KTH Electrical Engineering

Exam in EG2050 System Planning,
18 March 2010, 14:00–19:00, E31, E35, E36, E51-53

Allowed aids

In this exam in you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten**, **single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

PART I (MANDATORY)

Write all answers on the answer sheet provided. Motivations and calculations do not have to be presented.

Part I can yield 40 points in total. The examinee is guaranteed to pass if the score is at least 33 points. If the result in part I is at least 31 points, then there will be a possibility to complement for passing the exam with the grade E.

Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

a) (2 p) The consumers in a bilateral electricity market has the following choices: I) They can choose which system operator they want, II) They can choose which retailer they want, III) They can choose which player should manage their balance responsibility.

1. Only I is true.
2. Only III is true.
3. I and II are true but not III.
4. II and III are true but not I.
5. All the statements are true.

b) (2 p) Consider a balance responsible company which during one hour has generated 100 MWh, purchased 20 MWh from the power exchange, sold 120 MWh to consumers with take-and-pay contracts and sold 10 MWh regulation power to the system operator. What obligation does this player have in the post trading for this hour?

1. The company must buy 10 MWh balance power from the system operator.
2. The company must neither sell nor buy balance power.
3. The company must sell 10 MWh balance power to the system operator.
4. The company must sell 20 MWh balance power to the system operator.
5. None of the alternatives above is correct.

Problem 2 (6 p)

Assume that the electricity market in Land has perfect competition, perfect information and that there are neither capacity, transmission nor reservoir limitations. Data for the power plants in Land are shown in table 1. The variable operation costs are assumed to be linear within the intervals, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

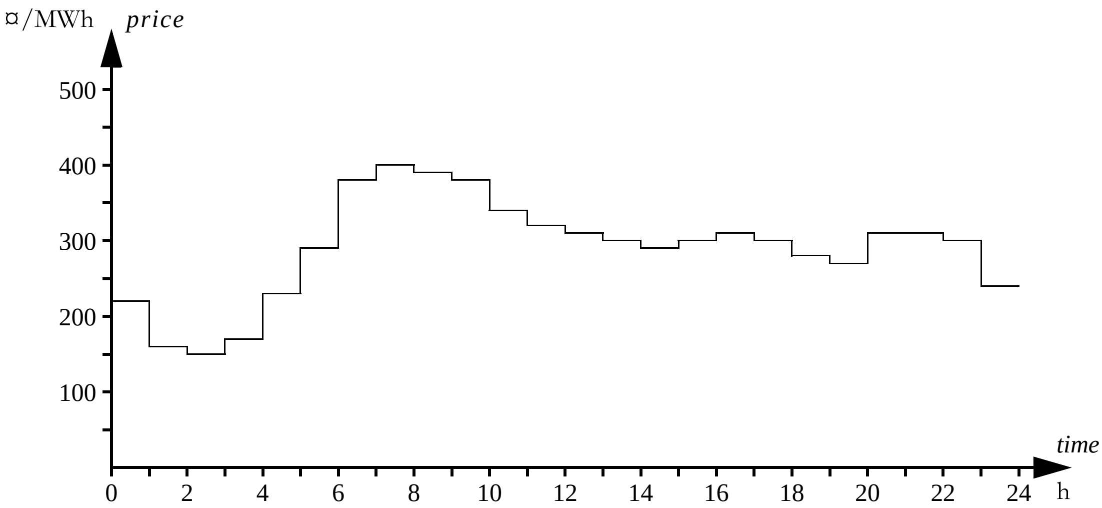
Table 1 Data for the electricity producers in Land.

Power source	Production capability [TWh/year]	Variable costs [¤/MWh]
Hydro power	60	5
Nuclear power	50	90–100
Biofuel	20	100–400
Fossil gas	10	200–300
Import from neighbouring countries	8	300–500

a) (2 p) How large is the electricity consumption in Land if the electricity price during a certain year is 325 ¤/MWh?

b) (1 p) Assume that fossil gas power plants with a generation capacity of 5 TWh are shut down and replaced by 5 TWh wind power. The variable generation cost in the wind power plants can be assumed to be 5 ¤/MWh. What will the electricity price be in Land if the conditions otherwise are the same as in part a?

c) (3 p) The figure below shows the electricity price in a certain electricity market during one day. Assume that there is perfect competition in this electricity market, that all players have access to perfect information, and that there are neither transmission nor reservoir limitations. How much will be generated during this day in a power plant with the variable operation cost 350 ¤/MWh and the installed capacity 150 MW?



Problem 3 (6 p)

Consider a power system divided in two areas, A and B. There is only one transmission line between these two areas. This line is a 220 kV AC line with a maximal capacity of 550 MW. The line is equipped with a protection system which after a short time delay disconnects the line if the maximal capacity is exceeded.

a) (2 p) The power plants in area A generate 4 200 MW when the frequency is 49.91 Hz and 3 900 MW when the frequency is 50.01 Hz. How large is the gain in area A?

b) (2 p) How much power is transferred on the transmission line when the frequency in the system is 49,96 Hz if the load in area A is 3 600 MW?

c) (2 p) At the occasion described in part b a 200 MW power plant in area B is started. The gain in area B is 2 000 MW/Hz. How large is the transmission from area A to area B then the primary control has stabilised the frequency in the system? (Answer 0 MW if the connection is disconnected due to overloading.)

Problem 4 (12 p)

AB Elkraft owns the hydro power plant Fjärd, the bio-fuelled power plant Flisinge and the wind farm Fjället. All electricity generated by the company is sold to the local power exchange ElKräng. The following symbols have been introduced in a short-term planning problem for the power plants of the company:

β = variable costs in Flisinge,
C^+ = start-up cost in Flisinge,
γ = expected future production equivalent for water stored in the reservoir of Fjärd,
G_t = electricity generation in Flisinge, hour t, t = 1, …, 24,
λ_t = expected electricity price at ElKräng hour t, t = 1, …, 24,
λ_{25} = expected electricity price at ElKräng after the end of the planning period,
M_0 = contents of the reservoir of Fjärd at the beginning of the planning period,
M_t = contents of the reservoir of Fjärd at the end of hour t, t = 1, …, 24,
μ_j = marginal production equivalent in Fjärd, segment j, j = 1, …, 4,
$Q_{j,t}$ = discharge in Fjärd, segment j, hour t, j = 1, …, 4, t = 1, …, 24,
s_t^+ = start-up variable for Flisinge, hour t, t = 1, …, 24,
s_t^- = stop variable for Flisinge hour t, t = 1, …, 24,
u_0 = unit commitment of Flisinge at the beginning of the planning period,
u_t = unit commitment of Flisinge hour, t = 1, …, 24,
V_t = local inflow to the reservoir of Fjärd hour t, t = 1, …, 24,
W_t = expected generation of the wind farm in hour t, t = 1, …, 24.

a) (2 p) The following symbols in the planning problem of AB Elkraft denote optimisation variables: I) M_0, II) u_t, III) W_t.

1. None of the statements are true.
2. Only I is true.
3. Only II is true.
4. I and II are true but not III.
5. I and III are true but not II.

b) (6 p) Formulate the objective function if the aim of the planning is to maximise the income of sold electricity plus the value of stored water minus the total operation cost (i.e., including both variable costs and start-up costs) of Flisinge. Use the symbols defined above.

c) (2 p) The production equivalent in Fjärd is 0.4 MWh/HE when the power plant is generating 250 MW. How many m^3 water will be discharged between 14:00 and 14:30 from the reservoir of Fjärd if the power plant is constantly generating 250 MW during this period?

d) (2 p) Assume that it has been decided that Flisinge should not be committed for a shorter time than four hours, i.e., if the power plant is started 12:00 then it may not be shut down again before 16:00. How should the constraint describing the relation between s_t^+, s_{t+1}^-, s_{t+2}^- and s_{t+3}^- be formulated?

1. $s_t^+ - s_{t+1}^- - s_{t+2}^- - s_{t+3}^- = 0.$
2. $s_t^+ - s_{t+1}^- - s_{t+2}^- - s_{t+3}^- \geq 0.$
3. $s_t^+ - s_{t+1}^- - s_{t+2}^- - s_{t+3}^- = 1.$
4. $s_t^+ + s_{t+1}^- + s_{t+2}^- + s_{t+3}^- = 1.$
5. $s_t^+ + s_{t+1}^- + s_{t+2}^- + s_{t+3}^- \leq 1.$

Problem 5 (12 p)

Akabuga is a small town in Eastern Africa. The town is not connected to a national grid, but has a local system of its own. The local grid is supplied by a hydro power plant and two diesel generator sets. The hydro power plant is a run-of-the-river station and it has 400 kW capacity and the risk of failure is negligible. The natural flow in the river passing by the power plant is always sufficient to generate the installed capacity. The diesel generator sets have an installed capacity of 200 kW each, the availability is 80% and the operation cost is 10 ¤/kWh. The figures on the next page show the equivalent load duration curves when adding the three power plants. The figures also indicate the area below the duration curve for different intervals.

a) (2 p) Use probabilistic production cost simulation to calculate the expected unserved energy per hour in Akabuga.

b) (2 p) Use probabilistic production cost simulation to calculate the *total* expected generation energy per hour in the two diesel generator sets.

c) (1 p) Use probabilistic production cost simulation to calculate the expected operation cost per hour in Akabuga.

d) (1 p) How large is the risk of power deficit in Akabuga approximately?

1. The risk of power deficit is larger than 20%.
2. The risk of power deficit is less than 20% but larger than 5%.
3. The risk of power deficit is less than 5%.

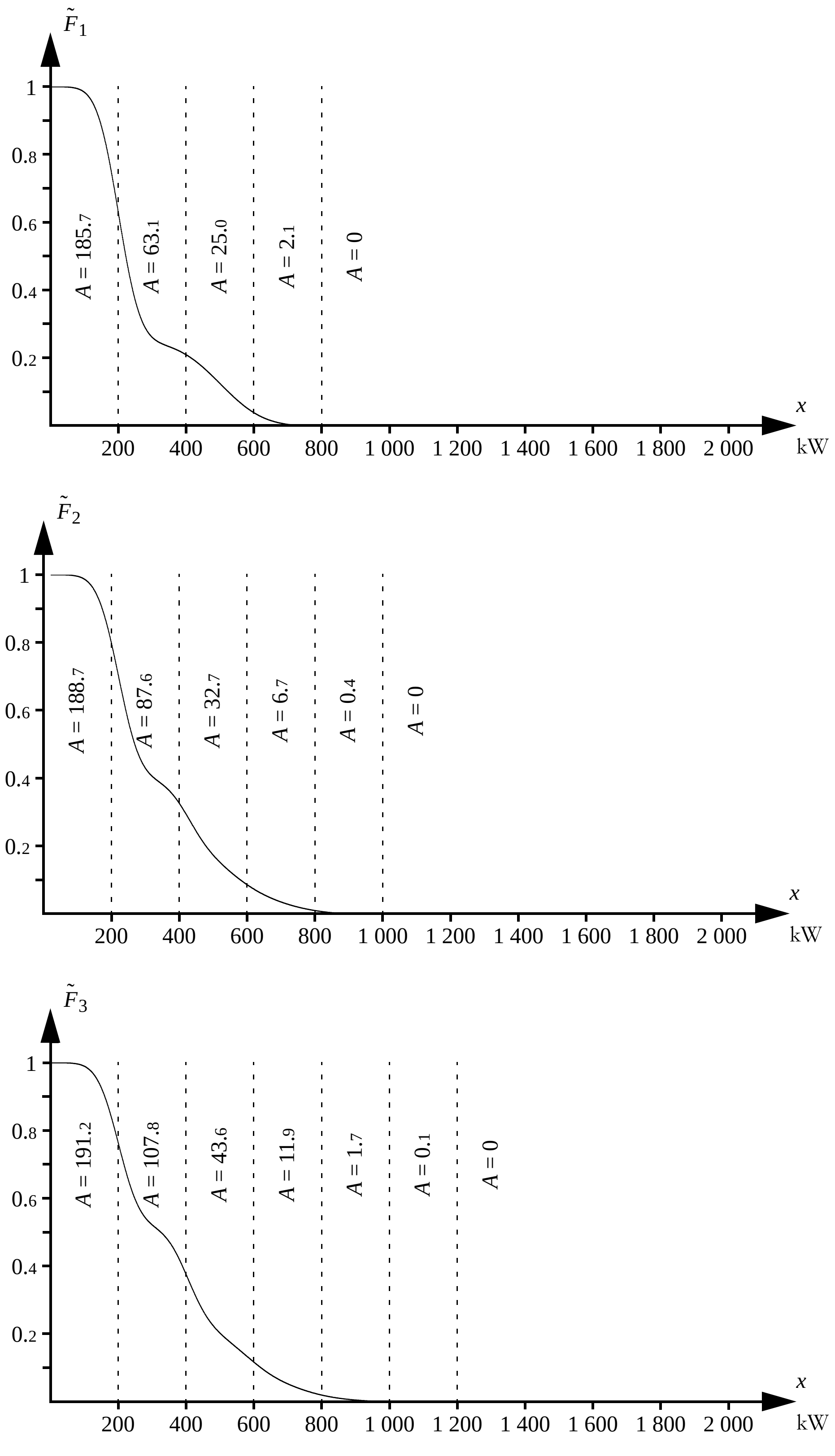
e) (2 p) Random values of the load must be generated for a Monte Carlo simulation of Akabuga. Assume that the random number U is in the interval 0.4 to 0.6. This value is transformed to a value of the load, D, between 200 and 240 kW. In which interval will the complementary random number, D^*, be found?

1. Less than 200 kW.
2. Between 200 and 240 kW.
3. Between 240 and 400 kW.
4. Larger than 400 kW.
5. Some other interval than above.

Table 2 Results from a Monte Carlo simulation of the power system in Akabuga.

Detailed model		Simplified model	
Total operation cost in the original scenarios, $\sum_{i=1}^{1000} g(y_i)$ [¤/h]	Total operation cost in the complementary scenarios, $\sum_{i=1}^{1000} g(y_i^*)$ [¤/h]	Total operation cost in the original scenarios, $\sum_{i=1}^{1000} \tilde{g}(y_i)$ [¤/h]	Total operation cost in the complementary scenarios, $\sum_{i=1}^{1000} \tilde{g}(y_i^*)$ [¤/h]
248 500	259 000	233 000	239 500

f) (4 p) Assume that the power system in Akabuga is simulated using a combination of complementary random numbers and control variates. The simulation comprises 1 000 original scenarios,



y_i, $i = 1, \ldots, 1\,000$. The corresponding complementary scenarios, y_i^*, $i = 1, \ldots, 1\,000$, have also been generated. The simplified model $\tilde{g}(Y)$, corresponds to the model used in probabilistic production cost simulation, whereas the detailed model, $g(Y)$, considers factors such as the losses being dependent on which power plants that are operated and that the load is varying in different parts of the system. The results are shown in table 2. Which estimate of *ETOC* is obtained for the detailed model?

PART II (FOR HIGHER GRADES)

All introduced symbols must be defined. Solutions should include sufficient detail that the argument and calculations can be easily followed.

The answer to each problem must begin on a new sheet, but answers to different parts of the same problem (a, b, c, etc.) can be written on the same sheet. The fields *Namn* (Name), *Blad nr* (Sheet number) and *Uppgift nr* (Problem number) must be filled out on every sheet.

Part II gives a total of 60 points, but this part will only be marked if the candidate has obtained at least 33 points in part I. Then the results of parts I and II and the bonus points will be added together to determine the examination grade (A, B, C, D, E).

Problem 6 (10 p)

The electricity market in Rike is dominated by three large companies. Moreover, there are several municipal electricity boards. The production capacity during a normal year and the variable costs are given in table 3 below. The variable operation costs are assumed to be linear within the intervals, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level. The fixed costs are given in table 4. The electricity consumption is not price sensitive and amounts to 20 TWh/year.

Table 3 Production capacity during a normal year and variable costs in Rike.

Power source	Production capacity [TWh/year]				Variable costs [¤/MWh]
	AB Vattenkraft	Elektrum AB	Strålinge kraft AB	Various municipal electricity boards	
Hydro	6	2.5		2	5
Nuclear			7		100
Coal condensing		4		4	315–555
Gas turbines	2	2		2.5	800–1 000

Table 4 Fixed costs of the power plants in Rike [M¤/year].

Power source	AB Vattenkraft	Elektrum AB	Strålinge kraft AB	Various municipal electricity boards
Hydro	200	80		60
Nuclear			1 800	
Coal condensing		400		500
Gas turbines	10	10		15

a) (6 p) The nuclear power plant Strålinge is during a normal year closed two months for maintenance works. However, this year the work has been delayed and the power plant has been closed for four months. This means that the annual generation capacity is decreased by 1.5 TWh compared to table 3. Moreover, the fixed costs of the power plant is increased by 100 M¤. Assume that the electricity market has perfect competition, perfect information and that there are neither capacity, transmission nor reservoir limitations. How are the profit of Strålinge kraft AB affected by the delayed maintenance works?

b) (4 p) Assume that AB Vattenkraft owns 50% of Strålinge kraft AB. Would it be profitable for AB Vattenkraft to deliberately delay the maintenance works in Strålinge nuclear power plant?

Problem 7 (10 p)

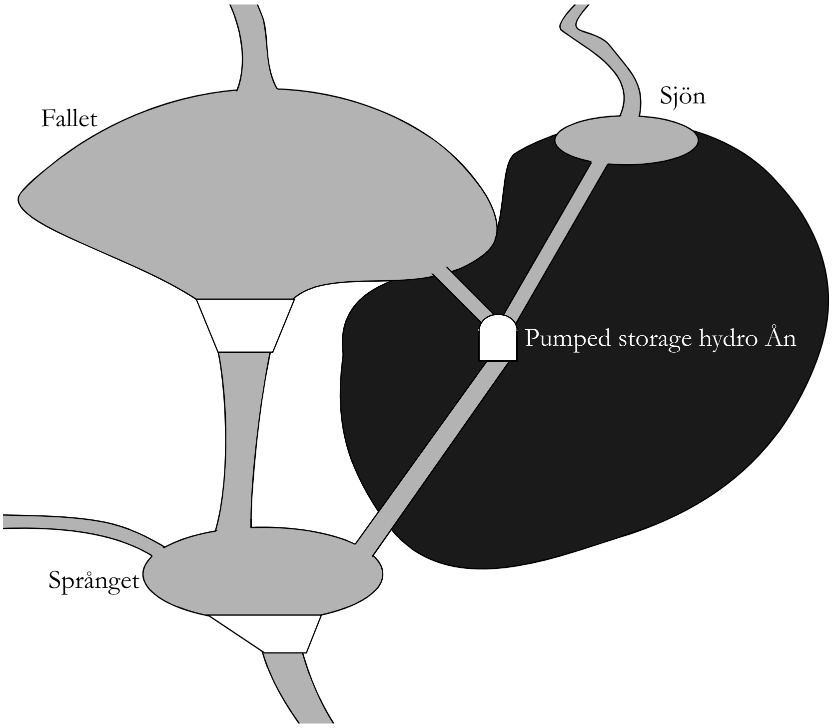
The power system in Rike is divided in two price areas. There is a lot of hydro power in the northern part of the system, but most of the load is located in the southern part. There are several parallel AC transmission lines between the two areas. The maximal flow from north to south is 1 000 MW—if this limit is exceeded, the power system becomes unstable and there is a risk that there will be extensive blackouts in the entire or parts of the power system. Out of these 1 000 MW, the market has been given access to 920 MW, i.e., the maximal planned flow from north to south (or vice versa) when the frequency is 50.0 Hz is 920 MW.

Riksnät is the system operator in Rike. Since Riksnät does not own any power plants of its own, they are forced to buy primary control capacity. Table 5 shows the bid for gain that has been submitted to Riksnät. The requirements on primary control is that at nominal frequency the primary control should be able to manage a change of 100 MW (out of which 90 MW are located in the southern part of the system and 10 MW in northern Rike) without resulting in a frequency outside the interval 50±0.1 Hz and without overloading the interconnection between the two price areas. Which bids should Riksnät accept if they want to minimise the costs of the primary control capacity?

Table 5 Bid s for gain in Rike.

Bid	Gain [MW/Hz]	Frequency range [Hz]	Price area	Price [¤/((MW/Hz) · year)]
1	250	49.9–50.1	North	6 000
2	250	49.9–50.1	North	6 300
3	200	49.9–50.1	North	6 500
4	100	49.9–50.1	North	6 800
5	100	49.9–50.1	North	6 900
6	100	49.9–50.1	North	7 000
7	100	49.9–50.1	South	7 500
8	100	49.9–50.1	North	7 500
9	100	49.9–50.1	South	8 500
10	100	49.9–50.1	South	10 000

Problem 8 (20 p)



AB Vattenkraft owns the two hydro power plants Ån and Språnget located as in the figure above. Ån is a pumped storage hydro with two reservoirs, Fallet and Sjön. This power plant can either be used for electricity generation or to pump water from Fallet to Sjön. Notice that it is not possible to generate electricity at the same time as water is pumped, which means that for every hour it must be decided whether the hydro power plant is to be used for generation or pumping. When generating electricity in the turbines of Ån, it is possible to use water from the reservoir Fallet or from Sjön or a combination of both. The maximal generation in Ån may however not exceed the installed capacity, which is 76 MW. The maximal pumping is 48 HE and the electricity consumption is 1.25 MWh/HE.

Other data for the hydro power plants are given in table 6. The company has a firm power contract of 100 MWh/h with AB Elleverantören. To deliver this quantity, AB Vattenkraft is using their own hydro power plants, but the company has also the possibility to trade at the local power exchange ElKräng. It is assumed that the company can buy and sell unlimited amounts of electricity for the prices stated in table 7. After that, the future electricity price is estimated to 750 ¤/MWh. The water delay time between the power plants can be neglected.

a) (2 p) How should simultaneous generation and pumping in Ån be prevented in the optimal solution? Do not forget to motivate your answer!

b) (18 p) Formulate the planning problem of AB Vattenkraft as a linear optimisation problem. Use the notation in table 8 for the parameters (it is however permitted to add further symbols if you consider it necessary).

NOTICE! The following is required to get full score for this problem:

- The symbols for the optimisation variables must be clearly defined.
- The optimisation problem should be formulated so that it is easy to determine what the objective function is, which constraints there are and which limits there are.
- The possible values for all indices should be clearly stated for each equation.

Table 6 Data for the hydro power plants of AB Vattenkraft.

Reservoir	Start contents of reservoir [HE]	Maximal contents of reservoir [HE]	Production equivalent [MWh/HE]	Maximal discharge [HE]	Maximal discharge capacity of spillways [HE]	Local inflow [HE]
Fallet	6 000	13 900	0.2	140	430	25
Sjön	0	1 100	1.0	60	0	0.1
Språnget	14 000	27 500	0.3	240	250	1

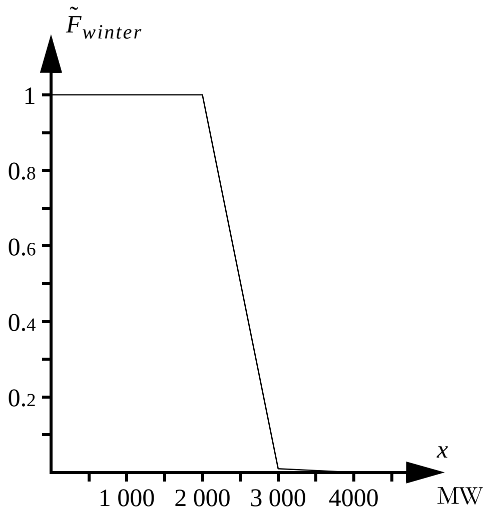
Table 7 Expected prices at ElKräng.

Hour	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Price at ElKräng [¤/MWh]	690	610	630	680	710	750	810	930	1 120	1 000	930	900
Hour	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24
Price at ElKräng [¤/MWh]	820	790	750	720	750	810	900	790	700	680	670	650

Table 8 Notation for the planning problem of AB Vattenkraft.

Symbol	Explanation	Value
$\overline{H}$	Maximal generation in pumped storage hydro Ån	76
$\overline{Q}_P$	Maximal pumping	48
γ_P	Electricity consumption for pumping	1.25
D	Contracted load	100
λ_f	Expected future electricity price	750
$M_{i,0}$	Start contents of reservoir i	See table 6
$\overline{M}_i$	Maximal contents of reservoir i	See table 6
γ_i	Production equivalent for discharge from reservoir i	See table 6
$\overline{Q}_i$	Maximal discharge from reservoir i	See table 6
$\overline{S}_i$	Maximal spillage from reservoir i	See table 6
V_i	Local inflow to reservoir i	See table 6
λ_t	Expected price at ElKräng hour t	See table 7

Problem 9 (20 p)



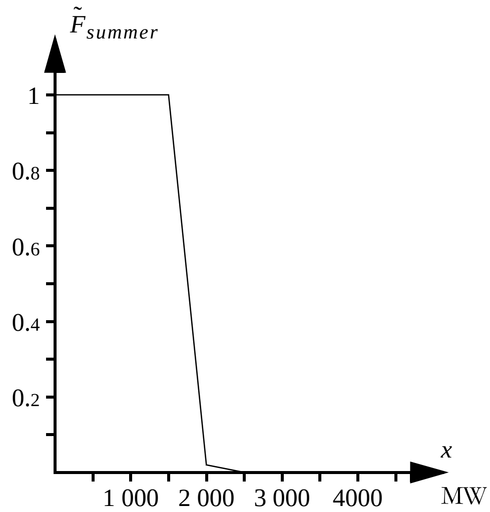


Table 9 Some values of the load duration curves of Rike.

x	1 500	2 000	2 500	3 000	3 500	4 000
$\tilde{F}_{winter}(x)$	1	1	0.506	0.012	0.006	0
$\tilde{F}_{summer}(x)$	1	0.024	0	0	0	0

The electricity generation in Rike is supplied by a number of hydro power plants (total installed capacity 1 200 MW), the nuclear power plant Strålinge of 1 000 MW, the coal condensing power plant Röksta of in total 1 000 MW as well as a number of gas turbines with a total installed capacity of 800 MW. The hydro power plant, Strålinge and the gas turbines are assumed to have 100% availability, whereas Röksta has two similar blocks, each having the installed capacity 500 MW and an availability of 95%. The load in Rike is different during winter time and summer time, as in the duration curves above. Eight months a year are considered winter time and the remaining four months are considered summer time. As mention in problem 6, the nuclear power plant Strålinge is normally closed during two months every year. The maintenance is performed when the demand is as low as possible, i.e., during summer time.

a) (8 p) What is the risk of power deficit (seen over a year) in Rike if Strålinge is closed for two months during summer time?

b) (2 p) What is the risk of power deficit (seen over a year) in Rike if the maintenance works are delayed and the nuclear power plant is closed for four months during summer time?

c) (8 p) To consider the transmission limitation between northern and southern Rike it is desirable to calculate the risk of power deficit using a multi-area model. Assume that the maximal unused generation capacity due to transmission congestion is 500 MW. Suggest a strata tree in order to use stratified sampling to estimate the risk of power deficit in Rike when the system with normal maintenance time in Strålinge is simulated. You should also compute the stratum weight with an accuracy of five decimals.

d) (2 p) Can the same strata tree as in part c be used to simulate the system when Strålinge is closed during the entire summer period? If not, what has to be changed in the strata tree? Do not forget to motivate your answer!

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Answer sheet for part I

Name: ....

Personal number: ....

Problem 1

a) Alternative .... is correct.

b) Alternative .... is correct.

Problem 2

a) .... TWh/year **b)** .... ¤/MWh

c) .... MWh

Problem 3

a) .... MW/Hz **b)** .... MW

c) .... MW

Problem 4

a) Alternative .... is correct.

b) ....

....

c) .... m^3

d) Alternative .... is correct.

Problem 5

a) .... kWh/h **b)** .... kWh/h

c) .... ¤/h

d) Alternative .... is correct.

e) Alternative .... is correct.

f) .... ¤/h

Problem 1

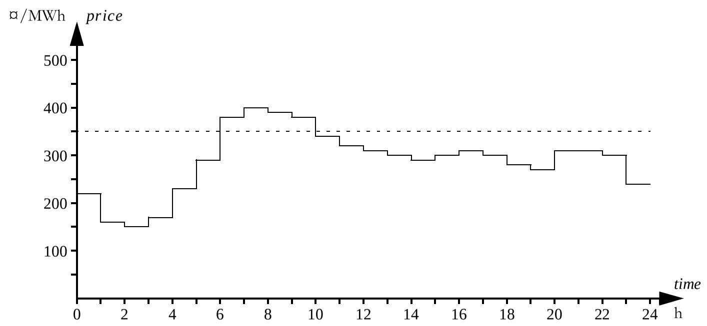
b) 4, **b)** 1.

Problem 2

a) Both hydro, nuclear and fossil gas are generating as much as possible, i.e., 60 + 50 + 10 = 120 TWh, when the electricity price is 325 ¤/MWh. The contribution from bio fuels is equal to (325 – 100)/(400 – 100) · 20 = 15 TWh and the contribution from import is (325 – 200)/(500 – 300) · 8 = 1 TWh. All in all, 136 TWh is generated in a year and the electricity consumption is of course equally large.

b) The fossil gas power plants are fully utilised, but as the shut down units are replaced by the same amount of wind power generation (which has a lower variable cost than the fossil gas power plants and therefore also will be fully utilised), the same units as before (i.e., bio fuel and import) will set the price. Consequently, the electricity price will remain 325 ¤/MWh.

c) The power plant will generate its installed capacity during those hours when the electricity price is higher than 350 ¤/MWh. Drawing a line at the level 350 ¤/MWh shows that the power plant will generate 150 MW during 4 hours, which results in a total generation of 600 MWh.



Problem 3

a) $R = \Delta G/\Delta f$ = 300/0.1 = 3 000 MW/Hz.

b) The generation in area A can be determined by comparing to for example the generation at the frequency 50,01 Hz: $\Delta G = R \cdot \Delta f$ = 3 000 · 0.05 = 150. Thus, the generation at 49.96 Hz is 3 900 + 150 = 4 050 MW. As only 3 600 MW is consumed within area A, the export to area B must be 450 MW.

c) When the system is supplied another 200 MW, the primary control must reduce the generation by p 200 MW. As area B has 40% of the gain in the system, 40% (i.e., 80 MW) of the down regulation will be in area B. The generation in area A is decreased by 120 MW (down to 3 930 MW) and hence the export to area B must decrease by 120 MW, which means that the export will now be 330 MW.

Problem 4

a) 3.

b) maximise $\sum_{t=1}^{24} \lambda_t \Big(G_t + \sum_{j=1}^{4} \mu_j Q_{j,t} \Big) + \lambda_f \gamma M_{24} - \sum_{t=1}^{24} (C^+ s_t^+ + \beta G_t)$.

c) The power plant generates 250 MWh at a production equivalent of 0.4 MWh/HE, which means that a discharge of 250/0.4 = 625 HE is necessary. This corresponds to 625 · 1 800 = 1 125 000 m^3 during a half hour.

d) 5.

Problem 5

a) The expected energy not served is given by

$$EENS_3 = \int_{800}^{\infty} \tilde{F}_3(x)dx = 1.7 + 0.1 = 1.8 \text{ kWh/h}.$$

b) The total generation is given by

$$EG_{23} = EENS_1 - EENS_3 = \int_{400}^{\infty} \tilde{F}_1(x)dx - \int_{800}^{\infty} \tilde{F}_3(x)dx = 25 + 2.1 - 1.8 = 25.3 \text{ kWh/h}.$$

c) The expected total operation cost is given by

$$ETOC = 10EG_{23} = 10 \cdot 25.3 = 253 \text{ ¤/h}.$$

d) 3.

e) 2.

f) In practice there is no need to differentiate between observations based on the original scenarios and the complementary scenarios; the expected difference between the detailed and simplified models is given by

$$m_{(TOC - \tilde{TOC})} = \frac{1}{2000}\Big(\sum_{i=1}^{1000} (g(y_i) - \tilde{g}(y_i)) + \sum_{i=1}^{1000} (g(y_i^*) - \tilde{g}(y_i^*)) \Big) = 17.5 \text{ ¤/h}.$$

The expectation value of the simplified model (which corresponds to a PPC model) was calculated to 253 ¤/h in part c. The estimate of *ETOC* for the detailed model is thus

$$m_{TOC} = m_{(TOC - \tilde{TOC})} + \mu_{\tilde{TOC}} = 17.5 + 253 = 270.5 \text{ ¤/h}.$$

Problem 6

a) Hydro power and nuclear power can generate 17.5 TWh during a normal year. The remaining a 2.5 TWh must come from coal condensing. This means that 31.25% of the coal condensing capacity is used; hence, 31,25% of the price interval for coal condensing is used. The electricity price must then be 390 ¤/MWh. The profit of Strålinge kraftverk AB is therefore 390·7 (income of sold electricity) – 100·7 (variable costs) – 1 800 (fixed costs) = 230 M¤/year.

The longer maintenance period reduces the combined capacity of hydro and nuclear to 16 TWh. Thus, half of the coal condensing is needed and the electricity prise must be 435 ¤/MWh. The profit of Strålinge is then 435·5.5 (income of sold electricity) – 100·5.5 (variable costs) – 1 800 (fixed costs) – 100 (extra maintenance cost) = –57.5 M¤/year. Although the electricity price is increasing, the company is nevertheless losing 287.5 M¤ due to the delayed maintenance work.

b) The profit of AB Vattenkraft during a normal year is 390·6 (income of sold electricity) – 5·6 (variable costs) – 210 (fixed costs) + 0.5·230 (share of profit of Strålinge kraft AB) = 2 215 M¤/year. When the maintenance works are delayed, the ´profits is changed to 435·6 (income of sold electricity) – 5·6 (variable costs) – 210 (fixed costs) + 0.5·(–57.5) (share of loss of Strålinge kraft AB) = 2 341.25 M¤/year. Hence, it would be profitable for AB Vattenkraft to delay the maintenance of Strålinge. (How this should be done without the other partners or the authorities reacting is another question.)

Problem 7

To manage a change of 100 MW without the frequency changing more than 0.1 Hz requires a gain of 100/0.1 = 1 000 MW/Hz. The least expensive alternative would thus be to accept bids 1–6, but as all the gain would be in the northern part of the system, a change of 90 MW in south and 10 MW in north would cause the transmission flow between the areas to increase by 90 MW. However, in worst case there is only an 80 MW margin; therefore, 10 MW of the primary control reserve (i.e., a gain of 100 MW/Hz) must be in the southern part of Rike. Hence, Riksnät should accept 900 MW/Hz of the least expensive bids in northern Rike (bids 1–5) as well as the least expensive bid in southern Rike (bid 7).

Problem 8

a) A binary variable, which for example is equal to 1 when pumping and 0 otherwise, can be used in order to be guarantee that there is no discharge from neither Fallet nor Sjön during pumping hours. However, in practice it is not reasonable that the optimal solution should include simultaneous pumping and discharge; discharge is done when the electricity price is high and pumping when the price is low. It is therefore most likely that an optimal discharge plan is obtained also without using integer variables. Avoiding integer variables is normally desirable, but in this case it is probable that a MILP solver will find optimal values of the integer variables without having to build a large search tree. Therefore, from a computational efficiency it does not matter which problem formulation that is used.

b) In the solution below, we have chosen to use an integer variable to determine whether Ån is used for generation or pumping. The problem we want to solve is then

maximise *value of stored water + value of sales – cost of purchase*,

subject to *hydrological balance of the reservoirs*,

generation limitation in Ån,

load balance,

limitations in reservoir contents, discharge and spillage.

Indices of reservoirs

Fallet 1, Sjön 2, Språnget 3.

Parameters

The parameters are defined in table 8 of the problem text.

Optimisation variables

$Q_{i,t}$ = discharge in power plant i during hour t, i = 1, 2, 3, t = 1, …, 24,
Q_{Pt} = pumping from Fallet to Sjön during hour t, t = 1, …, 24,
$S_{i,t}$ = spillage from reservoir i during hour t, i = 1, 3, t = 1, …, 24,
$M_{i,t}$ = contents of reservoir i at the end of hour t, i = 1, 2, 3, t = 1, …, 24,
u_{Pt} = status of Ån during hour t (1 if pumping, 0 otherwise), t = 1, …, 24,
p_t = purchase from ElKräng during hour t, t = 1, …, 24,
r_t = sales to ElKräng during hour t, t = 1, …, 24.

Objective function

$$\text{maximise} \quad \lambda_f((\gamma_1+\gamma_3)M_{1,24}+(\gamma_2+\gamma_3)M_{2,24}+\gamma_3 M_{3,24})+\sum_{t=1}^{24}\lambda_t(r_t-p_t).$$

Constraints

Hydrological balance for Fallet:

$$M_{1,t}=M_{1,t-1}-Q_{1,t}-S_{1,t}-Q_{Pt}+V_{1,t}, \qquad t = 1, \dots 24.$$

Hydrological balance for Sjön:

$$M_{2,t}=M_{2,t-1}-Q_{2,t}-S_{2,t}+Q_{Pt}+V_{2,t}, \qquad t = 1, \dots 24.$$

Hydrological balance for Språnget:

$$M_{3,t}=M_{3,t-1}-Q_{3,t}-S_{3,t}+Q_{1t}+Q_{2,t}+S_{1,t}+V_{2,t}, \qquad t = 1, \dots 24.$$

Relation between discharge and pumping in Fallet and Sjön:

$$Q_{i,t}\le(1-u_{Pt})\bar{Q}_i, \qquad i = 1, 2, t = 1, \dots, 24.$$

$$Q_{Pt}\le u_{Pt}\bar{Q}_P, \qquad t = 1, \dots, 24.$$

Maximal generation in Ån:

$$\sum_{i=1}^{2}\gamma_i Q_{i,t}\le\bar{H}, \qquad t = 1, \dots, 24.$$

Load balance:

$$\sum_{i=1}^{3}\gamma_i Q_{i,t}+p_t=D_t+r_t+\gamma_P Q_{Pt}, \qquad t = 1, \dots, 24.$$

Variable limits

$0 \le Q_{i,t} \le \overline{Q}_i$, $i = 1, 2, 3, t = 1, \ldots, 24$,

$0 \le Q_{Pt} \le \overline{Q}_P$, $t = 1, \ldots, 24$,

$0 \le S_{i,t} \le \overline{S}_i$, $i = 1, 3, t = 1, \ldots, 24$,

$0 \le M_{i,t} \le \overline{M}_i$, $i = 1, 2, 3, t = 1, \ldots, 24$,

$u_{Pt} \in \{0, 1\}$, $t = 1, \ldots, 24$,

$0 \le p_t$, $t = 1, \ldots, 24$,

$0 \le r_t$, $t = 1, \ldots, 24$.

Problem 9

a) Since we are only evaluating the risk of power deficit, the merit order of the power plants does not matter. For the sake of simplicity, consider hydro power, nuclear power and gas turbines as one 3 000 MW power plant, which is added first into the equivalent load duration curve.

$$\tilde{F}_1(x) = \tilde{F}_0(x).$$

Then, the two blocks of Röksta are added:

$$\tilde{F}_2(x) = 0.95\tilde{F}_1(x) + 0.05\tilde{F}_1(x - 500),$$
$$\tilde{F}_3(x) = 0.95\tilde{F}_2(x) + 0.05\tilde{F}_2(x - 500).$$

During winter time, the risk of power deficit is

$$LOLP_W = \tilde{F}_3(4\,000) = 0.95\tilde{F}_2(4\,000) + 0.05\tilde{F}_2(3\,500) =$$
$$= 0.95(0.95\tilde{F}_{winter}(4\,000) + 0.05\tilde{F}_{winter}(3\,500))$$
$$+ 0.05(0.95\tilde{F}_{winter}(3\,500) + 0.05\tilde{F}_{winter}(3\,000)) =$$
$$= 0.95(0.95 \cdot 0 + 0.05 \cdot 0.006) + 0.05(0.95 \cdot 0.006 + 0.05 \cdot 0.012) = 0.06\%.$$

During the part of the summer when the nuclear power plant is operational, the available generation capacity will be at least 3 000 MW, whereas the load will not exceed 2 500 MW. Hence, during this period we get $LOLP_{S+N} = 0\%$. During the remainder of the summer period, the risk of power deficit is given by

$$LOLP_{S-N} = \tilde{F}_4(3\,000) = 0.95\tilde{F}_3(3\,000) + 0.05\tilde{F}_2(2\,500) =$$
$$= 0.95(0.95\tilde{F}_{summer}(3\,000) + 0.05\tilde{F}_{summer}(2\,500))$$
$$+ 0.05(0.95\tilde{F}_{summer}(2\,500) + 0.05\tilde{F}_{summer}(2\,000)) =$$
$$= 0.95(0.95 \cdot 0 + 0.05 \cdot 0) + 0.05(0.95 \cdot 0 + 0.05 \cdot 0.024) = 0.006\%.$$

The risk of power deficit for the entire year is obtained as a weighted sum of the above results:

$$LOLP = \frac{8}{12}LOLP_W + \frac{2}{12}LOLP_{S+N} + \frac{2}{12}LOLP_{S-N} = 0.041\%.$$

b) In this case, the risk of power deficit for the entire year is

$$LOLP = \frac{8}{12}LOLP_W + \frac{4}{12}LOLP_{S-N} = 0.042\%.$$

c) There will be a need for three levels beneath the root of the strata tree, since both the load and the available generation capacity have different probability distributions during different time periods. At the top level we have time period, then generation capacity and finally the load. The intervals of the load nodes should be chosen so that we can differentiate between scenarios where there is no doubt that the load can be covered, scenarios where load shedding might be necessary due to generation capacity being confined by the transmission limitation, and scenarios where the load exceeds the available generation capacity.

The node weights of the time period depend on the length of the time period, i.e., 8/12 for the winter period and 2/12 each for the two summer periods (with and without Strålinge). The node weights of the available generation capacity depend on the probability of having both blocks in Röksta available(0.95·0.95), one block available (0.95·0.05 + 0.05·0.95) as well as no block being available (0.05·0.05). The node weights of the load are equal to the probability that the load belongs to the stated interval, and can be computed using table 9.

The resulting strata tree and the stratum weights are shown in the table below.

Stratum	Time period	Available generation capacity [MW]	Load [MW]	Stratum weight	*LOLO*
1	Winter time	3 000 (hydro, Strålinge and gas turbines)	2 000–2 500	8/12·0.05·0.05·0.494 ≈ 0.00082	0
2			2 500–3 000	8/12·0.05·0.05·0.494 ≈ 0.00082	0/1
3			3 000–4 000	8/12·0.05·0.05·0.0012 ≈ 0.00002	1
4		3 500 (everything except one block in Röksta)	2 000–3 000	8/12·2·0.95·0.05·0.9988 ≈ 0.06257	0
5			3 000–3 500	8/12·2·0.95·0.05·0.0006 ≈ 0.00038	0/1
6			3 500–4 000	8/12·2·0.95·0.05·0.0006 ≈ 0.00038	1
7		4 000 (all)	2 000–3 500	8/12·0.95·0.95·0.994 ≈ 0.59806	0
8			3 500–4 000	8/12·0.95·0.95·0.006 ≈ 0.00361	0/1
9	Summer time, Strålinge operational	4 000 (all)	1 500–2 500	2/12·1·1 ≈ 0.16667	0
10	Summer time, Strålinge closed	2 000 (hydro and gas turbines)	1 500– 2 000	2/12·0.05·0.05·0.976 ≈ 0.00041	0/1
11			2 000–2 500	2/12·0.05·0.05·0.024 ≈ 0.00001	1
12		2 500 (hydro, one block in Röksta, gas turbines)	1 500– 2 000	2/12·2·0.95·0.05·0.976 ≈ 0.01545	0
13			2 000–2 500	2/12·2·0.95·0.05·0.024 ≈ 0.00038	0/1
14		3 000 (all except Strålinge)	1 500–2 500	2/12·0.95·0.95·1 ≈ 0.15042	0

d) No, the same strata tree cannot be used, because the node weights on the time level are changing. The node for summer time with Strålinge operational should be given the node weight 0 (i.e., stratum 7 is excluded from the strata tree) whereas the node for summer time with Strålinge closed should be given the node weight 4/12 (i.e., strata 8–12 gets twice as high stratum weights).