KTH Electrical Engineering

Exam in EG2050/2C118 System Planning, 9 June 2010, 8:00–13:00, V34, V35

Allowed aids

In this exam you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten**, **single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

PART I (MANDATORY)

Write all answers on the answer sheet provided. Motivations and calculations do not have to be presented.

Part I can yield 40 points in total. The examinee is guaranteed to pass if the score is at least 33 points. If the result in part I is at least 31 points, then there will be a possibility to complement for passing the exam with the grade E.

Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

a) (1 p) Which players have the responsibility to maintain the frequency of the power system within nominal values (for example 49.9–50.1 Hz in the Nordel system)?

1. The system operator.
2. The balance responsible players.
3. The producers.

b) (1 p) Which players are economically responsible that during a particular trading period (for example one hour) the system is supplied as much energy as the consumption?

1. The system operator.
2. The balance responsible players.
3. The consumers.

c) (2 p) We use the notion "post trading" to describe all the trading which occurs after the hour of delivery (or any other trading period). The following types of contracts are traded in the post market: I) Balance power, i.e., when a balance responsible player is selling any surplus in their balance to the system operator, or when a balance responsible player is buying from the system operator to cover for any deficit in their balance, II) Firm power, i.e., the customer buys the same amount of energy in each trading period as long as the contract is valid, III) Regulation power, i.e., when a player at request from the system operator is supplying more power to the system (up-regulation) or when a player at request from the system operator is supplying less power to the system (down-regulation).

1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. I and III are true but not II.

Problem 2 (6 p)

Assume that the electricity market in Land has perfect competition, all players have perfect information, and there are neither transmission nor capacity limitations. Data for the electricity market in Land are given in table 1 below. The variable costs are assumed to be linear in the given interval, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

On 1 January the reservoirs holds in total 25 TWh and according to the long-term forecast for the electricity market (which as already mentioned is assumed to be faultless), the reservoirs should hold 20 TWh on 31 December.

Table 1 Data for the electricity market in Land.

Power source	Production capability [TWh/year]		Variable cost [¤/MWh]
	1 January to 30 April	1 May to 31 December	
Hydro (inflow)	55	10	30–60
Nuclear	30	50	100–120
Coal condensing	10	20	350–500
Gas turbines	5	10	800–1 000
Electricity consumption [TWh]	55	110	

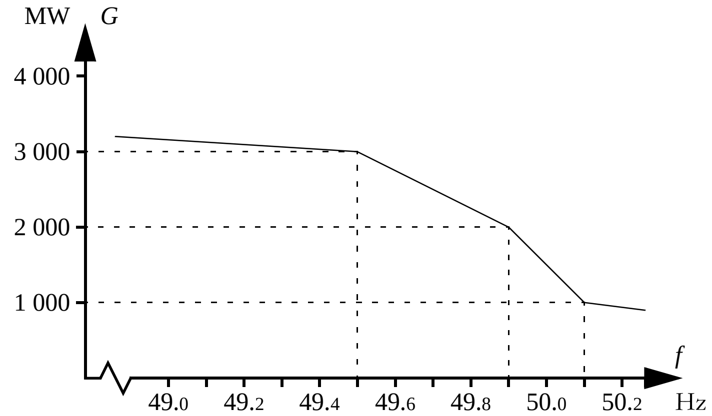
a) (2 p) What would the price be in the electricity market of Land if there is no reservoir limitation, i.e., if the reservoirs had infinite storage capacity and the entire inflow can be used during the year?

b) (2 p) How much would the reservoirs hold at midnight between 30 April and 1 May if there was no reservoir limitation, i.e., if the reservoirs had infinite storage capacity?

c) (2 p) Assume that reservoirs in Land has limited storage capacity. How large is this storage capacity if the electricity price is 395 ¤/MWh between 1 January and 30 April, and that it is 440 ¤/MWh between 1 May and 31 December?

Problem 3 (6 p)

The figure below shows the total generation in the power plants participating in the primary control as a function of the frequency in a certain power system.



a) (2 p) At 11:01 there is balance between production and consumption in the system and the frequency is 50.01 Hz. At this time the thermal power plant Sotinge is increasing its generation by 200 MW. The power plant Sotinge is not participating in the primary control. What will the frequency be when the primary control has restored the balance between generation and consumption?

b) (2 p) At 11:03 there is balance between production and consumption in the system and the frequency is 50.04 Hz. At this time the load of the system is increased by 200 MW. What will the frequency be when the primary control has restored the balance between generation and consumption?

c) (2 p) At 11:07 there is balance between production and consumption in the system and the frequency is 49.92 Hz. At this time the load of the system is increased by 200 MW. What will the frequency be when the primary control has restored the balance between generation and consumption?

Problem 4 (12 p)

Stads energi AB owns a thermal power plant with three blocks. Assume that the company has formulated their short-term planning problem as a MILP problem and that the following symbols have been introduced:

Indices for the power plants: Block I - 1, Block II - 2, Block III - 3.

β_{Gg} = variable operation cost in power plant g,
C_g^+ = start-up cost in power plant g, g = 1, 2, 3,
$G_{g,t}$ = generation in power plant g, hour t, g = 1, 2, 3, t = 1, …, 24,
$\bar{G}_g$ = installed capacity in power plant g, g = 1, 2, 3,
$\underline{G}_g$ = minimal generation when power plant g is committed, g = 1, 2, 3,
λ_t = expected electricity price at ElKräng hour t, t = 1, …, 24,
$s_{g,t}^+$ = start-up variable for power plant g, hour t, g = 1, 2, 3, t = 1, …, 24,
$u_{g,0}$ = unit commitment of power plant g at the beginning of the planning period, g = 1, 2, 3,
$u_{g,t}$ = unit commitment of power plant g, hour t, g = 1, 2, 3, t = 1, …, 24.

a) (3 p) Which of the symbols above represent optimisation variables and parameters respectively?

b) (4 p) Stads energi AB sells power to the local power exchange ElKräng.. Formulate the objective function if the aim of the planning problem is to maximise the income of sold electricity minus the costs of the thermal power plant. Use the symbols defined above.

c) (1 p) How should the relation between unit commitment and minimal generation in power plant g, hour t be formulated?

1. $G_{g,t} - \underline{G}_g \cdot u_{g,t} \leq 0$.
2. $G_{g,t} - \underline{G}_g \cdot u_{g,t} = 0$.
3. $G_{g,t} - \underline{G}_g \cdot u_{g,t} \geq 0$.

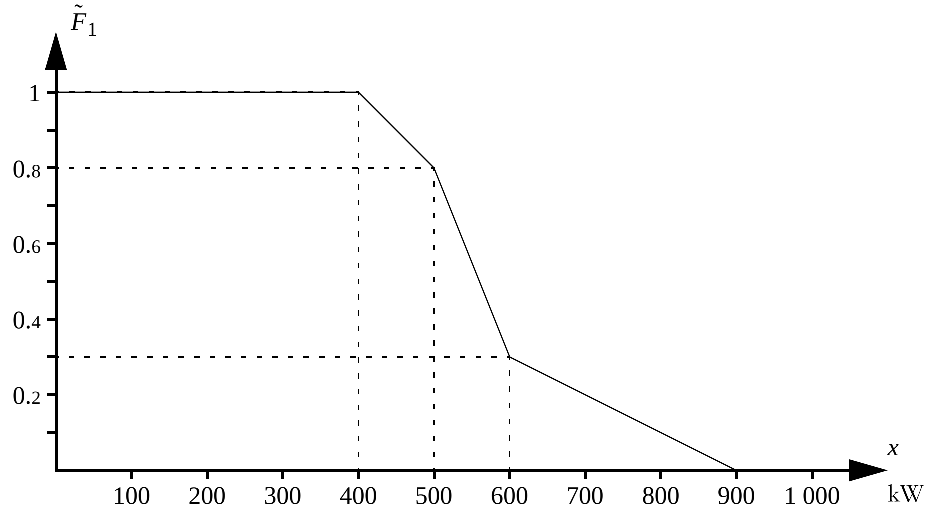
d) (2 p) Consider a hydro power plant where the maximal discharge is 100 m^3/s. At maximal discharge, the relative efficiency is 96% and the power plant generates 36 MW. The best efficiency is obtained at the discharge 68 m^3/s. How large is the maximal production equivalent in this power plant?

e) (2 p) Assume that a short-term planning problem for three hydro power plants has been formulated as an LP problem, where the objective is to maximise the value of stored water. The optimisation problem is then solved using commercial software (for example GAMS). Let $\nu_{i,t}$ denote the dual variable of the hydrological constraint for power plant i, hour t. The dual variables have reasonable values in the following situations: I) If all $\nu_{i,t} < 0$, II) If all $\nu_{i,t} \geq 0$, III) If power plant j is located directly downstream power plant i and $\nu_{j,t} > \nu_{i,t}$.

1. Only I is true.
2. Only II is true.
3. Only III is true.
4. II and III are true but not I.
5. All the statements are true.

Problem 5 (12 p)

Ekibuga is a town in East Africa. The town is not connected to a national grid, but has a local system of its own, which is supplied by a hydro power plant at nearby Ekiyira. The local grid also includes some smaller villages along the road between Ekiyira and Ekibuga. The hydro power plant does not have a reservoir, but the water flow is always sufficient to generate the installed capacity (800 kW) and the risk for outages in the power plant is negligible. The duration curve of the total load of the system is shown below.



a) (2 p) How large is the expected energy not served per hour for the system with only the hydro power plant?

b) (2 p) To improve the security of supply in the system, a diesel generator set is considered in Ekibuga. The planned diesel generator set would have 100 kW installed capacity, 75% availability and the operation cost 2 ¤/kWh. The expected energy not served when including both the hydro power plant and the diesel generator set is 1.25 kWh/h. Calculate the expected total operation cost per hour for the system with a hydro power plant and a diesel generator set.

c) (3 p) Another option to improve the security of supply is to build a wind power plant in Kasozi. The wind power plant would have 400 kW installed capacity and the available generation capacity is modelled according to table 2. What is the *LOLP* of the system with a hydro power plant and a wind power plant?

Hint: The convolution formula for a multi-state model reads

$$\tilde{F}_g(x) = \sum_{i=1}^{N_g} p_{g,i}\tilde{F}_{g-1}(x - x_{g,i}).$$

Table 2 Model of the wind power plant in Kasozi.

Available generation capacity [kW]	Probability [%]
0	15
100	40
200	20
300	15
400	10

d) (2 p) In order to take into account the relatively high losses on the lines between Ekiyira and Ekibuga, it is desirable to use a multi-area model. Assume that complementary random numbers are used to improve the simulation. What is the value of the complementary random number, D^*, if the total load of the system is randomised to D = 500 kW?

e) (2 p) Assume that the power system in Ekibuga is simulated using a combination of stratified sampling and control variates. The results of 2 000 scenarios in a simulation of the system with hydro power and diesel generator set are presented in table 3. How large is the expected difference between the multi-area model and the PPC model?

f) (1 p) Which estimate of *LOLP* is obtained from the Monte Carlo simulation in table 3 if the expected operation cost according to a probabilistic production cost simulation is 2.5%?

Table 3 Results from a Monte Carlo simulation of the power system in Ekibuga.

Stratum, h	Stratum weight, ω_h	Number of scenarios, n_h	Results from multi-area model, $\sum_{i=1}^{h} x_{i,h}$ (where $x_{i,h}$ is the observed value of *LOLO* in scenario i, stratum h)	Results from PPC model, $\sum_{i=1}^{h} z_{i,h}$ (where $z_{i,h}$ is the observed value of $LOLO_{PPC}$ in scenario i, stratum h)
1	0,90	0	0*	0*
2	0,075	1 000	300	0
3	0,025	1 000	1 000	1 000

* In these scenarios, the available generation capacity is always sufficient to cover the maximal load plus the maximal losses; hence, it is known that *LOLO* = 0 for all these scenarios.

PART II (FOR HIGHER GRADES)

All introduced symbols must be defined. Solutions should include sufficient detail that the argument and calculations can be easily followed.

The answer to each problem must begin on a new sheet, but answers to different parts of the same problem (a, b, c, etc.) can be written on the same sheet. The fields *Namn* (Name), *Blad nr* (Sheet number) and *Uppgift nr* (Problem number) must be filled out on every sheet.

Part II gives a total of 60 points, but this part will only be marked if the candidate has obtained at least 33 points in part I. Then the results of parts I and II and the bonus points will be added together to determine the examination grade (A, B, C, D, E).

Problem 6 (10 p)

Consider a simplified model of the electricity market in Rike, where it is assumed that there is perfect competition, that all players have access to perfect information and that there are neither reservoir nor capacity limitations. Data for electricity generation and consumption are given in table 4. The variable operation costs are assumed to be linear within the intervals, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level. Moreover, assume that the benefit of each consumed MWh can be valued to 1 000 ¤.

Table 4 Data for generation and consumption on the electricity market in Rike.

Power source	Production capability [TWh/year]		Variable costs [¤/MWh]
	Northern Rike	Southern Rike	
Hydro power	60	10	5
Nuclear power	0	70	80–120
Fossil fuels	8	12	300–540
Consumption	15	130	

a) (4 p) Currently the system operator Riksnät is using counter trading to manage limitations in the transmission capability between the northern and southern parts of Rike. This means that the electricity pricing is not affected by transmission limitations. Compute the following:

- The total producer surplus, i.e., the value of all electricity that is sold minus the variable generation costs.
- The total consumer surplus, i.e., the value of all electricity that is consumed minus the costs to buy electricity.

b) (4 p) Assume that the system operator Riksnät would introduce two price areas: one for northern Rike and one for southern Rike. The maximal transmission capability between the tow areas is 46 TWh/year. In this case, the transmission limitation will influence the electricity pricing. Compute the total producer and the total consumer surplus respectively.

Notice that all electricity that is supplied in the northern part of the country (i.e., including the part that is exported to the south) obtain the area price of northern Rike. Similarly, all consumers in southern Rike (i.e., including those that buy electricity imported from northern Rike) must pay the area price of southern Rike.

c) (2 p) Assume that Riksnät would save 2 325 M¤/year if price areas are introduced. Which of the alternatives is best from a society point of view? Do not forget to motivate your answer!

Problem 7 (10 p)

Consider a power system composed of the three countries Aland, Beland and Celand. The primary control in the three countries is divided in a normal operation reserve and a disturbance reserve. The normal operation reserve is designed to manage normal variations in for example load and wind power generation. The disturbance reserve is designed to manage outages in larger power plants. The distribution of the gain for these reserves is shown in table 5.

At one occasion, the frequency is 49.9 Hz and there is balance between generation and consumption. Data for the transmission lines between the countries are shown in table 6. Each transmission line is equipped with a protection system which after a short time delay disconnects the line if the power flow exceeds the maximal capacity of the line. The power flows on the HVDC line are not affected by the frequency of the system, but can only be controlled manually.

Table 5 Distribution of gain.

Country	Normal operation reserve (available between 49.9 and 50.1 Hz) [MW/Hz]	Disturbance reserve (available between 49.5 and 49.9 Hz) [MW/Hz]
Aland	2 250	1 000
Beland	3 250	1 500
Celand	500	500

Table 6 Data for the transmission lines.

Connection	Type	Current transmission [MW]	Maximal capacity [MW]
Aland ↔ Beland	Alternating current	1 000 MW from Aland to Beland	1 300
Aland ↔ Celand	Direct current (HVDC)	400 MW from Aland to Celand	600
Beland ↔ Celand	Alternating current	100 MW from Beland to Celand	1 000

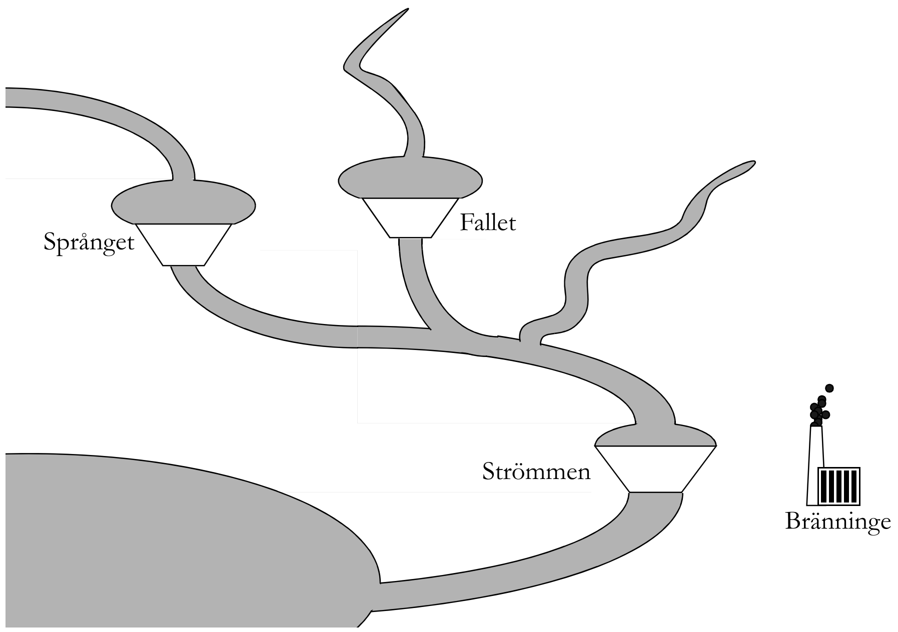
a) (2 p) Assume that there is an outage in a large power plant in Aland. How large may the outage be if the frequency should not decrease below 49.5 Hz and if no transmission lines is overloaded?

b) (2 p) Assume that the outage is in a large power plant in Beland. How large may the outage be if the frequency should not decrease below 49.5 Hz and if no transmission lines is overloaded?

c) (2 p) Assume that the outage is in a large power plant in Celand. How large may the outage be if the frequency should not decrease below 49.5 Hz and if no transmission lines is overloaded?

d) (4 p) The largest power plant in Celand is a nuclear power plant with 1 050 MW installed capacity. Assume that the disturbance reserve should be able to manage the loss of the entire generation capacity in this unit for the situation described above. How would it be possible to quickly arrange (i.e., without investing in new power plants, improving the transmission capacity or purchasing more gain) so that the system fulfils these requirements?

Problem 8 (20 p)



AB Elkraft owns three hydro power plants, located as shown in the figure above. Data for the power plants are given in table 7. The water delay time between the power plants can be neglected. The company also owns the thermal power plant Bränninge. When Bränninge is committed, the generation must be kept in the interval 50–300 MW. The variable operation cost is 450 SEK/MWh and the start-up cost is 13 000 SEK if the power plant has been decommitted for at least two hours and 5 000 SEK if the power plant has been decommitted for one hour. According to the current operation plan, Bränninge will be decommitted at 23:00 on Wednesday evening.

The electricity produced by the company is sold to the local power pool, ElKräng. AB Elkraft must submit bids for every hour of the next day no later than 12:00 on Wednesday. Table 8 shows the price forecast of AB Elkraft for Thursday. The company assume that they can sell unlimited amounts electricity to these prices. After that, the future electricity price is estimated to 400 SEK/MWh.

a) (15 p) Formulate the planning problem of AB Elkraft as a MILP problem. Use the notation in table 10 for the parameters (it is however permitted to add further symbols if you consider it necessary).

NOTICE! The following is required to get full score for this problem:

- The symbols for the optimisation variables must be clearly defined.
- The optimisation problem should be formulated so that it is easy to determine what the objective function is, which constraints there are and which limits there are.
- The possible values for all indices should be clearly stated for each equation.

b) (5 p) Assume that there is some uncertainty about the prices on Thursday. In addition to the price forecast in table 8 there is also an alternative price forecast as shown in table 9. Assume that AB Elkraft for each hour estimates the probability of the first forecast (table 8) to be 80% and that there is a 20% chance for the alternative forecast. How must the planning problem of AB Elkraft be formulated if the objective of the planning is to maximise the expected income minus the expected costs? Do not forget to define all new variables and parameters that you introduce!

Table 7 Data for the hydro power plants of AB Elkraft.

Power plant	Start contents of reservoir [HE]	Maximal contents of reservoir [HE]	Marginal production equivalents [MWh/HE]		Maximal discharge [HE]		Local inflow [HE]
			Segment 1	Segment 2	Segment 1	Segment 2	
Språnget	850	1 000	0.68	0.60	100	40	80
Fallet	1 100	1 400	0.72	0.62	80	40	60
Strömmen	750	800	0.40	0.35	150	75	10

Table 8 Price forecast of AB Elkraft for Thursday.

Hour	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Price at ElKräng [SEK/MWh]	325	265	255	255	245	300	370	395	420	425	425	425
Hour	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24
Price at ElKräng [SEK/MWh]	420	425	415	415	410	415	415	410	395	385	395	365

Table 9 Alternative price forecasts for Thursday.

Hour	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Price at ElKräng [SEK/MWh]	325	265	255	255	245	300	365	385	415	420	420	420
Hour	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24
Price at ElKräng [SEK/MWh]	415	410	400	405	395	400	390	370	340	330	325	295

Table 10 Notation for the planning problem of AB Elkraft.

Symbol	Explanation	Value
$M_{i,0}$	Start contents of reservoir i	See table 7
$\overline{M}_i$	Maximal contents of reservoir i	See table 7
$\mu_{i,j}$	Marginal production equivalent in power plant i, segment j	See table 7
$\overline{Q}_{i,j}$	Maximal discharge in power plant i, segment j	See table 7
V_i	Local inflow to reservoir i	See table 7
$\overline{G}$	Maximal generation when Bränninge is committed	300
$\underline{G}$	Minimal generation when Bränninge is committed	50
β	Variable operation cost in Bränninge	400
C^*	Start-up cost in Bränninge after one hour down-time	5 000
C^{**}	Start-up cost in Bränninge after at least two hours down-time	13 000
$\lambda_{1,t}$	Expected price at ElKräng hour t	See table 8
$\lambda_{2,t}$	Alternative forecast for the price at ElKräng hour t	See table 9
λ_f	Expected future electricity price	400

Problem 9 (20 p)

Inga and Inge Kol are planning to move to a new house. The house is very energy efficient and the couple have studied all energy going into and out from the house (they have even considered that their two cats generate 20 W heat energy and therefore reduce the need for heating in the winter). Their calculations have resulted in the duration curve of the electricity consumption in the house shown below.

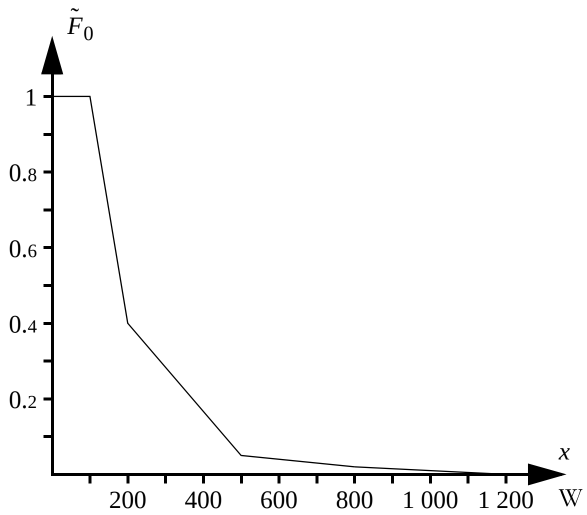


Table 11 Some values from the load duration curve of the new house.

x [kW]	$\tilde{F}_0(x)$
0.1	1
0.2	0.4
0.5	0.05
0.8	0.02
1.2	0

There are two options to supply electricity to the house:

- **Grid connection.** With a grid connection, there would be no problem to supply the house, not even at maximal electricity consumption. The annual cost of the investments and the grid fees amount to 10 000 ¤.The electricity purchased from the grid is assumed to cost 2 ¤/kWh in the foreseeable future.
- **Self-sufficient house.** This solution consists of photovoltaic modules with 3 kW peak capacity, which yield about 2 500 kWh/year. A battery bank is needed to get electricity also when the sun is not shining. The battery bank can supply at most 1 kW and has enough storage capacity to supply the house for two weeks without recharging. The investment cost of the system is assumed to be 20 000 ¤/year.

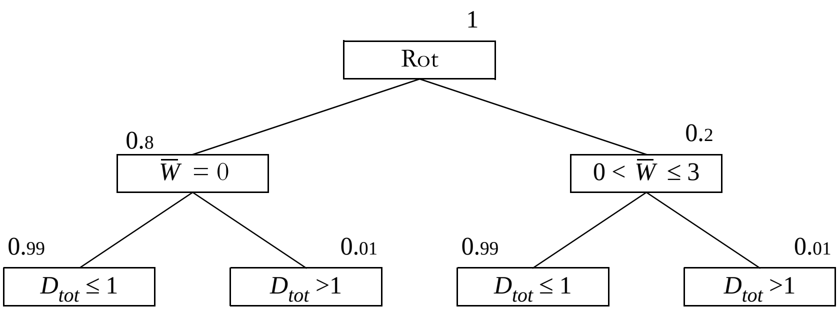
a) (8 p) Assume that the couple chooses the self-sufficient house. How many hours per year would they in average have to reduce their consumption due to power deficit? It can be assumed that the risk that the battery bank is completely discharged is negligible. The generation in the photovoltaic modules and the electricity consumption in the house are in reality correlated, but assume for this calculation that they are independent and use the probability distribution of the photovoltaic generation given in table 12.

Table 12 Simplified model of the available generation capacity in the photovoltaic modules.

Probability [%]	Available capacity [W]
80	0
10	100
10	200 or more

Table 13 The first batch of scenarios for the simulation of the Kol house.

$\overline{W}$ [W]	D_{tot} [W]	*LOLO*	$\overline{W}$ [W]	D_{tot} [W]	*LOLO*
0	119	0	15	1 168	1
0	144	0	87	197	0
0	177	0	208	142	0
0	192	0	298	184	0
0	321	0	298	1 004	0
0	1 004	1	304	1 136	1
0	1 089	1	408	303	0
0	1 091	1	529	1 076	0
0	1 152	1	1 220	164	0
0	1 164	1	1 906	1 166	0



$\overline{W}$ = available capacity in the photovoltaic modules [kW]

D_{tot} = total consumption in the house (including losses) [kW]

b) (8 p) Since the couple has made such a detailed energy balance for the house, they would like to use a more detailed model (especially concerning the generation in the photovoltaic modules) when computing the risk of power deficit. Therefore, they choose to simulate the house using stratified sampling. Above, you find a strata tree as well as a table with the first batch of scenarios. Assume that the simulation should include 10 000 samples. How should these samples be distributed between the strata?

c) (4 p) As the couple take an active interest in climate change, they would prefer if their house is self-sufficient; therefore, they are willing to choose the self-sufficient option even if the total annual costs are up to 50% higher than the total cost for grid connection. Which option will the couple choose, considering the conditions described above?

KTH Electrical Engineering

Answer sheet for part I

Name: ....

Personal number: ....

Problem 1

a) Alternative .... is correct.

b) Alternative .... is correct.

c) Alternative .... is correct.

Problem 2

a) .... ¤/MWh **b)** .... TWh

c) .... TWh

Problem 3

a) .... Hz **b)** .... Hz

c) .... Hz

Problem 4

a) Parameters: ....

Optimisation variables: ....

b) ....

c) Alternative .... is correct.

d) .... MWh/HE

e) Alternative .... is correct.

Problem 5

a) .... kWh/h **b)** .... ¤/h

c) .... % **d)** .... MW

e) .... % **f)** .... %

Problem 1

a) 1, **b)** 2, **c)** 2.

Problem 2

a) The total load in Land during the year is 165 TWh. Hydro and nuclear can in total generate 65 (inflow of this year) + 5 (stored water that is used during this year) + 80 (nuclear) = 150 TWh, which means that 15 TWh coal condensing will be needed. Hence, 15/30 of the price interval for coal condensing is used, i.e., the electricity price must be 350 + 15/30 · 150 = 425 ¤/MWh.

b) During January to April, 30 TWh nuclear and half the coal condensing capacity (i.e, 5 TWh), which means that hydro power must generate the coal condensing is generating 25 TWh, which means that the hydro power generates 20 TWh. As the inflow is 55 TWh, the reservoirs will be filled by 35 TWh during this period. The reservoirs held 25 TWh at the beginning, and they must then hold 60 TWh at the end of the period.

c) As the price is lower during the first half of the year, the reservoir will be filled between 30 April and 1 May (if there had not been a reservoir limitation then the electricity price would have been the same for the entire year). All nuclear and 45/150 = 30% of the coal condensing is used at the electricity price 395 ¤/MWh; in total this gives 33 TWh. The hydro generation must then be 33 TWh, as the electricity consumption is 55 TWh. The reservoir contents between 30 April and 1 May must therefore be 25 (start contents) + 55 TWh (inflow) – 22 (generation) = 58 TWh. Hence, we can conclude that the reservoirs can store 58 TWh.

Problem 3

a) The power plants participating in the primary control are generating 1 450 MW when the frequency is 50.01 Hz (according to the figure). These power plants have to decrease their generation to 1 250 MW when Sotinge increases the generation by 200 MW, which means that the frequency must increase to 50.05 Hz.

b) The power plants participating in the primary control are generating 1 300 MW when the frequency is 50.04 Hz (according to the figure). These power plants have to increase their generation to 1 500 MW when the load increases by 200 MW, which means that the frequency must decrease to 50.00 Hz.

c) The power plants participating in the primary control are generating 1 900 MW when the frequency is 49.92 Hz (according to the figure). These power plants have to increase their generation to 2 100 MW when the load increases by 200 MW, which means that the frequency must decrease to 49.86 Hz.

Problem 4

a) Parameters: β_{Gg}, C_g^+, $\overline{G}_g$, $\underline{G}_g$, λ_t, and $u_{g,0}$. Optimisation variables: $G_{g,t}$, $s_{g,t}^+$, and $u_{g,t}$.

b) maximise $\sum_{t=1}^{24} \sum_{g=1}^{3} ((\lambda_t - \beta_{Gg})G_{g,t} - C_g^+ s_{g,t}^+)$.

c) 3.

d) The generation as a function of discharge can be written as $H(Q) = \eta(Q) \cdot \gamma_{max} \cdot Q$. Solving for γ_{max} and using the values for maximal discharge, we get $\gamma_{max} = 36/(0.96 \cdot 100) = 0.375$ MWh/HE.

e) 2.

Problem 5

a) The unserved energy during an hour is given by

$$EENS_1 = 1 \cdot \int_{800}^{\infty} \tilde{F}_1(x)dx = 0.1 \cdot 100/2 = 5 \text{ kWh/h}.$$

b) The expected generation in the diesel generator set is $EG_2 = EENS_1 - EENS_2 = 3.75$ kWh/h. Hence, the expected operation cost is f $ETOC = 2EG_2 = 7.5$ ¤/h.

c) $LOLP = \tilde{F}_2(1\,200) = 0.1\tilde{F}_1(1\,200) + 0.15\tilde{F}_1(1\,100) + 0.2\tilde{F}_1(1\,000) + 0.4\tilde{F}_1(900) + 0.15\tilde{F}_1(800) = 0.1 \cdot 0 + 0.15 \cdot 0 + 0.2 \cdot 0 + 0.4 \cdot 0 + 0.15 \cdot 0.1 = 1.5\%$.

d) The inverse transform method states that $D = F_D^{-1}(U)$, where U is a $U(0, 1)$-distributed random number. Since it is the duration curve that is given in the problem, we may as well use the transform $D = \tilde{F}_D^{-1}(U)$. The original random number must then have been $U = \tilde{F}_D(500) = 0.8$. Hence, $U^* = 1 - U = 0.2$, which results in $D^* = \tilde{F}_D^{-1}(U^*) = 700$ MW.

e) We start by computing the expected difference between the multi-area model and the PPC model in each stratum:

$$LOLD_h = \frac{1}{n_h}\sum_{i=1}^{n_h}(x_{i,h} - z_{i,h}) = \frac{1}{n_h}\left(\sum_{i=1}^{n_h} x_{i,h} - \sum_{i=1}^{n_h} z_{i,h}\right)$$

$$\Rightarrow \quad LOLD_1 = 0,$$
$$LOLD_2 = (300 - 0)/1\,000 = 0.3,$$
$$LOLD_3 = (1\,000 - 1\,000)/1\,000 = 0.$$

We can now combine the results of each stratum weighted by their stratum weights:

$$LOLD = \sum_{h=1}^{3} \omega_h LOLD_h = 0.90 \cdot 0 + 0.075 \cdot 0.3 + 0.025 \cdot 0 = 2.25\%.$$

f) The $LOLP$ of the multi-area model is given by the expected difference plus the result of the PPC model:

$$LOLP = LOLD + LOLP_{PPC} = 2.25 + 2.5 = 4.75\%.$$

Problem 6

a) The total load is 145 TWh/year. Hydro and nuclear can supply 140 TWh/year, which means that 5/20 of the fossil fuel capacity is needed. The electricity price must then be 360 ¤/MWh.

Total producer surplus: 145 · 360 (income of sold electricity) – 70 · 5 (variable cost of hydro) – 70 · (80 + 120)/2 (variable cost of nuclear) – 5 · (300 + 360)/2 (variable cost of fossil fuels) = 43 200 M¤/year.

Total consumer surplus: 145 · 1 000 (value of electricity consumption) – 145 · 360 (cost of purchased electricity) = 92 800 M¤/year.

b) The division in price areas results in a lower electricity price in northern Rike and a higher price in the southern part of the country. In northern Rike is the demand 15 (local load) + 46 (maximal export) = 61 TWh/year. Hydro can supply 60 TWh/year, which means that 1/8 of the fossil fuel capacity is needed. The electricity price in northern Rike must then be 330 ¤/MWh. In northern Rike is the demand 130 (local load) – 46 (maximal import) = 84 TWh/year. Hydro and nuclear can supply 80 TWh/year, which means that 4/12 of the fossil fuel capacity is needed. The electricity price in northern Rike must then be 380 ¤/MWh.

Total producer surplus 61 · 330 (income of sales in northern Rike) + 84 · 380 (income of sales in southern Rike) – 70 · 5 (variable cost of hydro) – 70 · (80 + 120)/2 (variable cost of nuclear) – 1 · (300 + 330)/2 (variable cost of fossil fuels in northern Rike) – 4 · (300 + 380)/2 variable cost of fossil fuels in southern Rike) = 43 025 M¤/year.

Total consumer surplus: 145 · 1 000 (value of electricity consumption) – 15 · 330 (cost of purchased electricity in northern Rike) – 130 · 380 (cost of purchased electricity in southern Rike) = 90 650 M¤/year.

c) The producers loose 175 M¤/year and the consumers 2 150 M¤/year when price areas are introduced. The total loss is thus 2 325 M¤/year, which is as much as the savings of the system operator. Hence, it is not possible to say that one option is better than the other; it is a question about how the total surplus is distributed between the different players.

Problem 7

a) If the frequency decreases from 49.9 Hz to 49.5 Hz, the disturbance reserve in Aland will supply 400 MW. Another 600 MW and 200 MW are supplied from Beland and Celand respectively. There are is no problem that the power flow decreases by 800 MW on the AC interconnection from Aland to Beland. Similarly, there is no problem that the power flow decreases by 200 MW from Beland to Celand. Hence, the entire disturbance reserve can be used, and the outage can be at most 1 200 MW.

b) The power flow from Aland to Beland will increase when the outage occurs in Beland. As there is only 300 MW transmission capacity available, only 300 MW of the disturbance reserve may be utilised. This means that the system frequency can only decrease to 49.6 Hz without overloading a transmission line. The disturbance reserves in Beland and Celand will then supply 450 MW and 150 MW respectively. There is no problem to decrease the flow from Beland to Celand by 150 MW. The outage can thus be at most 900 MW.

c) The power flow from Aland to Beland will increase when the outage occurs in Celand. As there is only 300 MW transmission capacity available, only 300 MW of the disturbance reserve may be utilised. This means that the system frequency can only decrease to 49.6 Hz without overloading a transmission line. The disturbance reserves in Beland and Celand will then supply 450 MW and 150 MW respectively. There is no problem to increase the flow from Beland to Celand by 750 MW. The outage can thus be at most 900 MW.

d) In order to fulfil the requirement, a larger share of the disturbance reserve in Aland must be available in case of an outage in Celand; hence, there must be larger margins on the AC interconnection between Aland and Beland. This can be achieved by increasing the power flow on the HVDC line between Aland and Celand. When 1 050 MW is lost, the disturbance reserve in Aland must be able to supply 400/1200 · 1050 = 350 MW. Therefore, another 50 MW unused capacity between Aland and Beland is necessary. The power flow on the HVDC link should therefore be increased to 450 MW, which is possible as the maximal capacity is 600 MW.

Problem 8

a) The problem we want to solve is

maximise *value of sold electricity – costs of Bränninge + value of stored water,*

subject to *hydrological balance of the hydro reservoirs,,*
limitations in generation at Bränninge,
relation between unit commitment and start-up time in Bränninge,
limitations in reservoir contents, discharge and spillage.

Indices for the power plants

Språnget 1, Fallet 2, Strömmen 3.

Parameters

The parameters are defined in table 10 in the problem text. We also introduce the following parameters:

u_{-1} = unit commitment in Bränninge between 22:00 and 23:00 on Wednesday = 1,
u_0 = unit commitment in Bränninge between 23:00 and 24:00 on Wednesday = 0.

(Without these parameters, it would be necessary to modify the unit commitment constraints for hours 1 and 2.)

Optimisation variables

$Q_{i,j,t}$ = discharge in power plant i, segment j, during hour t, i = 1, 2, 3, j = 1, 2, t = 1, …, 24,
$S_{i,t}$ = spillage from reservoir i during hour t, i = 1, 2, 3, t = 1, …, 24,
$M_{i,t}$ = contents of reservoir i at the end of hour t, i = 1, 2, 3, t = 1, …, 24,
G_t = generation in Bränninge hour t, t = 1, …, 24,
s_t^* = start-up of Bränninge in hour t, after one hour down-time, t = 1, …, 24,
s_t^{**} = start-up of Bränninge in hour t, after at least two hours down-time, t = 1, …, 24,
u_t = unit commitment in Bränninge during hour t, t = 1, …, 24.

Objective function

$$\text{maximise} \quad \sum_{t=1}^{24} \lambda_{1,t}\left(G_t + \sum_{i=1}^{3}\sum_{j=1}^{2}\mu_{i,j}Q_{i,j,t}\right) - \sum_{t=1}^{24}(\beta G_t + C^* s_t^* + C^{**} s_t^{**}) + \lambda_f((\mu_{1,1} + \mu_{3,1})M_{1,24} + (\mu_{2,1} + \mu_{3,1})M_{2,24} + \mu_{3,1}M_{3,24}).$$

Constraints

Hydrological balance for Språnget and Fallet:

$$M_{i,t} = M_{i,,t-1} - Q_{i,1,t} - Q_{i,2,t} - S_{i,t} + V_{i,t}, \qquad i = 1, 2, t = 1, \ldots 24.$$

Hydrological balance for Strömmen:

$$M_{2,t} = M_{2,t-1} - Q_{3,1,t} - Q_{3,2,t} - S_{3,t} + Q_{1,1,t} + Q_{1,2,t} + S_{1,t} + Q_{2,1,t} + Q_{2,2,t} + S_{2,t} + V_{2,t}, \qquad t = 1, \ldots 24.$$

Maximal generation in Bränninge:

$$G_t \le u_t \overline{G}_g, \qquad t = 1, \ldots, 24.$$

Minimal generation in Bränninge:

$$u_t \underline{G} \le G_t, \qquad t = 1, \ldots, 24.$$

Relation between unit commitment and start-up after a certain down-time:

$$u_t - u_{t-1} - u_{t-2} - s_t^{**} \le 0, \qquad t = 1, \ldots, 24,$$

$$u_t - u_{t-1} - s_t^{**} - s_t^{*} \le 0, \qquad t = 1, \ldots, 24.$$

Variable limits

$0 \le Q_{i,j,t} \le \overline{Q}_{i,j}$, $i = 1, 2, 3, j = 1, 2, t = 1, \ldots, 24$,
$0 \le S_{i,t} \le \overline{S}_i$, $i = 1, 2, 3, t = 1, \ldots, 24$,
$0 \le M_{i,t} \le \overline{M}_i$, $i = 1, 2, 3, t = 1, \ldots, 24$,
$s_t^* \in \{0, 1\}$, $t = 1, \ldots, 24$,
$s_t^{**} \in \{0, 1\}$, $t = 1, \ldots, 24$,
$u_t \in \{0, 1\}$, $t = 1, \ldots, 24$.

b) In this case, the price is a random variable, which we can denote λ_t. However, the generation should be decided before the outcome of the price is known and the objective of the planning is to choose an optimal generation level for each hour. We may therefore use the same optimisation variables as in part a. The cost of generation in Bränninge is a deterministic function of the chosen generation plan, and the value of water is a function of how much is discharged during the planning period. The income is on the other hand a random variable, since the income during a certain hour is the product of the price (which is a random variable) and the planned generation (which is an optimisation variable) The expected income can therefore be formulated as

$$E\left[\sum_{t=1}^{24} \lambda_t \left(G_t + \sum_{i=1}^{3}\sum_{j=1}^{2} \mu_{i,j} Q_{i,j,t}\right)\right].$$

Using the rules that $E[X + Y] = E[X] + E[Y]$ and $E[aX] = a \cdot E[X]$ we can rewrite the income as

$$\sum_{t=1}^{24} E[\lambda_t]\left(G_t + \sum_{i=1}^{3}\sum_{j=1}^{2} \mu_{i,j} Q_{i,j,t}\right).$$

Since the first forecast has 80% probability and the alternative has 20% probability, we get that $E[\lambda_t] = 0.8\lambda_{1,t} + 0.2\lambda_{1,t}$. The only change of the planning problem that is necessary is thus that the objective function should be reformulated as

$$\text{maximise} \quad \sum_{t=1}^{24} (0.8\lambda_{1,t} + 0.2\lambda_{2,t})\left(G_t + \sum_{i=1}^{3}\sum_{j=1}^{2} \mu_{i,j} Q_{i,j,t}\right) - \sum_{t=1}^{24} (\beta G_t + C^* s_t^* + C^{**} s_t^{**}) + \lambda_f((\mu_{1,1} + \mu_{3,1})M_{1,24} + (\mu_{2,1} + \mu_{3,1})M_{2,24} + \mu_{3,1} M_{3,24}).$$

Problem 9

a) The battery bank can be seen as a 100% available power plant with the installed capacity 1 kW. The model of the photovoltaic modules identifies three states: 3 kW (80%), 2,9 kW (10%) and 0–2.8 kW (10%). To fit into the convolution formula we assume that the outage is 2.8 kW in the last state. The risk of power deficit is then given be

$$\tilde{F}_2(4) = 0.8\tilde{F}_1(4-3) + 0.1\tilde{F}_1(4-2.9) + 0.1\tilde{F}_1(4-2.8) = \{\tilde{F}_1(x) = \tilde{F}_0(x)\} =$$

$$= 0.8\tilde{F}_0(1) + 0.1\tilde{F}_0(1.1) + 0.1\tilde{F}_0(1.2) = \{\text{read the figure and table}\} =$$
$$= 0.8 \cdot 0.01 + 0.1 \cdot 0.005 + 0.1 \cdot 0 = 0.0085 = 0.85\%.$$

Hence, during one year they can expect to have power deficit about 75 hours.

b) An appropriate sample allocation can be computed using the Neyman allocation after the first batch of samples. However, if we look closer at the strata tree, we find that the value of *LOLO* can be predicted in three of the strata:

- In the first branch of the tree, the load is always less than the capacity of the battery bank. Thus, in these scenarios there will never be any power deficit, i.e., *LOLO* = 0 in all scenarios.
- In the second branch of the tree is the load always higher than the capacity of the battery bank. As no photovoltaic generation is available, there will always be power deficit, i.e., *LOLO* = 1 in all scenarios.
- In the third branch of the tree, the load is always less than the capacity of the battery bank. Thus, in these scenarios there will never be any power deficit, i.e., *LOLO* = 0 in all scenarios.

In the fourth stratum is it on the other hand uncertain what will happen, whether the photovoltaic modules will have sufficient capacity to cover load or if it is going to be a power deficit. As these are the only scenarios that cannot be studied analytically, all the remaining scenarios should be allocated to stratum 4.

c) The operation and maintenance costs of the self-sufficient house can be assumed to be negligible, which means that the total cost equals the investment costs, i.e., 20 000 ¤.

The total cost of grid connection consists of the fixed costs (10 000 ¤) and the energy cost. The expected electricity consumption is given by

$$E[D] = 8\,760 \int_0^\infty \tilde{F}_0(x)dx = 8\,760(0.1 \cdot 1 + 0.1 \cdot (1 + 0.4)/2 + 0.3 \cdot (0.4 + 0.05)/2 + 0.3 \cdot (0.05 + 0.02)/2 + 0.4 \cdot 0.02/2) = 2\,207.5 \text{ kWh/year}.$$

With the electricity price 2 ¤/kWh the energy cost is 4 415 ¤/year. The total cost of grid connection is then 14 415 ¤/year.

As the couple are willing to pay up to 50% extra for a self-sufficient house (i.e., up to about 21 600 ¤) they will choose the option with their own power generation.