KTH Electrical Engineering

Exam in EG2050 System Planning, 16 March 2011, 14:00–19:00, E35, E51-53

Allowed aids

In this exam in you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten**, **single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

PART I (MANDATORY)

Write all answers on the answer sheet provided. Motivations and calculations do not have to be presented.

Part I can yield 40 points in total. The examinee is guaranteed to pass if the score is at least 33 points. If the result in part I is at least 31 points, then there will be a possibility to complement for passing the exam with the grade E.

Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

a) (2 p) In a modern, restructured ("deregulated") electricity market, the system operator is responsible for the short-term balance between generation and consumption. This means that I) The system operator has to ensure that the frequency is kept within given limits, II) If the system operator does not ensure that the system in every moment is supplied as much power as is consumed, then the system operator will have to pay a penalty fee to the balance responsible players, III) If the system operator does not ensure that the system in every trading period (for example an hour) is supplied as much energy as is consumed, then the system operator will have to pay a penalty fee to the balance responsible players.

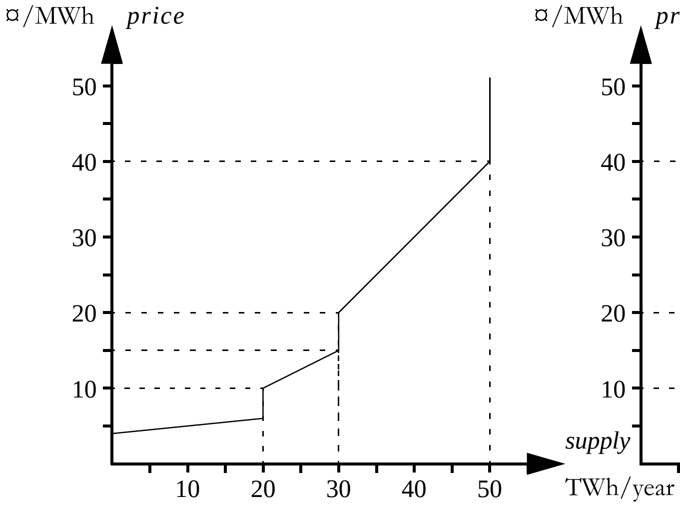
1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. I and II are true but not III.

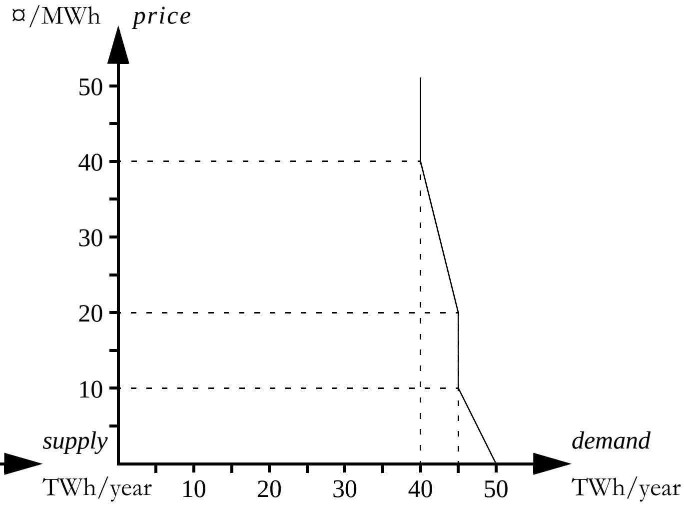
b) (2 p) The consumers in a vertically integrated electricity market has the following choices: I) They can choose which system operator they want, II) They can choose which retailer they want, III) The can choose which player should manage their balance responsibility.

1. None of the statements is true.
2. Only I is true.
3. Only III is true.
4. I and II are true but not III.
5. II and III are true but not I.

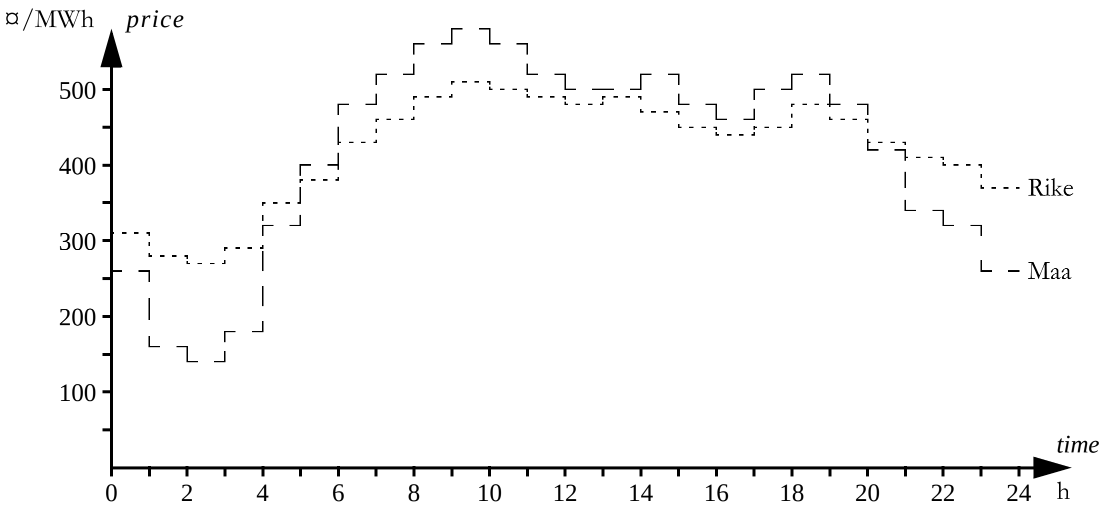
Problem 2 (6 p)

a) (3 p) The figures below shows the supply and demand curves of a certain electricity market. What will the electricity price become in this electricity market if we assume perfect competition, perfect information and that there are neither transmission, reservoir nor capacity limitations?





b) (2 p) Consider the common electricity market of the two countries Rike and Maa. Assume that there is perfect competition and that all players have perfect information. The power systems of Rike and Maa are connected by an HVDC line, which can transfer at most 1 000 MW. The figure below shows the electricity prices in Rike and Maa during a day. During how many hours will Rike export to Maa?



c) (1 p) How large is the net trading between the two countries, i.e., which country will export most energy during the day and how large is the total export minus the total import for this country?

Problem 3 (6 p)

Consider a power system divided in five areas. At a certain occasion there is balance between production and consumption in the system and the frequency is exactly equal to 50 Hz. Data for the primary control in the system are given in table 1. Data for the transmission lines between the countries are shown in table 2. Each transmission line is equipped with a protection system which after a short time delay disconnects the line if the power flow exceeds the maximal capacity of the line. The power flow on the HVDC line are not affected by the frequency of the system, but can only be controlled manually..

Table 1 Data for the primary control.

Area	Gain (available between 49.9 and 50.1 Hz) [MW/Hz]
A	2 000
B	2 000
C	1 000
D	500
E	500

Table 2 Data for the interconnections.

Connection	Type	Current transmission [MW]	Maximal capacity [MW]
A ↔ B	Alternating current	1 000 MW from A to B	2 000
A ↔ C	Direct current (HVDC)	600 MW from A to C	600
A ↔ D	Direct current (HVDC)	400 MW from A to D	400
A ↔ E	Alternating current	1 000 MW from A to E	1 500
B ↔ D	Alternating current	500 MW from B to D	1 200

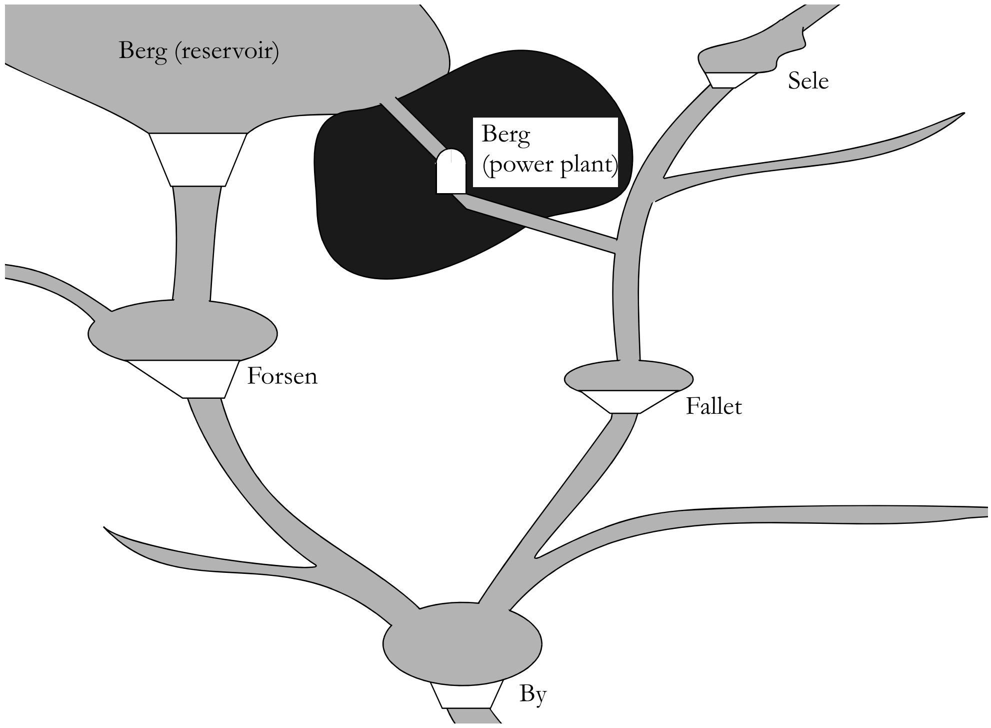
a) (3 p) At this occasion, the generation is increased by 150 MW in a thermal power plant in area A. The power plant is not part of the primary control. What will the frequency be in area A when the primary control has restored the balance between generation and consumption?

b) (1 p) What will the frequency be in area B after the event in area A?

c) (1 p) What will the frequency be in area C after the event in area A?

d) (1 p) What will the frequency be in area D after the event in area A?

Problem 4 (12 p)



a) (5 p) AB Vattenkraft owns five hydro power plant located as in the figure above. Notice that Berg is an underground power plant and that water that is discharged through the turbine is flowing to Fallet, whereas spillage ends up in Forsen. The following symbols have been introduced in a short-term planning problem for these hydro power plants:

Indices for the power plants: Berg - 1, Sele - 2, Forsen - 3, Fallet - 4, By - 5.

γ_i = expected future production equivalent for water stored in reservoir i, 1, …, 5,
λ_t = expected electricity price hour t, t = 1, …, 24,
λ_{25} = expected electricity price after the end of the planning period,
$M_{i,\,0}$ = contents of reservoir i at the beginning of the planning period,1, …, 5,
$M_{i,\,t}$ = contents of reservoir i at the end of hour t, i = 1, …, 5, t = 1, …, 24,
$\mu_{i,\,j}$ = marginal production equivalent in power plant i, segment j, i = 1, …, 5, j = 1, 2,
$Q_{i,\,j,\,t}$ = discharge in power plant i, segment j, during hour t, 1, …, 5, j = 1, 2, t = 1, …, 24,
$S_{i,\,t}$ = spillage from reservoir i during hour t, i = 1, …, 5, t = 1, …, 24,
$V_{i,\,t}$ = local inflow to reservoir i during hour t, i = 1, …, 5, t = 1, …, 24.

Formulate the objective function if the purpose of the planning is to maximise the income of generated hydro power plus the value of stored water. Use the symbols defined above.

b) (3 p) The hydro power plant Språnget has a maximal discharge of 240 m^3/s. The best efficiency is obtained for the discharge 180 m^3/s. At maximal discharge the power plants generates its installed capacity, which is 153 MW. At best efficiency the power plant generates 117 MW.

Assume that we need a piecewise linear model of electricity generation as function of the discharge in Språnget. The model should have two segments and the breakpoint between them should be located at the best efficiency. Calculate the following parameters:

μ_j = marginal production equivalent in Språnget, segment j,
$\overline{Q}_j$ = maximal discharge in Språnget, segment j.

c) (4 p) Consider a thermal power plant, where the unit commitment is modelled using the following variables:

s_t^+ = start-up of the power plant in the beginning of hour t,
u_t = unit commitment of the power plant during hour t, t = 1, …, 24.

Formulate the constraint that sets the relation between u_t, u_{t-1} and s_t^+ for hour t. Notice that the constraint must be formulated without using any additional optimisation variables!

Problem 5 (12 p)

a) (2 p) Assume that the electricity market in Land has been simulated with data for next year (i.e., the availabilities and operation costs of the power plants, and the load duration curve) and that the resulting *ETOC* is equal to 1 000 k¤/h. How large will the total operation cost become in Land next year?

1. The total operation cost will be exactly 1 000 M¤ during the year.
2. The total operation cost will be less than or equal to 8 760 M¤ during the year.
3. The total operation cost will be exactly 8 760 M¤ during the year.
4. The total operation cost will be larger than or equal to 8 760 M¤ during the year.
5. It is not possible to exactly predict the total operation cost next year, but it is likely that it will be around 8 760 M¤.

b) (2 p) Consider an electricity market with three power plants. The following values of the unserved energy has been obtained from a probabilistic production cost simulation:

$EENS_0$ = 500 MWh/h, $EENS_1$ = 290 MWh/h,
$EENS_2$ = 120 MWh/h, $EENS_3$ = 2 MWh/h.

What is the *ETOC* of the system if the variable costs of the power plants are 10 ¤/MWh, 15 ¤/MWh and 20 ¤/MWh respectively?

c) (3 p) The figure below shows the equivalent load duration curve of a system, where the total installed capacity is 900 MW. Assume that a wind farm is added to the system, and that the available generation capacity of the wind farm can be estimated by the model in table 3. What is the risk of power deficit for this system?

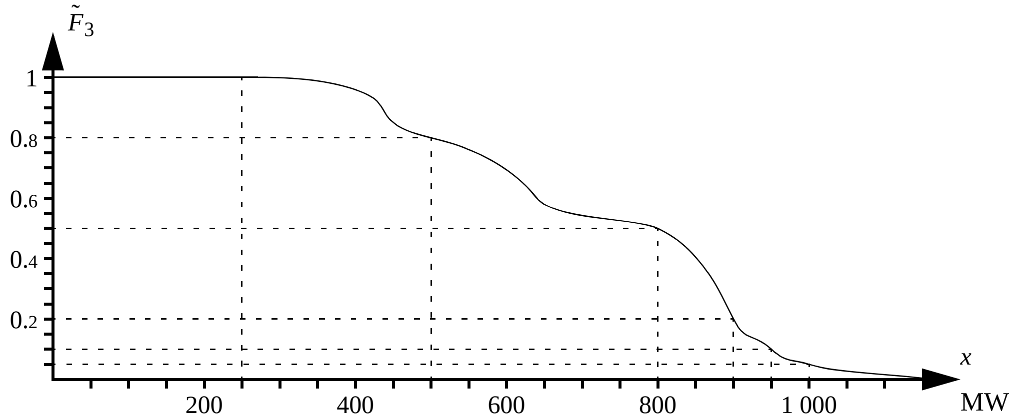
Hint: The convolution formula for a multi-state model reads

$$\tilde{F}_g(x) = \sum_{i=1}^{N_g} p_{g,i}\tilde{F}_{g-1}(x - x_{g,i}).$$

Table 3 Model of the wind farm in problem 5c.

Available generation capacity [MW]	Probability [%]
0	50
50	40
100	10

d) (4 p) Assume that a power system is simulated using stratified sampling and complementary random numbers. A pilot study is performed in order to decide how to distribute the samples between the strata. In this pilot study, 100 scenarios and 100 complementary scenarios are generated for each stratum. The results of the pilot study are shown in table 4. Which estimate of *ETOC*



is obtained after the pilot study?

Table 4 Results from a Monte Carlo simulation of the power system in problem 5d.

Stratum, h	Stratum weight, ω_h	Total operation cost in the original scenarios, $\sum_{i=1}^{100} g(y_i)$ [¤/h]	Total operation cost in the complementary scenarios, $\sum_{i=1}^{100} g(y_i{}^*)$ [¤/h]
1	0.95	900 000	1 100 000
2	0.01	1 500 000	1 400 000
3	0.04	4 010 000	3 990 000

e) (1 p) Assume that a batch of 1 000 scenarios is simulated after the pilot study in part d, and that the samples are distributed as close to the Neyman allocation as possible. What estimate of *ETOC* will be obtained after this batch? The true value of the expected total operation cost of the system is 12 200 ¤/h.

1. The result will be worse, i.e., the new estimate will be further away from the true value compared to the estimate from part d.
2. The result will be improved, i.e., the new estimate will be closer to the true value compared to the estimate from part d.
3. It is likely that the result will be improved, but it is also possible that the result is worse.

PART II (FOR HIGHER GRADES)

All introduced symbols must be defined. Solutions should include sufficient detail that the argument and calculations can be easily followed.

The answer to each problem must begin on a new sheet, but answers to different parts of the same problem (a, b, c, etc.) can be written on the same sheet. The fields *Namn* (Name), *Blad nr* (Sheet number) and *Uppgift nr* (Problem number) must be filled out on every sheet.

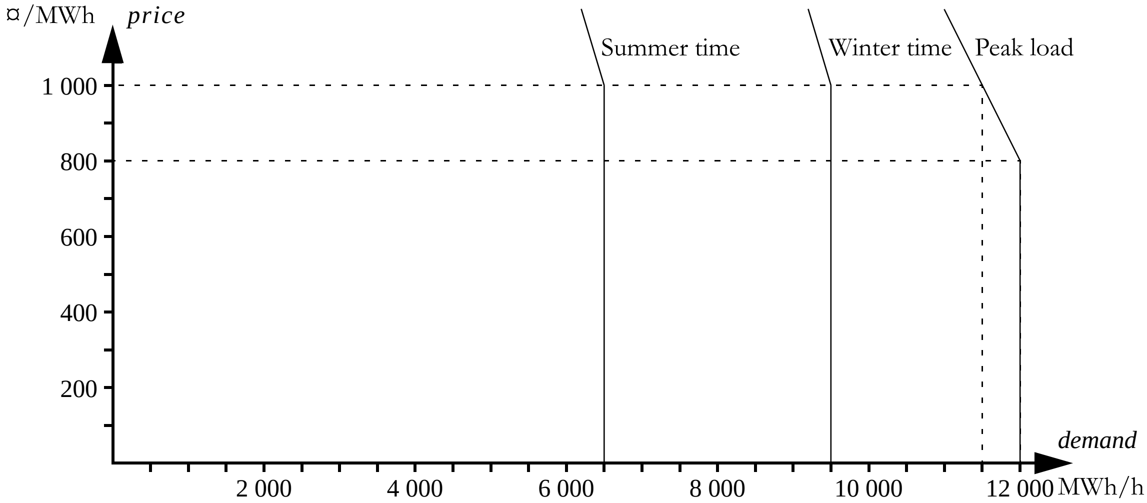
Part II gives a total of 60 points, but this part will only be marked if the candidate has obtained at least 33 points in part I. Then the results of parts I and II and the bonus points will be added together to determine the examination grade (A, B, C, D, E).

Problem 6 (10 p)

Consider a simplified model of the electricity market in Land, where it is assumed that there is perfect competition, that all players have access to perfect information and that there are neither transmission nor reservoir limitations. However, generation capacity as well as demand are varying during the year. To get a rough estimate of the electricity price in Land, the year can be divided in peak load periods (60 hours per year), winter time (5 700 hours per year) and summer time (3 000 hours per year). Data for the electricity generation are given in table 5. The variable operation costs are assumed to be linear within the intervals, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level. The demand is presented in the figure below.

Table 5 Generation capacity in the electricity market of Land.

Power source	Production capability [MWh/h]			Variable costs [¤/MWh]
	Peak load	Winter time	Summer time	
Hydro	5 000	4 500	3 000	5
Nuclear	4 000	4 000	3 000	80–120
Fossil fuels	2 500	2 500	2 500	300–550



a) (6 p) Calculate how the electricity price is varying over the year in the electricity market of Land.

b) (4 p) Would it be profitable to invest in a gas turbine with a production capacity of 125 MWh/h, variable costs 800 ¤/MWh and fixed costs of 42 M¤/year?

Problem 7 (10 p)

The power system in Rike is divided in three price areas. There is a lot of hydro power in the northern part of the system, but most of the load is located in the central and southern parts of the country. There are several parallel AC transmission lines between the three areas. Table 6 shows the maximal flow on these lines (if this limit is exceeded, the power system becomes unstable and there is a risk that there will be extensive blackouts in the entire or parts of the power system) and the trading capacity (the part of the maximal transmission capacity that have been assigned to the market, i.e., the maximal planned flow when the frequency is exactly 50 Hz).

The primary control of Rike is divided in a normal operation reserve and a disturbance reserve. Riksnät, who is the system operator in Rike, does not own any power plants of its own, but are forced to buy primary control capacity. The purchase is composed of a mix of long-term and short-term contracts, because the need for gain varies depending on which power plants that are available and what the demand is. Table 7 shows an overview of the current long-term contracts. Before next week, Riksnät needs to purchase additional disturbance reserves to guarantee that the system can manage an outage of 1 000 MW in southern Rike without the frequency dropping below 49.5 Hz and without overloading any interconnection. This requirement must be fulfilled even in a situation when the normal operation reserve is exhausted (i.e., when the frequency is 49.9 Hz).Which bids should Riksnät accept if they want to minimise the costs of the increased disturbance reserve?

Table 6 Interconnections between the price areas in Rike.

Interconnection	Maximal transmission capacity [MW]	Trading capacity MW]
North ↔ Central	4 000	3 500
Central ↔ South	2 500	1 700

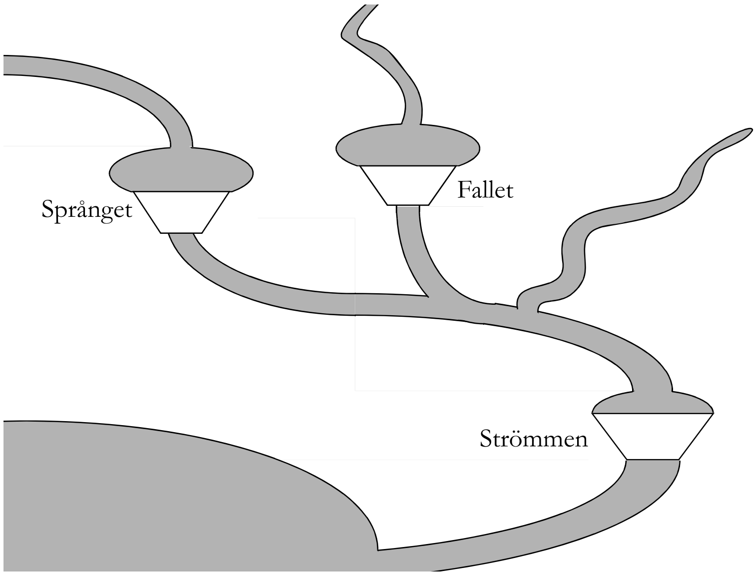
Table 7 Primary control capacity according to long-term contracts.

Price area	Gain [MW/Hz]	
	Normal operation reserve	Disturbance reserve
North	1 500	750
Central	300	550
South	200	700

Table 8 Bids to the disturbance reserve in Rike.

Bid	Gain [MW/Hz]	Frequency range [Hz]	Price area	Price [¤/((MW/Hz) · week)]
1	125	49.5–49.9	North	120
2	125	49.5–49.9	Central	125
3	125	49.5–49.9	North	130
4	125	49.5–49.9	South	135
5	125	49.5–49.9	Central	140
6	125	49.5–49.9	South	145

Problem 8 (20 p)



AB Vattenkraft owns three hydro power plants, located as shown in the figure above. The Environment Court has decided a minimal flow downstream each power plant, which means that the corresponding discharge or spillage must be released every hour. Data for the power plants are given in table 9. The electricity produced by the company is sold to the local power exchange, ElKräng. The company has during Wednesday morning submitted bids for every hour of the following Thursday and at two pm. ElKräng has notified which bids that have been accepted (see table 10). Future generation is assumed to be sold for 500 SEK/MWh and stored water is assumed to be used for electricity generation at average production equivalent, i.e, installed capacity in MW divided by maximal discharge in HE. The water delay time between the power plants can be neglected.

Rainy weather is expected to pass the power plants of AB Vattenkraft during the first hours on Thursday. This rainy weather is expected to result in a significant increase of the local inflow during a couple of hours. Forecasts for the local inflow are shown in table 11.

a) (12 p) Assume that the original forecast of the local inflow during Thursday is valid. Formulate the planning problem of AB Vattenkraft as an LP problem. Use the notation in table 12 for the parameters (it is however permitted to add further symbols if you consider it necessary).

NOTICE! The following is required to get full score for this problem:

- The symbols for the optimisation variables must be clearly defined.
- The optimisation problem should be formulated so that it is easy to determine what the objective function is, which constraints there are and which limits there are.
- The possible values for all indices should be clearly stated for each equation.

b) (8 p) The forecast of the local inflow is quite certain considering the amount of increased inflow in connection with the rain. However, there is some uncertainty concerning when the rainy weather will pass. Assume that there is a 25% probability that it will arrive one hour earlier (alternative forecast 1) than in the original forecast, and 25% probability that it will arrive one hour later (alternative forecast 2). AB Vattenkraft want to set a plan for the discharge and spillage already on Wednesday afternoon, in order to be certain that the contracted sales to ElKräng can be delivered and that the decision of the Environment Court concerning minimal flows can be maintained. Discharge and spillage should thus not depend on the local inflow—there should be one decision on discharge and spillage for each power plant and for each hour. This means that the reservoir levels must be allowed to vary as a consequence of discharge, spillage and local inflow, i.e., there will be different reservoir levels for the three possible alternatives for the local inflow. depending

Table 9 Data for the hydro power plants of AB Vattenkraft.

Power plant	Start contents of reservoir [HE]	Maximal contents of reservoir [HE]	Marginal production equivalents [MWh/HE]		Maximal discharge [HE]		Minimal flow downstream the power plant [HE]
			Segment 1	Segment 2	Segment 1	Segment 2	
Språnget	850	1 000	0.68	0.60	100	40	25
Fallet	1 100	1 400	0.70	0.62	80	40	20
Strömmen	750	800	0.40	0.35	150	80	50

Table 10 The sales of AB Vattenkraft to ElKräng during Thursday.

Hour	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Accepted bids [MWh]	184	142	142	162	128.4	170.4	212	212	267	267	236.8	212
Hour	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24
Accepted bids [MWh]	212	212	198	184	184	184	212	212	184	184	55	51

Table 11 Forecast concerning local inflow during Thursday.

Time	0-1	1-2	2-3	3-4	4-5	5-24
Local inflow according to original forecast [HE]						
Språnget	80	80	200	200	80	80
Fallet	60	60	150	150	60	60
Strömmen	10	10	30	30	10	10
Local inflow according to alternative forecast 1 [HE]						
Språnget	80	200	200	80	80	80
Fallet	60	150	150	60	60	60
Strömmen	10	30	30	10	10	10
Local inflow according to alternative forecast 1 [HE]						
Språnget	80	80	80	200	200	80
Fallet	60	60	60	150	150	60
Strömmen	10	10	10	30	30	10

Table 12 Notation for the planning problem of AB Vattenkraft.

Symbol	Explanation	Value
$M_{i,0}$	Start contents of reservoir *i*	See table 9
$\overline{M}_i$	Maximal contents of reservoir *i*	See table 9
$\mu_{i,j}$	Marginal production equivalent in power plant *i*, segment *j*	See table 9
$\overline{Q}_{i,j}$	Maximal discharge in power plant *i*, segment *j*	See table 9
$\underline{Q}_i$	Minimal flow downstream of power plant *i*	See table 9
D_t	Sales to ElKräng hour *t*	See table 10
$V_{i,t,1}$	Expected local inflow to reservoir *i* during hour *t* according to original forecast	See table 11
$V_{i,t,2}$	Expected local inflow to reservoir *i* during hour *t* according to alternative forecast 1	See table 11
$V_{i,t,3}$	Expected local inflow to reservoir *i* during hour *t* according to alternative forecast 1	See table 11
λ_f	Expected future electricity price	500

on the local inflow. How must the planning problem of AB Vattenkraft be reformulated if the objective is to maximise the value of the expected reservoir content? Do not forget to define all new variables and parameters that you introduce!

Hint: Introduce separate reservoir contents variables for the three alternatives for local inflow, i.e.,

$M_{i,t,p}$ = contents of reservoir i at the end of hour t if forecast p occurs,
$i = 1, 2, 3$, $t = 1, \ldots, 24$, $p = 1, 2, 3$.

Problem 9 (20 p)

Öarna is an autonomous group of islands in the Atlantic Ocean. The power system of Öarna is not large enough to make it meaningful to restructure the electricity market and introduce competition; therefore, Öarna still has a vertically integrated electricity market, where a government owned utility operates all generation, transmission and distribution. All consumers pay an electricity price of 150 ¤/MWh.

The generation on Öarna is based on hydro power and wind power, located in the sparsely populated coastal area Berg. The total installed capacity is 160 MW and the variable operation cost in these power plants is negligible. A duration curve for the total available generation capacity, $\tilde{F}_{\overline{W}}(x)$, is shown on the next page. The major part of the load is located in the two larger villages, Hamn and Vik. The total load duration curve of the system, $\tilde{F}_D(x)$, is also shown on the next page. Hamn constitutes between 60 and 80% of the total load and the remainder is in Vik.

In order to simulate the power system on Öarna, one can use a model with three areas: Berg, Hamn and Vik. The interconnection between Berg and Hamn is composed by two parallel overhead lines. Each line has a capacity of 100 MW and the losses on each line can be expressed as $L = 5\cdot 10^{-4}P^2$, where L is the losses on the line and P is the power injected into the line. For normal weather, the available transmission capacity on these two lines is practically independent, but during storms there is a risk that both lines are disconnected simultaneously. In average, there are storms during 8.76 h per year on Öarna. A model of the available transmission capacity on these two lines are given in table 13. The interconnection between Hamn and Vik is a submarine cable, which has a capacity of 50 MW and which can be assumed to have the loss function $L = 8\cdot 10^{-4}P^2$. The risk of outages on the submarine cable is negligible.

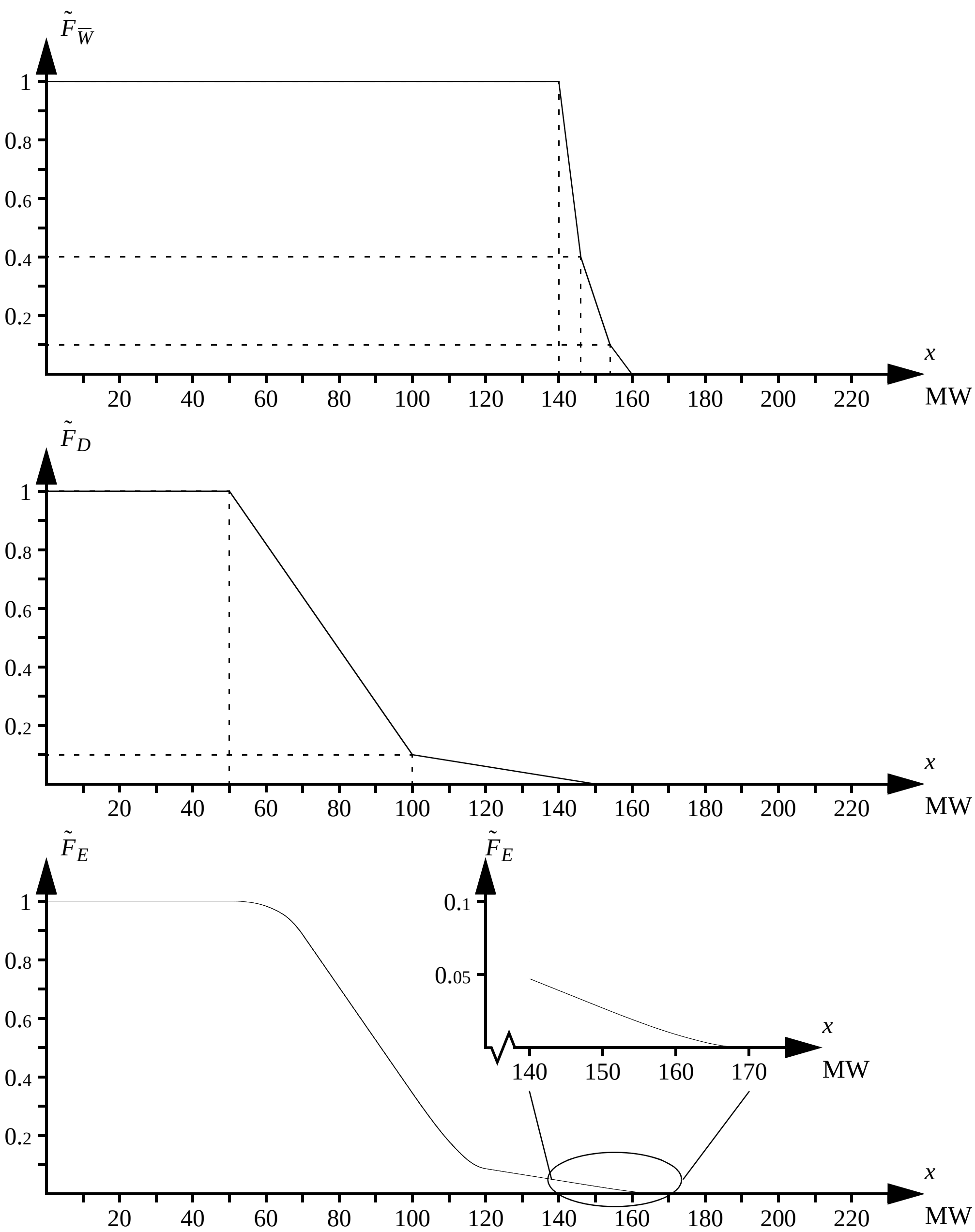
Table 13 Model of the interconnection between Berg and Hamn.

Available transmission capacity [MW]	Probability [%]	
	Normal weather	Storm
0	Negligible	10
100	0.2	20
200	99.8	70

Table 14 Random numbers.

Probability distribution	Value									
Total generation capacity, $\tilde{F}_{\overline{W}}$	152	154	141	155	147	141	143	145	157	158
	142	158	157	145	151	141	144	155	151	158
Total load. $\tilde{F}_D$	74	71	92	94	60	77	75	86	89	92
	65	88	86	59	57	78	130	69	82	62
$U(0, 1),\ \tilde{F}_U(x) = \begin{cases} 1 & \text{if } x < 0, \\ 1 - x & \text{if } 0 \le x \le 1, \\ 0 & \text{if } 1 < x. \end{cases}$	0.35	0.83	0.59	0.55	0.92	0.11	0.08	0.18	0.55	0.40
	0.29	0.76	0.75	0.38	0.57	0.96	0.40	0.26	0.14	0.08
	0.08	0.05	0.53	0.78	0.93	0.00	0.26	0.15	0.85	0.24
	0.13	0.57	0.47	0.01	0.34	0.77	0.80	0.14	0.62	0.12
	0.16	0.79	0.31	0.53	0.17	0.82	0.43	0.87	0.35	0.18
	0.60	0.26	0.65	0.69	0.75	0.87	0.91	0.58	0.51	0.24

a) (8 p) The figure on next page shows the equivalent load duration curve, $\tilde{F}_E(x)$, of Öarna (transmission losses have not been included in the equivalent load) and table 14 provides some random numbers from different distributions. Suggest a simulation method for the electricity market on Öarna. Describe briefly which assumptions you do and which computations that are necessary. Describe how you do to obtain a result that is as accurate as possible considering the limited



data given above and the limited time you have at disposal.

b) (4 p) Use the simulation method you have suggested to determine an estimate of the risk of power deficit on Öarna.

c) (8 p) Assume that the social cost of disconnected load is1 000 ¤/MWh. Would it be beneficial for the society to make an investment in a diesel generator set in Hamn that costs 1.2 M¤/year? The new unit would have the installed capacity 2.5 MW, availability 90% and a variable cost of 100 ¤/MWh.

KTH Electrical Engineering

Answer sheet for part I

Name: ....

Personal number: ....

Problem 1

a) Alternative .... is correct.

b) Alternative .... is correct.

Problem 2

a) .... ¤/MWh **b)** .... hours

c) .... will export .... MWh

3

a) .... Hz **b)** .... Hz

c) .... Hz **d)** .... Hz

Problem 4

a) ....

....

b) μ_1 .... MWh/HE μ_2 .... MWh/HE

$\bar{Q}_1$ .... HE $\bar{Q}_2$ .... HE

c) ....

Problem 5

a) Alternative .... is correct.

b) .... ¤/h **c)** .... %

d) .... ¤/h

e) Alternative .... is correct.

Problem 1

a) 2, **b)** 1.

Problem 2

a) The electricity price is determined by the intersection of the supply and demand curves. The intersection can be found graphically by drawing both curves in the same figure. An alternative method of solution is to assume an electricity price, λ, between 20 and 40 ¤/MWh. The supply at these price levels can be written as $30 + (\lambda - 20)$ and the demand can be written $45 - (\lambda - 20)/4$. These two expressions should be equal, which results in the electricity price $\lambda = 32$ ¤/MWh.

b) The electricity price is lower in Rike between five in the morning until eight in the night, i.e., Rike will export during 15 hours.

c) During 15 hours the export from Rike is 15·1 000 = 15 000 MWh and during the remaining 9 hours Maa exports 9·1 000 = 9 000 MWh. Thus, the net export is 6 000 MWh from Rike to Maa.

Problem 3

a) Area A is part of the same synchronous grid as areas B, D and E, which means that the total gain of the system is 5 000 MW/Hz. The increase in electricity generation results in a frequency increase $\Delta f = \Delta G/R = 150/5\,000 = 0.03$ Hz, i.e., the new frequency is 50 + 0.03 = 50.03 Hz.

b) Since area B is part of the same synchronous grid as area A, the frequency must be the same, i.e., 50.03 Hz.

c) Since area C does not belong to the same synchronous grid as area A, the frequency remains the same, i.e., it is still exactly 50 Hz.

d) Since area D is part of the same synchronous grid as area A, the frequency must be the same, i.e., 50.03 Hz.

Problem 4

a) maximise $\sum_{t=1}^{24} \lambda_t \sum_{i=1}^{5} \sum_{j=1}^{2} \mu_{i,j} Q_{i,j,t}$

$+ \lambda_{25}((\gamma_1 + \gamma_4 + \gamma_5)M_{1,24} + (\gamma_2 + \gamma_4 + \gamma_5)M_{2,24} + (\gamma_3 + \gamma_5)M_{3,24} + (\gamma_4 + \gamma_5)M_{4,24} + \gamma_5 M_{5,24})$.

b) The following data are given in the problem text:

$\overline{Q}$ = maximal discharge in Spränget = 240,
$\hat{Q}$ = discharge in Spränget at best efficiency = 180,
$\ddot{H}$ = generation in Spränget at best efficiency = 117,
$\overline{H}_i$ = maximal generation in Spränget = 153.

The marginal production equivalents can now be calculated according to

$$\mu_1 = \frac{\ddot{H}}{\hat{Q}}$$

and

$$\mu_2 = \frac{\overline{H} - \ddot{H}}{\overline{Q} - \hat{Q}},$$

which results in the following linear models of the power plant:

μ_j = marginal production equivalent in Spränget, segment j =

$$= \begin{cases} 0{,}65 & j = 1, \\ 0{,}6 & j = 2, \end{cases}$$

$$\overline{Q}_j = \text{maximal discharge in Spränget, segment } j = \begin{cases} 180 & j = 1, \\ 60 & j = 2. \end{cases}$$

c) $u_t - u_{t-1} \le s_t^+$.

Problem 5

a) 5.

b) The expected generation in the three power plants can be calculated as follows:

$EG_1 = EENS_0 - EENS_1 = 210$ MWh/h,
$EG_2 = EENS_1 - EENS_2 = 170$ MWh/h,
$EG_3 = EENS_2 - EENS_3 = 118$ MWh/h.

The expected operation cost is then $ETOC = 10 \cdot 210 + 15 \cdot 170 + 20 \cdot 118 = 7\,010$ ¤/h.

c) $LOLP = \tilde{F}_4(1\,000) = 0.1\tilde{F}_3(1\,000) + 0.4\tilde{F}_3(950) + 0.5\tilde{F}_3(900) =$
$= 0.1 \cdot 0.05 + 0.4 \cdot 0.1 + 0.5 \cdot 0.2 = 14.5\%$.

d) We start by computing the expectation value for each stratum. In practice there is no need to differentiate between observations based on the original scenarios and the complementary scenarios; hence, we get the following estimates:

$$m_{Xh} = \frac{1}{200}\left(\sum_{i=1}^{100} g(y_i) + \sum_{i=1}^{100} g(y_i^*)\right) = \begin{cases} 10\,000 & h = 1, \\ 14\,500 & h = 2, \\ 40\,000 & h = 3. \end{cases}$$

We can now combine the results of each stratum weighted by their stratum weights:

$$m_X = \sum_{h=1}^{3} \omega_h m_{Xh} = 0.95 \cdot 10\,000 + 0.01 \cdot 14\,500 + 0.04 \cdot 40\,000 = 11\,245 \text{ ¤/h}.$$

e) 3.

Problem 6

a) The maximal generation capacity is 11 500 MWh/h during peak load periods. The electricity price must then be high enough to reduce the demand to 11 500 MWh/h; this level will be reached when the electricity price is 1 000 ¤/MWh.

During winter time the electricity price will be less than 1 000 ¤/MWh, which means that the demand is 9 500 MWh/h. Hence, all hydro power and nuclear power as well as 1 000/2 500 = 40% of the fossil fuelled generation will be used. This means that 40% of the price interval for fossil fuels is used, i.e., the electricity price will be 400 ¤/MWh.

During summer time the electricity price will be less than 1 000 ¤/MWh, which means that the demand is 6 500 MWh/h. Hence, all hydro power and nuclear power as well as 500/2 500 = 20% of the fossil fuelled generation will be used. This means that 20% of the price interval for fossil fuels is used, i.e., the electricity price will be 350 ¤/MWh.

b) The variable costs of the gas turbine are so high that it will only be used during peak load periods. Including the gas turbine, the generation capacity will be 11 625 MWh/h at peak load, which reduces the electricity price to 950 ¤/MWh. The income of sold electricity is then 950 ¤/MWh · 125 MWh/h · 60 h/year = 7.125 M¤/year. The total costs are 800 · 125 · 60 (variable costs) + 42 000 000 (fixed costs) = 48 M¤/year. Such a gas turbine is consequently far from profitable—then the electricity prices during peak load periods would have to be much higher.

Problem 7

To manage an outage of 1 000 MW without a frequency decrease larger than 0,4 Hz it is necessary to have a gain equal to 1 000/0.4 = 2 500 MW/Hz. The existing disturbance reserve is however only 2 000 MW/Hz. The margins on the transmission lines must be considered when selecting bids corresponding to the remaining 500 MW/Hz. The worst possible case is when the entire trading capacity to the south is used, at the same time as there is a load increase corresponding to the entire normal operation reserve (2 000 · 0.1 = 200 MW) in the southern price area and the dimensioning fault occurs. In this case, the flow from the northern to the central price area os increased by 150 (normal operation reserve) + 300 (existing disturbance reserve) = 450 MW. As the margin on this interconnection is 500 MW, the extra disturbance reserve may not be larger than 50/0.4 = 125 MW. Thus, it is only possible to accept the least expensive bid from the northern price area. The flow from the central to the southern price area is then increased by 500 (increase of the flow from north including the extra disturbance reserve) + 30 (normal operation reserve) + 220 (existing disturbance reserve) = 750 MW. In this case there is 800 MW margin on the interconnection, which means that also in this price area it is only possible to accept one bid. The remaining disturbance reserve must therefore be located in the southern price area. Thus, Riksnät should accept bids 1, 2, 4 and 6.

Problem 8

a) The problem we want to solve is

maximise *value of stored water*,

subject to *hydrological balance for the reservoirs*,

minimal flow downstream of the power plants,

load balance (bids accepted by ElKräng must be fulfilled),

limitations in reservoirs, discharge and spillage.

Indices for the power plants

Språnget 1, Fallet 2, Strömmen 3.

Parameters

Most parameters are defined in table 12 in the problem text. In addition to that, we also need to calculate the average production equivalents according to the instruction in the problem:

γ_i = expected future production equivalent in power plant i =

$$= \frac{\mu_{i,1}Q_{i,1} + \mu_{i,2}Q_{i,2}}{Q_{i,1} + Q_{i,2}} \approx \begin{cases} 0.657 & i = 1, \\ 0.669 & i = 2, \\ 0.383 & i = 3. \end{cases}$$

Optimisation variables

$Q_{i,j,t}$ = discharge in power plant i, segment j, during hour t, i = 1, 2, 3, j = 1, 2, t = 1, …, 24,
$S_{i,t}$ = spillage from reservoir i during hour t, i = 1, 2, 3, t = 1, …, 24,
$M_{i,t}$ = contents of reservoir i at the end of hour t, i = 1, 2, 3, t = 1, …, 24,

Objective function

maximise $\lambda_f((\gamma_1 + \gamma_3)M_{1,24} + (\gamma_2 + \gamma_3)M_{2,24} + \gamma_3 M_{3,24})$.

Constraints

Hydrological balance for Språnget and Fallet:

$$M_{i,t} = M_{i,t-1} - Q_{i,1,t} - Q_{i,2,t} - S_{i,t} + V_{i,t}, \qquad i = 1, 2, t = 1, \dots 24.$$

Hydrological balance for Strömmen:

$$M_{3,t} = M_{3,t-1} - Q_{3,1,t} - Q_{3,2,t} - S_{3,t} + Q_{1,1,t} + Q_{1,2,t} + S_{1,t} + Q_{2,1,t} + Q_{2,2,t} + S_{2,t} + V_{3,t}, \qquad t = 1, \dots 24.$$

Minimal flow downstream of the power plants:

$$Q_{i,1,t} + Q_{i,2,t} + S_{i,t} \geq \underline{Q}_i, \qquad i = 1, 2, 3, t = 1, \dots 24.$$

Load balance:

$$\sum_{i=1}^{3} \sum_{j=1}^{2} \mu_{i,j} Q_{i,j,t} \geq D_t, \qquad t = 1, \dots, 24.$$

Variable limits

$0 \leq Q_{i,j,t} \leq \overline{Q}_{i,j}$, i = 1, 2, 3, j = 1, 2, t = 1, …, 24,

$0 \leq S_{i,t}$, i = 1, 2, 3, t = 1, …, 24,

$0 \leq M_{i,t} \leq \overline{M}_i$, i = 1, 2, 3, t = 1, …, 24.

b) The difference compared to the previous part is that we must now have separate hydrological balance constraints for the different inflow alternatives and that we will not know the exact amount of water stored in the reservoirs at the end of the period; hence, we will have to maximise the value of the expected final reservoir contents.

First, we introduce index values for the different forecasts:

$p = 1$ original forecast,

$p = 2$ alternative forecast 1,
$p = 3$ alternative forecast 2.

Then we introduce the following new parameters:

$M_{i,0,p}$ = start contents of reservoir i if forecast p occurs = $M_{i,0}$,

$$\pi_p = \text{probability that forecast } p \text{ occurs} = \begin{cases} 0.5 & p = 1, \\ 0.25 & p = 2, \\ 0.25 & p = 3. \end{cases}$$

We also introduce new optimisation variables according to the hint:

$M_{i,t,p}$ = contents of reservoir i at the end of hour t if forecast p occurs, $i = 1, 2, 3$, $t = 1, \ldots, 24$, $p = 1, 2, 3$.

New objective function:

$$\text{maximise} \quad \sum_{p=1}^{3} \pi_p \lambda_f((\gamma_1 + \gamma_3)M_{1,24,p} + (\gamma_2 + \gamma_3)M_{2,24,p} + \gamma_3 M_{3,24,p}).$$

New hydrological balance for Språnget and Fallet:

$$M_{i,t,p} = M_{i,t-1,p} - Q_{i,1,t} - Q_{i,2,t} - S_{i,t} + V_{i,t,p}, \qquad i = 1, 2, t = 1, \ldots 24, p = 1, 2, 3,$$

New hydrological balance for Strömmen:

$$M_{3,t,p} = M_{3,t-1,p} - Q_{3,1,t} - Q_{3,2,t} - S_{3,t} + Q_{1,1,t} + Q_{1,2,t} + S_{1,t} + Q_{2,1,t} + Q_{2,2,t} + S_{2,t} + V_{3,t,p}, \qquad t = 1, \ldots 24, p = 1, 2, 3.$$

New variable limits:

$$0 \le M_{i,t,p} \le \overline{M}_i, \qquad i = 1, 2, 3, t = 1, \ldots, 24, p = 1, 2, 3.$$

Problem 9

a) It would not be sufficient to only apply probabilistic production cost simulation and completely neglect the interconnection between Berg and Hamn. An alternative would be to perform three separate simulations for different states of the available transmission capacity; the system is simulated as usual when 200 MW is available, consider Berg as a 100% available power plant with 100 MW installed capacity if 100 MW transmission is available, and if no transmission is available when there will be no power plants in the system. Another alternative is to perform a Monte Carlo simulation using control variates or stratified sampling. (It would also be possible to use a combination of control variates and stratified sampling, but that can be quite complicated if it is to be done in an efficient manner.) The solution below will apply stratified sampling.

Introduce the following index values for the areas: 1 – Hamn, 2 – Vik. As we do not have any load in the area Berg, we may exclude it from the model and take for granted that the transmission from Berg to Hamn is always equal to the generation in Berg. Moreover, we can observe that the power flow will always go from Berg to Hamn to Vik., and there is no need to model flows in the other direction. This results in the following electricity market model:

minimise $U_1 + U_2$,

subject to $W - 5 \cdot 10^{-4} W^2 = D_1 - U_1 + P$, (load balance of Hamn)

$P - 8 \cdot 10^{-4} P^2 = D_2 - U_2$, (load balance of Vik)

$0 \le P \le 40$,

$0 \le U_n \le D_n$, $n = 1, 2$,

$0 \le W \le \min(\overline{W}, \overline{P})$. ,

In this optimisation problem, we have the following scenario parameters:

D_n = load in area n, $n = 1, 2$,
$\overline{P}$ = maximal transmission from Berg to Hamn,
$\overline{W}$ = maximal total generation in hydro and wind.

When solving the optimisation problem (which can be done by simple manual calculations) we get or may compute the following result variables:

ENS = unserved energy = $U_1 + U_2$,

$$LOLO = \text{loss of load occasion} = \begin{cases} 0 & \text{if } ENS = 0, \\ 1 & \text{if } ENS > 0, \end{cases}$$

P = transmission from Hamn to Vik,
U_n = unserved power in area n, $n = 1, 2$,
W = total generation in hydro and wind.

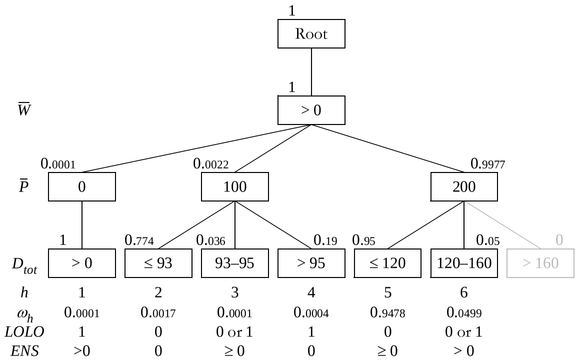
The strata can be defined using a strata tree. In the first level below the root we put the available generation capacity. Since available capacity is a continuous random variable in this case, we must state an interval for the possible values in each node. However, it is not easy to identify any intervals which makes it possible to predict the result variables $LOLO$ and ENS; therefore, we put all possible outcomes in one node, which then will have the node weight 1.

On the next level we put the available transmission capacity. Here we have three discrete outcomes, which we put in three separate nodes. The node weights are calculated in the following manner:

- There is a 10% probability that there is no transmission capacity during a storm (0.1% probability) ⇒ the node weight is 0.001·0.1 = 0.01%.
- There is a 0.2% probability that only one line is available during normal weather (99.9% probability) and a 20% probability during a storm (0.1% probability) ⇒ the node weight is 0.999·0.002 + 0.001·0.2 = 0.22%.
- There is a 99.8% probability that both lines are available during normal weather (99.9% probability) and a 70% probability during a storm (0.1% probability) ⇒ the node weight is 0.999·0.998 + 0.001·0.7 = 99.77%.

In the last level we put the total load. If there is no transmission capacity available then the total load has no importance and all outcomes of the total load can be placed in a node with the weight 1. There will always be possible to generate 100 MW in Berg when one line is available, but at most 95 MW can be transferred to Hamn, because the losses are $5 \cdot 10^{-4} \cdot 100^2 = 5$ MW if the flow is 100 MW between Berg and Hamn. There will not be any problem to supply a load that is less than 100 MW minus the maximal losses (which can be calculated according to $\bar{L}_{100} = 5 \cdot 10^{-4} \cdot 100^2 + 8 \cdot 10^{-4} \cdot 50^2 = 7$ MW). Load shedding cannot be avoided if the demand is larger than 95 MW. The results for the interval in between depend on the losses. The entire generation capacity in Berg can be transferred to Hamn when both overhead lines are available. However, in worst case the load in Vik is larger than 48 MW and then load shedding is unavoidable. This can only happen if the total load is larger than 48/0.4 = 120 MW, because the load in Vik can at most be 40% of the total load. There will not be any problem to supply the if it is less than this and if the load exceeds 160 MW then load shedding is unavoidable. For the interval in between, load shedding might occur due to transmission losses or the limitations in the cable between Hamn and Vik. The node weights on this level are computed using the given total load duration curve, $\tilde{F}_D(x)$.

The resulting strata tree with stratum weights and possible values of the result variables is shown in the figure below.



b) When computing $LOLP$ it is only strata 3 and 6 that are interesting, because it is only in those strata that there is an uncertainty regarding the result. For practical reasons, we may choose to generate ten scenarios each for these strata. The available generation capacity can be taken directly from the corresponding probability distribution in table 14. The probability distribution of the total load is different in the two strata and therefore we use uniformly distributed random numbers and the inverse transform method to generate these values. The share of the load that is in Hamn is assumed to be uniformly distributed between 60 and 80%; these values are also generated using the uniformly distributed random numbers. The resulting scenarios are shown in the table below. (The values are rounded and may therefore seem to be slightly incorrect.)

Scenario	1	2	3	4	5	6	7	8	9	10
Stratum 3										
$\overline{W}$ [MW]	152	154	141	155	147	141	143	145	157	158
$\overline{P}$ [MW]	100	100	100	100	100	100	100	100	100	100
D_{tot} [MW]	94	93	94	94	94	93	94	94	95	95
D_1 [MW]	62	70	70	64	67	74	64	62	59	58
D_2 [MW]	32	23	23	31	27	19	30	33	35	36
$LOLO$	1	0	0	1	0	0	0	1	1	1
Stratum 6										
$\overline{W}$ [MW]	142	158	157	145	151	141	144	155	151	158
$\overline{P}$ [MW]	200	200	200	200	200	200	200	200	200	200
D_{tot} [MW]	153	154	139	132	127	155	147	151	130	148
D_1 [MW]	96	110	97	79	85	117	112	95	94	92
D_2 [MW]	57	44	43	52	42	38	35	56	36	56
$LOLO$	1	1	0	1	0	1	1	1	0	1

The expectation value per stratum can now be computed or estimated as follows:

$$m_{LOLO1} = 1,\ m_{LOLO2} = 0,\ m_{LOLO3} = 0.5,\ m_{LOLO2} = 1,\ m_{LOLO2} = 0,\ m_{LOLO2} = 0.7.$$

The results of the strata are weighted according to their stratum weights:

$$m_{LOLO} = \sum_{h=1}^{3} \omega_h m_{LOLOh} = 0.0001\cdot 1 + 0 + 0.0001\cdot 0.5 + 0.0004\cdot 1 + 0 + 0.0499\cdot 0.7 \approx 3.55\%.$$

c) Before we start estimating the expected generation in the diesel genset using Monte Carlo simulation, we make a rough estimate using probabilistic production cost simulation. The expected generation can be computed as

$$EG = 0.9 \cdot T \int_{160}^{162.5} \tilde{F}_E(x)dx \approx 0.9 \cdot 8\,760 \cdot \frac{0.02 + 0.012}{2} 2.5 = 315 \text{ MWh/year.}$$

One MWh generated in the diesel genset will cost 100 ¤/MWh and reduces the social cost by 1 000 ¤/MWh, i.e., each MWh provides a benefit to the society equal to 900 ¤/MWh. The total benefit in a year is about 315 · 900 = 283 500 ¤/year. Although probabilistic production cost underestimates the need for the diesel genset (for example, the system without the diesel genset yields $LOLP = \tilde{F}_E(160) \approx 2\%$ compared to the result 3.55% from part b), but the difference between the benefit to the society and the investment cost is large enough to conclude that the diesel genset is not profitable for the society.