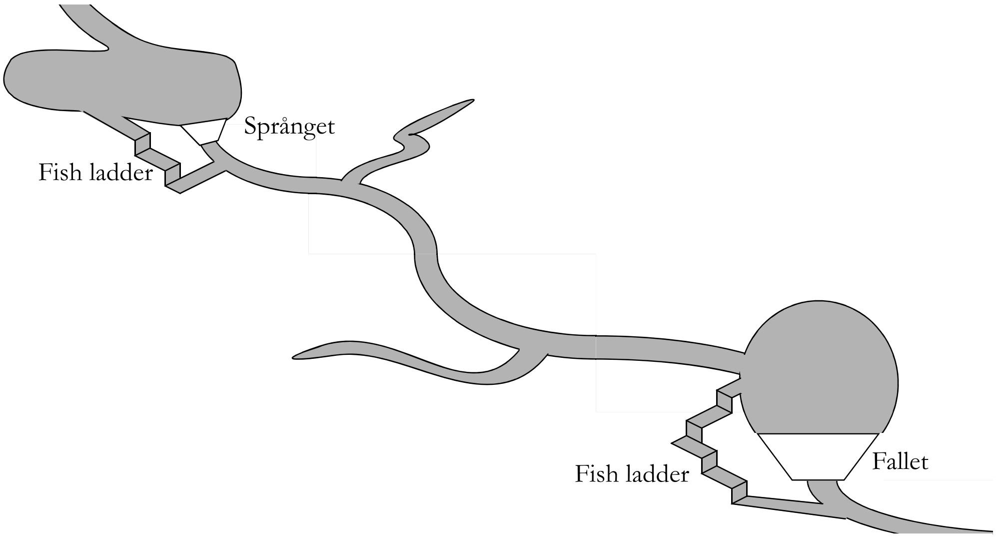
b) (2 p) Assume that Inge sets the base generation to 50 kW in diesel generator set I. Which is the highest gain R_I that he can choose, if the gain is to be available in the frequency interval 50 ± 1 Hz?

c) (6 p) Inge has concluded that diesel generator set I is less expensive for base generation as well as gain, but the difference between the two generators is quite small when it comes to the gain. It is therefore optimal to have as much base generation as possible in generator I. Which base generation and gain should he then set in each generator?.

Table 4 Data for the diesel generator sets on Skäret.

Generator	Installed capacity [kW]	Average operation cost at constant generation [SEK/kWh]	Average cost to supply gain [(SEK/h)/(kW/Hz)]
I	80	3.00	0.30
II	60	3.15	0.32

Problem 8 (20 p)



AB Vattenkraft owns two hydro power plants, located as shown in the figure above. Data for the power plants are given in table 5. The company is planning to sell their electricity generation at the local power pool, ElKräng. It is assumed that the company can sell as much as they want to the prices listed in table 6. After this period the average electricity price is estimated to 400 SEK/MWh and stored water is assumed to be used for generation at best efficiency. The water delay time between the power plants can be neglected.

Since the river where the power plants are located is an important breeding area for salmon, the Environment Court has judged that the company must build fish ladders which allow fish to pass the dams. The minimum water flow in each fish ladder must be 1 m^3/s; this water can thus not be used for power generation. Moreover, the Environment Court has decided that for tourism reasons the river flow in the river sections downstream each power plant may not be less than 10 m^3/s.

a) (10 p) Formulate the planning problem of AB Vattenkraft as an LP problem. Use the notation in table 7 for the parameters (it is however permitted to add further symbols if you consider it necessary).

NOTICE! The following is required to get full score for this problem:

- The symbols for the optimisation variables must be clearly defined.
- The optimisation problem should be formulated so that it is easy to determine what the objective function is, which constraints there are and which limits there are.
- The possible values for all indices should be clearly stated for each equation.

b) (10 p) The reservoirs of Språnget and Fallet have comparatively small surface area compared to the active storage. This means that then water is released from the reservoirs, the reservoir water level may significantly decrease; thus, the head of the power plants will decrease. Assume that AB Vattenkraft wants to consider this in the planning problem using a simplified model. If the reservoirs are filled to at least 50% the marginal production equivalents stated in table 5 should be used, but if the contents is less than 50% then 25% lower marginal production equivalents are to be used. How must the planning problem from part a be reformulated in order to consider this head dependence model? Do not forget to define all new variables and parameters that you introduce!

Hint: A binary variable $l_{i,t}$ can be assigned the value 0 if the reservoir level is lower than $\overline{M}_i/2$ and the value 1 if the reservoir level is higher than $\overline{M}_i/2$ by introducing the following constraints:

$$l_{i,t} \le \frac{M_{i,t}}{(\overline{M}_i/2)}, \qquad i = 1, 2,\ t = 1, \ldots, 24,$$

$$l_{i,t} \ge \frac{M_{i,t}}{(\overline{M}_i/2)} - 1, \qquad i = 1, 2,\ t = 1, \ldots, 24.$$

Table 5 Data for the power plants of AB Vattenkraft.

Power plant	Start contents of reservoir [HE]	Maximal contents of reservoir [HE]	Marginal production equivalents [MWh/HE]		Maximal discharge [HE]		Local inflow [HE]
			Segment 1	Segment 2	Segment 1	Segment 2	
Språnget	1 000	3 600	0.64	0.56	85	30	112
Fallet	1 200	3 900	0.72	0.64	125	40	2

Table 6 Expected prices at ElKräng.

Hour	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Price at ElKräng [SEK/MWh]	325	265	255	255	245	300	370	395	420	425	425	425
Hour	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24
Price at ElKräng [SEK/MWh]	420	425	415	415	410	415	415	410	395	385	395	365

Table 7 Notation for the planning problem of AB Vattenkraft.

Symbol	Explanation	Value
$\underline{S}_i$	Minimal flow through the fish ladder at reservoir *i*	1
$\underline{W}_i$	Minimal flow downstream of power plant *i*	10
$M_{i,0}$	Start contents of reservoir *i*	Se table 5
$\overline{M}_i$	Maximal contents of reservoir *i*	Se table 5
$\mu_{i,j}$	Marginal production equivalent in power plant *i*, segment *j*	Se table 5
$\overline{Q}_{i,j}$	Maximal discharge in power plant *i*, segment *j*	Se table 5
V_i	Local inflow to reservoir *i*	Se table 5
λ_t	Expected price at ElKräng hour *t*	Se table 6
λ_f	Expected future electricity price	400

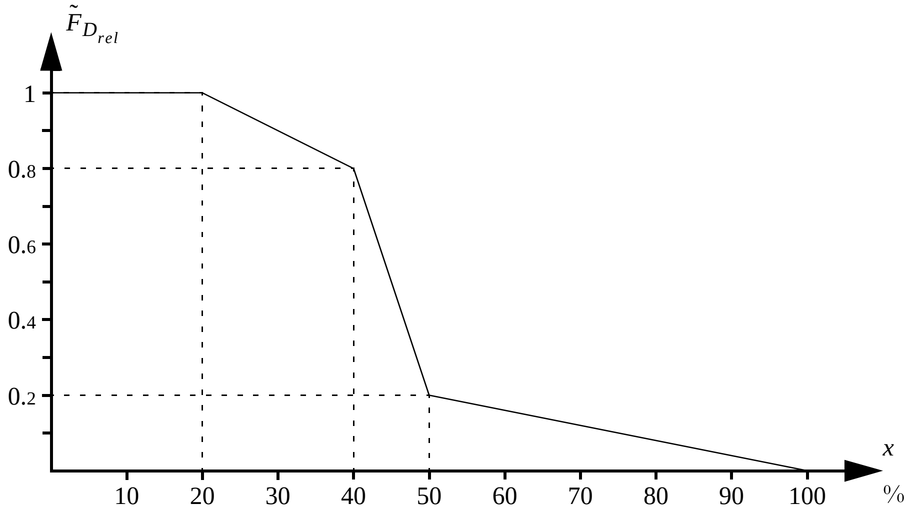
Problem 9 (20 p)

Large parts of the rural areas of Eggwanga have no access to electricity. The Eggwangan authorities have started a rural electrification programme, and as part of this programme, the Eggwanga National Electricity Supply Company Ltd. (ENESCO) has been given the task of electrifying the village Ekyaro and its surroundings. ENESCO is now investigating how much it would cost to build a local grid in Ekyaro. The local grid will not be connected to the national grid within a close future, but will be supplied by a hydro power plant and a diesel generator set.

ENESCO assumes that the demand for electricity in Ekyaro will depend on the tariff the consumers have to pay. Based on experience from similar areas in Eggwanga, ENESCO have estimated the relation between the peak load in Ekyaro, $\hat{D}$, and the tariff, λ, as

$$\hat{D} = 900 - 300\lambda.$$

The actual load at a random point of time, D, can be calculated according to $D = D_{rel} \cdot \hat{D}$, where D_{rel} is a random variable describing how large the load is compared to the peak load. It can be assumed that the electricity consumption will be distributed in the same way regardless of which tariff that is charged, i.e., if the peak load increases by 10% then the least load also increases by 10%. This means that the probability distribution of the relative load is the same regardless of the tariff. The figure below shows the relative load duration curve estimated by ENESCO.



a) (5 p) Assume that the hydro power plant would have an installed capacity of 400 kW and negligible operation costs. There are no plans for a reservoir, but on the other hand is the water flow always sufficient to generate the installed capacity. ENESCO expects a very high reliability in the hydro power plants and the risk of outages can therefore be neglected in the hydro power plant. The planned diesel generator set would have 200 kW installed capacity, variable operation cost 1 ¤/kWh and 80% availability.

The total investment cost for the local grid, the hydro power plant and the diesel generator set has been calculated to 1.8 M¤/year. In addition to that there will be fixed costs for maintenance, staff salary, etc., amounting to 0.5 M¤/year. Will the suggested system for Ekyaro carry its own costs if the tariff is set to 1 ¤/kWh?

b) (5 p) Which tariff should ENESCO set in order to get a risk of power deficit equal to 2%?

c) (10 p) A problem for rural grids in Eggwanga is that the losses become comparatively high. This is partly due to that the consumers are few and scattered and that ENESCO has limited resources for grid investments (resulting in a tendency to install too weak power lines and transformers), and partly due to the occurrence of people connecting illegally to the grid or manipulating their meters.

ENESCO has developed a more detailed model of the power system in Ekyaro. This model considers that parts of the distribution system can be overloaded in some cases, resulting in larger losses than normal. The model also includes "non-technical losses", i.e., electricity consumption that ENESCO will not be paid for. Assume that the total losses (technical and non-technical) can vary between 0 and 160 kW when the tariff is 1 ¤/kWh. The objective of the model is to use it to estimate *ETOC* and *LOLP* in a Monte Carlo simulation.

Suggest an appropriate strata tree for the power system in Ekyaro. State the stratum weights and indicate which values that are possible for the result variables *TOC* and *LOLO* in each stratum.

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Answer sheet for part I

Name: ....

Personal number: ....

Problem 1

a) Alternative .... is correct.

b) Alternative .... is correct.

c) Alternative .... is correct.

Problem 2

a) .... ¤/MWh

b) The electricity price would decrease by .... ¤/MWh

c) Alternative .... is correct.

Problem 3

a) .... Hz **b)** .... Hz

c) .... Hz

Problem 4

a) ....

....

b) ....

c) .... ¤/MWh **d)** .... HE

Problem 5

a) .... ¤/h **b)** .... %

c) *ETOC* .... ¤/h *LOLP* .... %

d) .... MW

Problem 1

a) 3, **b)** 2, **c)** 2.

Problem 2

a) The electricity price is determined by the intersection of the supply and demand curves. The intersection can be found graphically by drawing both curves in the same figure. In order to have a demand of 500 MWh/h the electricity price may not exceed 30 ¤/MWh, but the supply is only 350 MWh/h at that electricity price. Hence, the demand is n 400 MWh/h and then it the electricity price will have to be 35 ¤/MWh (half the interval for fossil fuel generation).

b) 20 MWh wind power replaces 20 MWh fossil fuels. Since 20 MWh is 10% of the fossil fuel capacity, the price decrease must correspond to 10% of the price interval of fossil fuels, i.e., 2 ¤/MWh.

c) 3.

Problem 3

a) The power plants participating in the primary control are generating 1 450 MW when the frequency is 50.01 Hz (according to the figure). These power plants have to decrease their generation to 1 350 MW when Sotinge increases the generation by 100 MW, which means that the frequency must increase to 50.03 Hz.

b) The power plants participating in the primary control are generating 1 300 MW when the frequency is 50.04 Hz (according to the figure). These power plants have to decrease their generation to 1 250 MW when the load decreases by 50 MW, which means that the frequency must increase to 50.05 Hz.

c) The power plants participating in the primary control are generating 1 750 MW when the frequency is 49.95 Hz (according to the figure). These power plants have to increase their generation to 2 500 MW when 750 MW generation is lost, which means that the frequency must decrease to 49.70 Hz.

Problem 4

a) maximise $\sum_{t=1}^{24}\left(\lambda_t(r_t - p_t) - \sum_{g=1}^{3}(C_g^+ s_{g,t}^+ + \beta_{Gg}G_{g,t})\right)$.

b) $G_{g,t} \le u_{g,t}\overline{G}_g$.

c) The heat contents of one m^3 fuel is 0.4 ton/m^3 · 5 MWh/ton = 2 MWh/m^3. Since the efficiency is 40% the electricity generation is 0.4 · 2 = 0.8 MWh/m^3. If the fuel costs 260 ¤/m^3 then the variable generation cost is 260/0.8 = 325 ¤/MWh.

d) The generation as a function of discharge can be expressed as $H(Q) = \eta(Q)\chi_{max}Q \Rightarrow$

$Q = H(Q)/\eta(Q)\chi_{max}$ = 14.7 MWh/(0.98 · 0.5 MWh/HE) = 30 HE.

Problem 5

a) Since all power plants have 100% availability, we get $\tilde{F}_2(x) = \tilde{F}_1(x) = \tilde{F}_0(x)$, which means that

$$EG_2 = EENS_1 - EENS_2 = \int_{750}^{\infty}\tilde{F}_1(x)dx - \int_{800}^{\infty}\tilde{F}_2(x)dx = \int_{750}^{800}\tilde{F}_0(x)dx =$$

= (0.075 + 0.05) · 50/2 = 3.125 MWh/h.

We can assume that the operation cost of hydro power is negligible; hence, the expected operation cost per hour is $ETOC = 10EG_2$ = 31.25 ¤/h.

b) $LOLP = \tilde{F}_2(800) = 0.9\tilde{F}_1(800) + 0.1\tilde{F}_1(800 - 50)$ = 0.9 · 0.05 + 0.1 · 0.075 = 5.25%.

c) $m_{TOC} = m_{(TOC - \tilde{TOC})} + \mu_{\tilde{TOC}} = \frac{1}{10}\sum_{i=1}^{10}(TOC_i - \tilde{TOC}_i) + 35$ = (20 + 180 + 400)/10 + 20 = =80 ¤/h.

$m_{LOLO} = m_{(LOLO - \tilde{LOLO})} + \mu_{\tilde{LOLO}} = \frac{1}{10}\sum_{i=1}^{10}(LOLO_i - \tilde{LOLO}_i) + 0.05$ = 1/10 + 0.03 = 13%.

d) The inverse transform method states that $D = F_D^{-1}(U)$, where U is a $U(0, 1)$-distributed random number. Since it is the duration curve that is given in the problem, we may as well use the transform $D = \tilde{F}_D^{-1}(U)$. The original random number must then have been $U = \tilde{F}_D(400)$ = 0.8. Hence, $U^* = 1 - U$ = 0.2, which results in $D^* = \tilde{F}_D^{-1}(U^*)$ = 600 MW.

Problem 6

a) The total demand in Unionen is 299 TWh/year. If the transmission limitations are ignored, the electricity price would be i 376 ¤/MWh, because hydro and nuclear can supply at most 280 TWh/year; hence, 19 TWh/year of fossil fuel generation will be needed, which means that 19/100 of the price interval for fossil fuel will be utilised. At this electricity price, the imports and exports of each country will become as follow:

- Aland generates 143.8 TWh/year and consumes 133 TWh/year
  ⇒ export 10.8 TWh/year, which is possible since Aland can export in total 40 TWh/year.
- Beland generates 120 TWh/year and consumes 97 TWh/year
  ⇒ export 23 TWh/year, which is possible since Beland can export in total 40 TWh/year.
- Celand generates 11,4 TWh/year and consumes 32 TWh/year
  ⇒ import 20,6 TWh/ear, which is possible since Celand can import in total 60 TWh/year.
- Deland generates 23.8 TWh/year and consumes 37 TWh/year
  ⇒ import 13.2 TWh/year, which is not possible since Deland only can import 10 TWh/year from Celand.

To obtain balance between supply and demand in Deland, the electricity price must therefore be higher compared to if there had not been a transmission limitation. At maximal import from Celand, the generation in Deland needs to be a27 TWh/year, which requires 7/20 of the fossil-fuelled power plants in Deland to be utilised. The electricity price in Deland must therefore be 440 ¤/MWh.

In the remaining parts of Unionen, the total demand is then 272 TWh/year (including 10 TWh/year export from Celand to Deland). Hydro and nuclear can generate 260 TWh/year in these countries, which means that 12/80 of the fossil-fuelled power plants will be needed. The electricity price in Aland, Beland and Celand will therefore be 360 ¤/MWh. At this electricity price, Aland exports 10 TWh/year and Beland 23 TWh/year.

b) The new transmission capacity between Celand and Deland is sufficient to obtain the electricity price 376 ¤/MWh throughout the entire Unionen. This means that the costs for the consumers in Deland would decrease by 64 ¤/MWh · 37 TWh/year = 2 368 M¤/year. The annual cost may not be larger than that if the investment is to be profitable for the consumers in Deland.

Problem 7

a) The diesel generator set can decrease its generation by 20 kW and increase it by 60 kW when the base generation is 20 kW. The maximal generation change is therefore 20 kW and since the maximal frequency change is 1 Hz, the gain cannot be larger than $R = \Delta G/\Delta f = 20$ kW/Hz.

b) The diesel generator set can decrease its generation by 50 kW and increase it by 60 kW when the base generation is 30 kW. The maximal generation change is therefore 30 kW and since the maximal frequency change is 1 Hz, the gain cannot be larger than $R = \Delta G/\Delta f = 30$ kW/Hz.

c) The total base generation of the two diesel generator sets should be 60 kW (because that is the load for which the frequency should be 50 Hz). If the entire base generation was in diesel generator then it would not be possible to set the gain in that unit higher than 20 kW/Hz (cf. the previous problems) But the load can deviate up to 40 kW form the load at 50 Hz and therefore the total gain of the system must be 40 kW/Hz if the frequency is to be kept in the interval 50 ± 1 Hz. Hence, diesel generator set II must contribute with a gain of 20 kW/Hz. This is however not possible if the base generation in diesel generator set II is zero, because then there would not be any margin to decrease the generation.

Thus, we can conclude that a part of the base generation must be supplied by diesel generator set II. If we assume that the base generation in generator will continue to be at least 40 kW then the margin to increase the generation will limit the gain in generator I, i.e., $R_I = 80 - G_I = \{G_I = 60 - G_{II}\} = 20 + G_{II}$. Moreover, if we assume that the base generation of diesel generator set II will not exceed 30 kW then it will be the margin to decrease generation which determines the possible gain in generator II; hence, we can set $R_{II} = G_{II}$. If the total gain should equal 40 kW/Hz then we must have

$$R_I + R_{II} = 20 + G_{II} + G_{II} = 40.$$

Solving this equation yields that the base generation in generator II should be 10 kW; the base generation in generator I must accordingly be 50 kW (which means that the assumptions above are fulfilled). The gain then becomes 30 kW/Hz in generator I and 10 kW/Hz i generator II respectively.

Problem 8

a) In words the planning problem can be formulated as

maximise	*value of sold electricity + value of stored water*,
subject to	*hydrological balance for Språnget and Fallet*,
	minimal flow in the river.

Indices for power plants

Språnget 1, Fallet 2.

Parameters

The parameters are defined in table 7 of the problem text.

Optimisation variables

$Q_{i,j,t}$ = discharge in power plant i, segment j, during hour t, $i = 1, 2, j = 1, 2, t = 1, \ldots, 24$,
$S_{i,t}$ = spillage from reservoir i during hour t, $i = 1, 2, t = 1, \ldots, 24$,
$M_{i,t}$ = contents of reservoir i at the end of hour t, $i = 1, 2, t = 1, \ldots, 24$.

Objective function

$$\text{maximise} \quad \sum_{t=1}^{24} \lambda_t \sum_{i=1}^{2} \sum_{j=1}^{2} \mu_{i,j} Q_{i,j,t} + \lambda_f((\mu_{1,1} + \mu_{2,1})M_{1,24} + \mu_{2,1} M_{2,24}).$$

Constraints

Hydrological balance for Språnget:

$$M_{1,t} = M_{1,t-1} - Q_{1,1,t} - Q_{1,2,t} - S_{1,t} + V_{1,t}, \qquad t = 1, \ldots 24.$$

Hydrological balance for Fallet:

$$M_{2,t} = M_{2,t-1} - Q_{2,1,t} - Q_{2,2,t} - S_{2,t} + Q_{1,1,t} + Q_{1,2,t} + S_{1,t} + V_{2,t}, \qquad t = 1, \ldots 24.$$

Minimal flow in the river:

$$Q_{i,1,t} + Q_{i,2,t} + S_{i,t} \geq \underline{W}_i, \qquad i = 1, 2, t = 1, \ldots 24.$$

Variable limits

$$0 \leq Q_{i,j,t} \leq \overline{Q}_{i,j}, \qquad i = 1, 2, j = 1, 2, t = 1, \ldots, 24,$$

$$\underline{S}_i \leq S_{i,t}, \qquad i = 1, 2, t = 1, \ldots, 24,$$

$$0 \leq M_{i,t} \leq \overline{M}_i, \qquad i = 1, 2, t = 1, \ldots, 24.$$

b) We start by introducing the binary variable suggested in the hint:

$l_{i,t}$ = level or reservoir i, hour t (0 if the level corresponds to low head, otherwise 1).

To manage the head dependence we introduce an additional index for the discharge:

$Q_{i,j,h,t}$ = discharge in power plant i, segment j, head h, during hour t, $i = 1, 2, j = 1, 2, h = 1, 2, t = 1, \ldots, 24$,

where we choose to interpret $h = 1$ as low head and $h = 2$ as high head. Consequently, we need another index also for the marginal production equivalents:

$\mu_{i,j}$ = marginal production equivalent in power plant i, segment j, head h =

$$= \begin{cases} 0.64 & i = 1, j = 1, h = 1, & 0.48 & i = 1, j = 1, h = 2, \\ 0.56 & i = 1, j = 2, h = 1, & 0.42 & i = 1, j = 2, h = 2, \\ 0.72 & i = 2, j = 1, h = 1, & 0.54 & i = 2, j = 1, h = 2, \\ 0.64 & i = 2, j = 2, h = 1, & 0.48 & i = 2, j = 2, h = 2. \end{cases}$$

As we have introduced new discharge variables, we must update the objective function and constraints from part a:

$$\text{maximise} \quad \sum_{t=1}^{24} \lambda_t \sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{h=1}^{2} \mu_{i,j,h} Q_{i,j,h,t} + \lambda_f((\mu_{1,1,1} + \mu_{2,1,1})M_{1,24} + \mu_{2,1,1}M_{2,24}).$$

$$M_{1,t} = M_{1,t-1} - \sum_{j=1}^{2} \sum_{h=1}^{2} Q_{1,j,h,t} - S_{1,t} + V_{1,t}, \qquad t = 1, \ldots 24.$$

$$M_{2,t} = M_{2,t-1} - \sum_{j=1}^{2} \sum_{h=1}^{2} Q_{2,j,h,t} - S_{2,t} + \sum_{j=1}^{2} \sum_{h=1}^{2} Q_{1,j,h,t} + S_{1,t} + V_{2,t}, \quad t = 1, \ldots 24.$$

$$\sum_{j=1}^{2} \sum_{h=1}^{2} Q_{i,j,h,t} + S_{i,t} \geq \underline{W}_i, \qquad i = 1, 2, t = 1, \ldots 24.$$

We must now make sure that only one of the head models are used in each hour. This can be achieved by setting the upper limit of discharge for a certain head to zero if the binary variable $l_{i,t}$ does not indicate that the reservoir has that head. At low head, the upper limit should be zero if $l_{i,t} = 1$ and equal to $\overline{Q}_{i,j}$ if $l_{i,t} = 0$:

$$Q_{i,j,1,t} \leq (1 - l_{i,t})\overline{Q}_{i,j} \qquad i = 1, 2, j = 1, 2, t = 1, \ldots 24.$$

Similarly, we get the following constraints for discharge at high head:

$$Q_{i,j,2,t} \leq l_{i,t}\overline{Q}_{i,j} \qquad i = 1, 2, j = 1, 2, t = 1, \ldots 24.$$

Finally, we need the constraints suggested in the problem text to assign correct values to $l_{i,t}$, and we need variable limits for the new variables. (Notice that the discharge variables do not need an upper limit, because the upper limit is controlled by the constraints above.)

$$l_{i,t} \leq \frac{M_{i,t}}{(\overline{M}_i/2)}, \qquad i = 1, 2, t = 1, \ldots, 24,$$

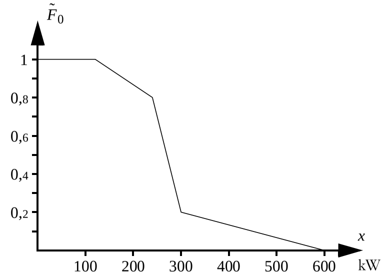
$$l_{i,t} \geq \frac{M_{i,t}}{(\overline{M}_i/2)} - 1, \qquad i = 1, 2, t = 1, \ldots, 24,$$

$$0 \leq Q_{i,j,h,t}, \qquad i = 1, 2, j = 1, 2, h = 1, 2, t = 1, \ldots, 24,$$

$$l_{i,t} \in \{0, 1\}, \qquad i = 1, 2, t = 1, \ldots, 24.$$

Problem 9

a) The peak load is 600 kW for the tariff 1 ¤/kWh. The load duration curve will therefore be as in the figure below:



Since the hydro power plant is assumed to have 100% availability we get that $\tilde{F}_1(x) = \tilde{F}_0(x)$. Thus, we can calculate the expected generation per year in the hydro power plant and the diesel generator set respectively:

$$EG_1 = 8\,760 \cdot 1 \int_0^{400} \tilde{F}_0(x)dx = 8\,760(1 \cdot 120 + (1 + 0.8)/2 \cdot 120 + (0.8 + 0.2)/2 \cdot 60$$

$$+ (0.2 + 0.133)/2 \cdot 200) \approx 2.41 \text{ GWh/year.}$$

$$EG_2 = 8\,760 \cdot 0{,}8 \int_{400}^{600} \tilde{F}_1(x)dx = 8\,760 \cdot 0.8 \cdot 0.133/2 \cdot 100) \approx 0.09 \text{ GWh/year.}$$

The income of sold electricity will therefore be $(EG_1 + EG_2) \cdot 1$ ¤/kWh ≈ 2.50 M¤/year. The total cost of the system will be 1.8 (investment cost) + 0.5 (salaries & maintenance) + 0.03 (operation cost in the diesel generator set) = 2.33 M¤/year. Hence, the system will provide a small surplus for ENESCO.

b) The risk of power deficit at the tariff 1 ¤/kWh is given by

$$LOLP = \tilde{F}_2(600) = 0.8\tilde{F}_0(600) + 0.2\tilde{F}_0(400) = 0.8 \cdot 0 + 0.2 \cdot 0.133 \approx 2.67\%.$$

If the tariff is set slightly higher, the peak load will decrease which results in a lower *LOLP*. For tariffs higher than 1 ¤/kWh it holds that $LOLP = 0.2\tilde{F}_0(400)$ which means that the tariff must be set so that $\tilde{F}_0(400) = 0.1$ if *LOLP* should be equal to 0.02. We can se in the relative load duration curve that the probability of the relative load exceeding 75% of the peak load equals 10%. Therefore, the peak load must be 400/0,75 ≈ 533 kW, which is obtained if the tariff is set to $\lambda = (900 - 533)/300 \approx 1.22$ ¤/kWh.

c) The properties of the scenarios can be predicted by comparing the available generation capacity and the total load. Therefore, we create a strata tree with available generation capacity below the root and total load at the lowest level, as shown in the figure below: