KTH Electrical Engineering

Exam in EG2050 System Planning,
10 June 2011, 8:00–13:00, V34, V35

Allowed aids

In this exam you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten**, **single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

PART I (MANDATORY)

Write all answers on the answer sheet provided. Motivations and calculations do not have to be presented.

Part I can yield 40 points in total. The examinee is guaranteed to pass if the score is at least 33 points. If the result in part I is at least 31 points, then there will be a possibility to complement for passing the exam with the grade E.

Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

a) (2 p) A balance responsible player has the following responsibilities: I) Physical responsibility that the system continuously is supplied as much power as consumed by the customers of the player, II) Economical responsibility that the system continuously is supplied as much power as consumed by the customers of the player, III) Economical responsibility that the system during each trading period (for example one hour) is supplied as much energy as consumed by the customers of the player.

1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. I and II are true but not III.

b) (2 p) The consumers can choose which retailer they want in the following types of electricity markets: I) Vertically integrated electricity market, II) Centralised electricity market, III) Bilateral electricity market.

1. Only I is true.
2. Only II is true.
3. Only III is true.
4. I and III are true but not II.
5. II and III are true but not I.

Problem 2 (6 p)

Consider the common electricity market of the two countries Rike and Maa. Assume that there is perfect competition, that all players have perfect information, and that there are neither reservoir nor capacity limitations in the power plants. The power systems of Rike and Maa are interconnected by an HVDC line, which can transfer a certain amount of TWh per year. Data for the common electricity market are shown in table 1. The variable production costs are assumed to be linear in the intervals, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

Table 1 Data for the electricity market in Rike and Maa.

Power source	Production capability [TWh/year]		Variable cost [¤/MWh]
	Rike	Maa	
Hydro power	50	10	30–60
Nuclear power	50	20	120
Coal condensing	10	20	300–480
Gas turbines	5	5	800–1 000
Electricity consumption [TWh/year]	100	50	

a) (2 p) Assume that the electricity price in Rike is 408 ¤/MWh, whereas the electricity price in Maa is 426 ¤/MWh. How large is the transmission capacity between Rike and Maa?

b) (2 p) The power company Strålinge owns a nuclear power plant in Rike. The power plant has a production capability of 10 TWh per year and the fixed costs are 2 900 M¤/year. How large is the profit of the company?

c) (2 p) Assume that the transmission capacity between Rike and Maa is upgraded to 10 TWh/year. The investment cost is paid by the system operators in Rike and Maa, and the cost is transferred to the players of the electricity market through the grid fees. This means that the fixed costs of Strålinge increases to 2 920 M¤/year. What is the profit of Strålinge then?

Problem 3 (6 p)

The hydro power plant Forsen is one of the units which participate in the primary control of Rike. At nominal frequency (50 Hz) Forsen is generating 120 MW. The gain in Forsen is set to 200 MW/Hz and it is available within the frequency range 50 ± 0.1 Hz.

a) (3 p) At a certain occasion, the power system in Rike has a total gain (i.e., including the gain in Forsen) of 2 560 MW/Hz and the frequency is 50.01 Hz. How much is generated in Forsen at this occasion?

b) (3 p) At the occasion described in part a, a failure occurs in a transformer at Forsen and the power plant is disconnected from the grid. What will the frequency be when the primary control has restored the balance between generation and consumption? Answer with three decimals!

Problem 4 (12 p)

Stads energi AB owns a thermal power plant with three blocks. Assume that the company has formulated their short-term planning problem as a MILP problem and that the following symbols have been introduced:

Indices for the power plants: Block I - 1, Block II - 2, Block III - 3.

β_{Gg} = variable operation cost in power plant g,
C_g^* = start-up cost of power plant g after one hour down-time, g = 1, 2, 3,
C_g^{**} = start-up cost of power plant g after at least two hours down-time, g = 1, 2, 3,
D_t = contracted load during hour t, t = 1, …, 24,
$G_{g,t}$ = generation in power plant g, hour t, g = 1, 2, 3, t = 1, …, 24,
p_t = purchase from ElKräng hour t, t = 1, …, 24,
r_t = sales to ElKräng hour t, t = 1, …, 24,
$s_{g,t}^*$ = start-up of power plant g, hour t, after one hour down-time, g = 1, 2, 3, t = 1, …, 24,
$s_{g,t}^{**}$ = start-up of power plant g, hour t, after at least two hours down-time, g = 1, 2, 3, t = 1, …, 24,
$u_{g,0}$ = unit commitment of power plant g at the beginning of the planning period, g = 1, 2, 3,
$u_{g,t}$ = unit commitment of power plant g, hour t, g = 1, 2, 3, t = 1, …, 24.

a) (3 p) Which of the symbols above represent optimisation variables and parameters respectively?

b) (4 p) Assume that Stads energi AB sells power to customers with firm power contracts, and is also trading at the local power exchange ElKräng, where the company has the possibility to both sell and purchase electricity. Formulate the load balance constraint of Stads energi AB. Use the symbols defined above.

c) (2 p) Two linear constraints are needed in order to get correct values of $s_{g,t}^*$ and $s_{g,t}^{**}$. The first constraint forces $s_{g,t}^{**}$ to be equal to one if the power plant is committed in hour t, while it has not been committed during hours $t - 1$ and $t - 2$ respectively:

$$s_{g,t}^{**} \geq u_{g,t} - u_{g,t-1} - u_{g,t-2}.$$

The second constraint should force $s_{g,t}^*$ to be equal to one if the power plant has been committed in hours t and $t - 2$ respectively, but has not been committed during hour $t - 1$. Which of the following relations can be used for the second constraint?

I) $s_{g,t}^* \geq u_{g,t} - u_{g,t-1} - s_{g,t}^{**}$.

II) $s_{g,t}^* \geq u_{g,t} - u_{g,t-1} - u_{g,t-2}$.

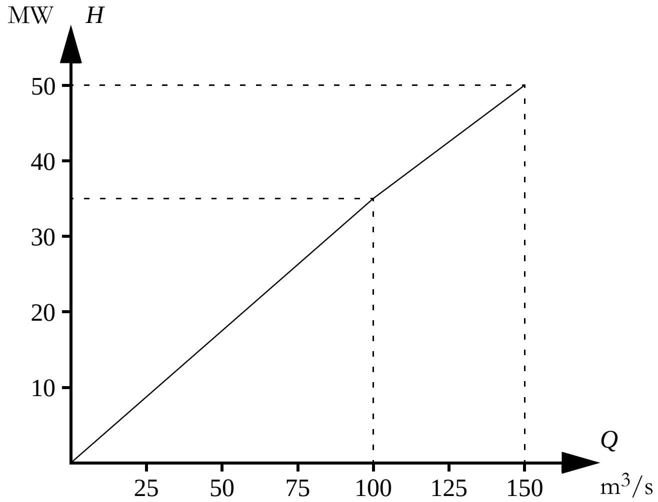
III) $s_{g,t}^* - u_{g,t-2} \geq u_{g,t} - u_{g,t-1}$.

1. None of the alternatives is correct.
2. Only alternative I is correct.
3. Only alternative II is correct.
4. Only alternative III is correct.
5. One can choose between using alternative I and alternative II.

d) (1 p) The best efficiency of the power plant Strömmen is achieved at the discharge 100 m^3/s. The power plant is then generating 35 MW. How large is the maximal production equivalent of the power plant?

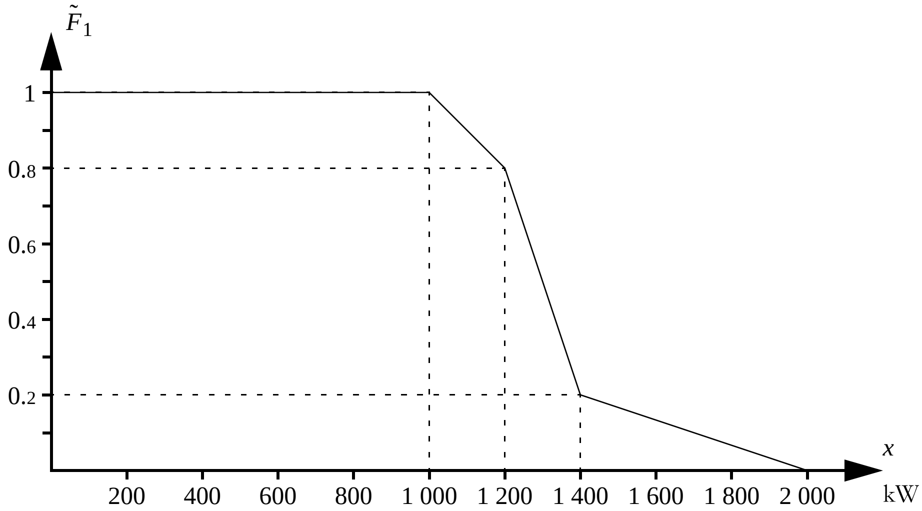
e) (2 p) The figure below shows a piecewise linear model of the electricity generation as a function of the discharge in Strömmen. Calculate the following parameters:

μ_j = marginal production equivalent in Strömmen, segment j,
$\overline{Q}_j$ = maximal discharge in Strömmen, segment j.



Problem 5 (12 p)

Mji is a town in East Africa. The town is not connected to a national grid, but has a local system of its own. The local grid is supplied by a hydro power plant and two diesel generator sets. The hydro power plant does not have a reservoir, but the water flow is always sufficient to generate the installed capacity (1 400 kW) and the risk for outages in the power plant is negligible. The diesel generator sets has a capacity of 150 kW each, the availability is 75% and the operation cost is 1 ¤/kWh.



a) (2 p) What is the expectation value of the load?
Hint: Study $EENS_0$!

b) (2 p) Using probabilistic production cost simulation it is calculated that the expected hydro power generation of this system is 1 280 kWh/h and that the unserved energy is 24.8 kWh/h. How large is the expected operation cost?

c) (3 p) Use probabilistic production cost simulation to calculate the risk of power deficit in the system.

d) (2 p) Generate a value of the load using the inverse transform method and the random number 0.35 from a $U(0, 1)$-distribution.

e) (3 p) To consider the losses in the grid, a Monte Carlo simulation has been performed of the power system in Mji. The simulation is using control variates. The simplified model corresponds to the model used in probabilistic production cost simulation, whereas the detailed model considers such factors as how the losses depends on which power plants that are operated, and how the load is distributed within the system. The results are shown in table 2. Which estimate of *LOLP* is obtained for the detailed model when using the control variate method?

Table 2 Results from a Monte Carlo simulation of the power system in Mji.

Number of scenarios, n	Results from the detailed model, $\sum_{i=1}^{n} lolo_i$	Results from the simplified model, $\sum_{i=1}^{n} \tilde{lolo}_i$
2 000	286	212

PART II (FOR HIGHER GRADES)

All introduced symbols must be defined. Solutions should include sufficient detail that the argument and calculations can be easily followed.

The answer to each problem must begin on a new sheet, but answers to different parts of the same problem (a, b, c, etc.) can be written on the same sheet. The fields *Namn* (Name), *Blad nr* (Sheet number) and *Uppgift nr* (Problem number) must be filled out on every sheet.

Part II gives a total of 60 points, but this part will only be marked if the candidate has obtained at least 33 points in part I. Then the results of parts I and II and the bonus points will be added together to determine the examination grade (A, B, C, D, E).

Problem 6 (10 p)

The power exchange ElKräng allows the following types of bids:

- **Sell bid.** A sell bid is valid for a specific hour and comprises a specific volume in MWh and a price interval in SEK/MWh. If the electricity price in the corresponding hour is higher than the highest price in the bid then the bid is accepted in whole. If the electricity price however is lower than the least price in the bid then the bid is rejected. If the electricity price is within the specified price range, then a linear relation is assumed between the electricity price and the quantity that is accepted. (If the price for example is in the middle of the price range then half of the bid is accepted.)
- **Purchase bid.** A purchase bid is valid for a specific hour and comprises a specific volume in MWh and a price interval in SEK/MWh. If the electricity price in the corresponding hour is lower than the least price in the bid then the bid is accepted in whole. If the electricity price however is higher than the highest price in the bid then the bid is rejected. If the electricity price is within the specified price range, then a linear relation is assumed between the electricity price and the quantity that is accepted. (If the price for example is in the middle of the price range then half of the bid is accepted.)
- **Block bid.** A block bid is a special type of sell bid, which is valid for several hours. The bid comprises a specific volume in MWh and a specific price in SEK/MWh. If the average price during the hours for which the bid is comprising is higher than the price in the bid then the bid is accepted in whole; otherwise, the bid is rejected.

The electricity price at ElKräng is calculated by combining sell and purchase bids into a supply and a demand curve for each hour. The electricity price for the corresponding hour is set by the intersection between the curves. Then the price for each hour has been calculated this way, it is checked if the conditions of the block bids are fulfilled. If that is the case, the block bids are included in the beginning of the supply curve (i.e., the bid price of the block bids is assumed to be zero in this calculation) and the price calculation is remade. After this computation, where is a risk that the prices have changed so that the conditions for some block bids are no longer fulfilled. In that case these block bids are removed and the price calculation is remade again.

Table 3 shows the bids to ElKräng for two hours. What are the resulting electricity prices for these two hours?

Table 3 Bids to ElKräng.

Bid	Volume [MWh]		Price interval [SEK/MWh]	
	Hour 1	Hour 2	Hour 1	Hour 2
Sell bid 1	1 500	1 500	450–600	450–600
Sell bid 2	800	800	520–680	520–680
Sell bid 3	500	500	540–640	540–640
Purchase bid 1	750	1 100	8 000–10 000	8 000–10 000
Purchase bid 1	850	1 500	8 000–10 000	8 000–10 000
Purchase bid 1	150	350	425–525	425–525
Block bid 1	200 MWh/h		550	
Block bid 2	500 MWh/h		600	

Problem 7 (10 p)

The three countries Aland, Beland and Celand have a common electricity market. The countries are interconnected by AC lines as shown in the figure below; therefore, the countries also have a common system for frequency control.

The primary control is divided in a normal operation reserve and a disturbance reserve. The normal operation reserve is available in the frequency range 49.9–50.1 Hz and is intended to manage load variations. The countries have agreed on their contribution to the normal operation reserve according to table 4.

The maximal transmission capacity between the countries is depending partly on thermal limits (i.e., the components of the transmission lines could fail due to overheating) as well as voltage stability. The players of the electricity market may however not have access to the entire transmission capacity, because the frequency control may change the flow between the countries. How large marginal must be reserved on the transmission lines if it should be possible for the system to manage the load variations listed in table 4? (Please notice that the system should be able to manage the maximal load variations occurring simultaneously in all three countries and that the load variations are considered uncorrelated.)

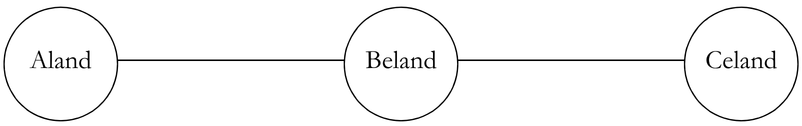
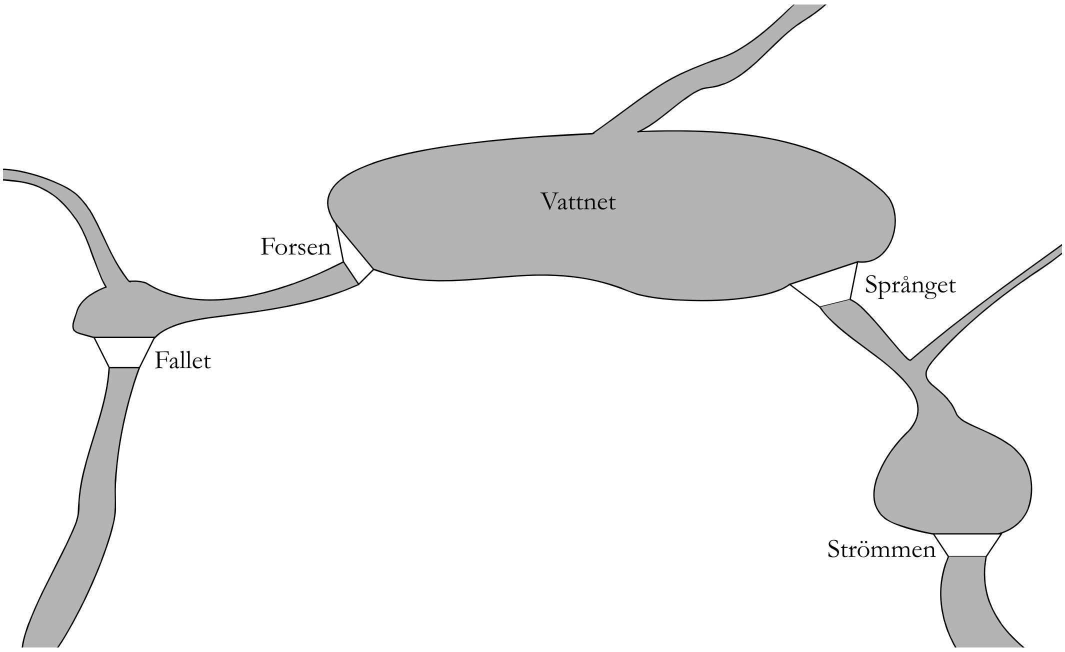


Table 4 Data for the three countries in problem 7.

Land	Contribution to normal operation reserve [MW/Hz]	Load variations [MW]
Aland	1 500	±120
Beland	2 000	±180
Celand	500	±90

Problem 8 (20 p)



AB Vattenkraft owns four hydro power plants, located as shown in the figure above. Data for the hydro power plants are given in tables 5 and 6. The electricity produced by the company is sold to the power exchange, ElKräng. The trading at ElKräng is divided in several price areas; the power plants Forsen and Fallet are located in price area 1, whereas Språnget and Strömmen are located in price area 2. It is assumed that the company can sell as much as they want to the prices listed in table 7. Stored water in Vattnet is assumed to be used for electricity generation in Språnget and Strömmen. The company expects the average future electricity prices to be 540 SEK/MWh in price area 1 and 550 SEK/MWh in price area 2. Stored water is assumed to be used for electricity generation at maximal efficiency. The water delay time between the power plants can be neglected.

a) (12 p) Formulate the planning problem of AB Vattenkraft as an LP problem. Use the notation in table 8 for the parameters (it is however permitted to add further symbols if you consider it necessary).

NOTICE! The following is required to get full score for this problem:

- The symbols for the optimisation variables must be clearly defined.
- The optimisation problem should be formulated so that it is easy to determine what the objective function is, which constraints there are and which limits there are.
- The possible values for all indices should be clearly stated for each equation.

b) (8 p) The efficiency of the hydro power plants is poor for low discharges. Moreover, there is risk that the turbines are damaged. How must the planning problem from part a be reformulated if AB Vattenkraft wants to avoid discharges below 20% of maximal discharge in all power plants? (It should still be possible to not produce anything in the hydro power plant, i.e., the discharge should either be zero or between 20% and 100% of maximal discharge.) Do not forget to define all new variables and parameters that you introduce!

Table 5 Data for the reservoirs of AB Vattenkraft.

Reservoir	Start contents of reservoir [HE]	Maximal contents of reservoir [HE]	Local inflow [HE]
Vattnet	5 000	8 000	300
Fallet	500	900	10
Strömmen	200	500	18

Table 6 Data for the hydro power plants of AB Vattenkraft.

Power plant	Marginal production equivalents [MWh/HE]		Maximal discharge [HE]	
	Segment 1	Segment 2	Segment 1	Segment 2
Forsen	0.64	0.59	180	65
Fallet	0.47	0.43	180	70
Språnget	0.78	0.69	200	175
Strömmen	0.32	0.28	190	100

Table 7 Expected prices at ElKräng.

Hour	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Price at ElKräng, price area 1 [SEK/MWh]	540	535	535	535	535	540	555	560	590	585	575	570
Price at ElKräng, price area 2 [SEK/MWh]	530	525	525	530	530	540	560	575	615	620	595	585
Hour	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24
Price at ElKräng, price area 1 [SEK/MWh]	565	570	560	550	545	545	550	550	545	530	525	525
Price at ElKräng, price area 2 [SEK/MWh]	580	580	575	560	555	550	580	580	565	535	515	510

Table 8 Notation for the planning problem of AB Vattenkraft.

Symbol	Explanation	Value
$M_{i,0}$	Start contents of reservoir *i*	See table 5
$\overline{M}_i$	Maximal contents of reservoir *i*	Se table 5e
V_i	Local inflow to reservoir *i*	See table 5
$\mu_{k,j}$	Marginal production equivalent in power plant *k*, segment *j*	See table 6
$\overline{Q}_{k,j}$	Maximal discharge in power plant *k*, segment *j*	See table 6
$\lambda_{n,t}$	Expected price at ElKräng, price area *n*, hour *t*	See table 7
λ_{f1}	Expected future electrictiy price in price area 1	540
λ_{f2}	Expected future electrictiy price in price area 2	550

Problem 9 (20 p)

The Ström family has recently moved to a house with electrical heating. Due to strained private economics, they are slightly worried about the electricity bills for the coming winter. Therefore, the family wants to calculate the estimated electricity cost for the period December to February.

The family has a take-and-pay contract with AB Elleverantören, where they pay a fixed price of 81 öre/kWh. In addition to that they have to pay the variable grid fee (19 öre/kWh) and energy tax (28 öre/kWh), as well as 25% VAT on the total amount (i.e., the electricity price plus variable grid fee plus energy tax). Besides this, the family have installed a small wind power plant—which earlier served the family's holiday house—on their garden. The maximal generation in the wind power plant is 2 kW and the generated electricity can be used to reduce the need for buying electricity from AB Elleverantören via the grid. If the wind power plant generates more than what is consumed in the house, the excess will be injected to the grid, but the family will not be paid for this surplus.

a) (10 p) The figures on the next page show the duration curves of the consumption, $\tilde{F}_D(x)$, and the wind power generation $\tilde{F}_W(x)$, for the period December to February. The probability that the load exceeds 7 kW depend on the average temperature during the period according to

$$\pi = \begin{cases} 0.1 - 0.01T & T \geq -2, \\ 0.12 - 0.02(T + 2) & T \leq -2, \end{cases}$$

where

π = probability that the load exceeds 7 kW = $\tilde{F}_D(7)$,
T = average temperature December to February [°C].

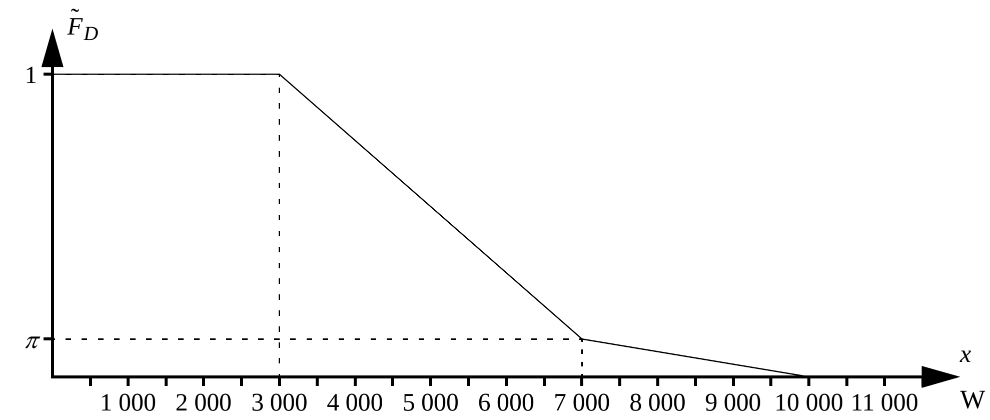
Determine the expected electricity bill of the family for the three months as a function of the average temperature.

b) (10 p) Assume that the average temperature is normally distributed with the mean 0°C and the standard deviation 1°C. Estimate the expected electricity bill of the family for the three months.

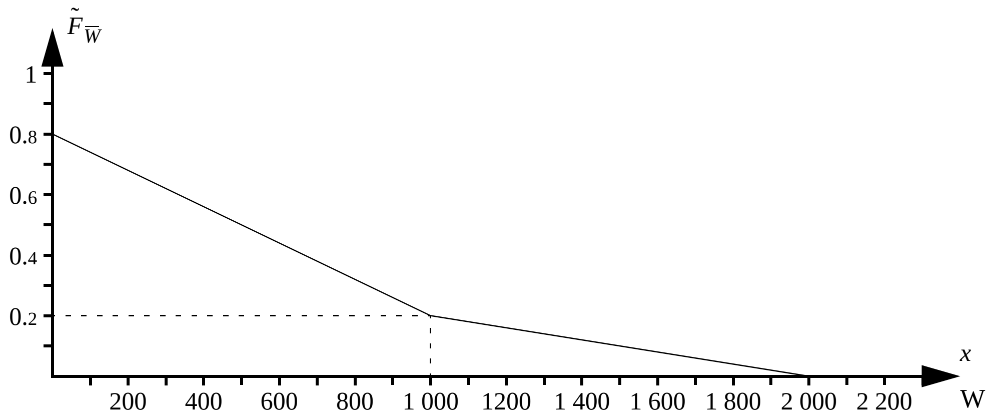
NOTICE! To get full score for this problem it is necessary to use some kind of variance reduction technique!

Table 9 Random numbers from an $N(0, 1)$-distribution.

0.5	–2.3	0.3	–0.4	3.6
1.8	0.9	–1.3	0.3	2.8



$\tilde{F}_D$
1
π
1 000
2 000
3 000
4 000
5 000
6 000
7 000
8 000
9 000
10 000
11 000
x
W



$\tilde{F}_{\overline{W}}$
1
0.8
0.6
0.4
0.2
200
400
600
800
1 000
1200
1 400
1 600
1 800
2 000
2 200
x
W

KTH Electrical Engineering

Answer sheet for part I

Name: ....

Personal number: ....

Problem 1

a) Alternative .... is correct.

b) Alternative .... is correct.

Problem 2

a) .... TWh/year **b)** .... M¤/year

c) .... M¤/year

Problem 3

a) .... MW **b)** .... Hz

Problem 4

a) Parameters: ....

Optimisation variables: ....

b) ....

c) Alternative .... is correct.

d) .... MWh/HE

e) μ_1 .... MWh/HE μ_2 .... MWh/HE

$\overline{Q}_1$ .... HE $\overline{Q}_2$ .... HE

Problem 5

a) .... kWh/h **b)** .... ¤/h

c) .... % **d)** .... kW

e) .... %

Problem 1

a) 4, **b)** 3.

Problem 2

a) The electricity price is lower in Rike than in Maa; hence, the interconnection between Rike and Maa will be fully utilised (if there was no transmission limitation, the price would be the same in both countries). The total generation in Rike is 106 TWh/year, whereas the consumption is 100 TWh/year. The surplus is exported to Maa. Thus, we can conclude that the maximal transmission capacity is 6 TWh/year.

b) The income of Strålinge amounts to 10 TWh/year · 408 ¤/MWh = 4 080 M¤/year. From the income we subtract the total variable costs (10 TWh/year · 120 ¤/MWh = 1 200 M¤/year) and the fixed costs, which gives a financial loss of 20 M¤/year.

c) Assume that the new transmission capacity is large enough to eliminate the price difference between the two countries. The total consumption of the two countries is 150 TWh. Hydro and nuclear can generate in total 130 TWh, which means that 20 TWh coal condensing will be needed. Hence, 20/30 of the price interval of coal condensing is utilised, i.e., the electricity price must be 300 + 20/30 · 180 = 420 ¤/MWh. At this price, the export from Rike is 50 (hydro) + 50 (nuclear) + 6.67 (coal condensing) – 100 (load) = 6.67 TWh, which means that the assumption about the price is correct. The resultat of Strålinge is then 420 · 10 – 120 · 10 – 2 920 = 80 M¤/year.

Problem 3

a) The generation in Forsen is calculated according to $G = G_0 - R(f - f_0)$ = 120 – 200(50.01 – 50) = 118 MW.

b) There is now a deficit of 118 MW, and at the same time the system gain is reduced to 2 360 MW/Hz. The frequency must then change by $\Delta f = \Delta G/R$ = 118/2 360 = 0.05 Hz. As we have a deficit, the frequency must decrease, i.e., the new frequency will be 50.01 – 0.05 = 49.96 Hz.

Problem 4

a) Parameters: β_{Gg}, C_g^*, C_g^{**}, D_t and $u_{g,0}$. Optimisation variables: $G_{g,t}$, p_t, r_t, $s_{g,t}^*$, $s_{g,t}^{**}$ and $u_{g,t}$

b) $\sum_{g=1}^{3} G_{g,t} + p_t = D_t + r_t$.

c) 2.

d) The maximal production equivalent is obtained at the discharge where we have the best efficiency. The definition of production equivalent then gives us $\gamma_{max} = H/Q$ = 0.35 MWh/HE.

e) The following data are given in the figure:

$\overline{Q}$ = maximal discharge in Strömmen = 150,

$\overline{H}_i$ = maximal generation in Strömmen =50,

$\hat{Q}$ = discharge in Strömmen at best efficiency = 100,

$\hat{H}$ = generation in Strömmen at best efficiency = 35.

The marginal production equivalents can now be calculated according to

$$\mu_1 = \frac{\hat{H}}{\hat{Q}}$$

and

$$\mu_2 = \frac{\overline{H} - \hat{H}}{\overline{Q} - \hat{Q}},$$

which results in the following linear models of the power plant:

μ_j = marginal production equivalent in Strömmen, segment j =

$$= \begin{cases} 0.22 & j = 1, \\ 0.18 & j = 2, \end{cases}$$

$$\overline{Q}_j = \text{maximal discharge in Strömmen, segment } j = \begin{cases} 70 & j = 1, \\ 30 & j = 2. \end{cases}$$

Problem 5

a) $E[D] = EENS_0 = \int_0^{\infty} \tilde{F}_0(x)dx$ = 1 000·1 + 200·(1 + 0.8)/2 + 200·(0.8 + 0.2)/2 + 600·0.2/2 = = 1 340 MWh/h.

b) The expected load is 1 340 kWh/h, which means that $EG_1 + EG_2 + EENS_2$ = 1 380. As EG_1 = 1 280 kWh/h and $EENS_2$ = 24.8 kWh/h it can be concluded that the expected generation in the diesel generator sets is 35.2 kWh/h. Hence, the exptected operation cost is $ETOC = ETOC$ = $1 \cdot (EG_2 + EG_3)$ = 35.2 ¤/h.

c) The risk of power deficit is given by

$$\tilde{F}_3(1\,700) = 0.75\tilde{F}_2(1\,700) + 0.25\tilde{F}_2(1\,550) =$$

$$= 0.75(0.75\tilde{F}_1(1\,700) + 0.25\tilde{F}_1(1\,550)) + 0.25(0.75\tilde{F}_1(1\,550) + 0.25\tilde{F}_1(1\,400)).$$

Since the hydro power plant has 100% availability, we get $\tilde{F}_1(x) = \tilde{F}_0(x)$, i.e., $LOLP$ = 0.75·(0.75·0.1 + 0.25·0.15) + 0.25·(0.75·0.15 + 0.25·0.2) = 12.5%.

d) If the given random number is denoted U then the load is calculated by $D = \tilde{F}_0^{-1}(U)$ = {use the figure} = 1 350 kW.

e) $m_{LOLO} = \eta_{(LOLO - \tilde{LOLO})} + \mu_{\tilde{LOLO}} = \frac{1}{n}\sum_{i=1}^{n}(lolo_i - \tilde{lolo}_i) + 0.125 = \frac{1}{2\,000}74 + 0.125$ = 16.2%.

Problem 6

Assume that the electricity price is higher than 525 SEK/MWh during both hours. In this case, purchase bids 1 and 2 will b e accepted, whereas purchase bid 3 is rejected; consequently, the demand will be 1 600 MWh in the first hour and 2 600 MWh in the second.

Assume that the electricity pridce in the first hour, λ_1, is in the interval 525–600 SEK/MWh. This means that all three sell bids are partically accepted. The accepted volume of each bid can be expressed as

$$\frac{\lambda_1 - 450}{600 - 450}1\,500 + \frac{\lambda_1 - 520}{680 - 520}800 + \frac{\lambda_1 - 540}{640 - 540}500.$$

Setting this volume to be equal to 1 600 MWh and solving the equation yields the electricity price 570 SEK/MWh for the first hour. The demand in the second hour is significantly larger and it therefore be assumed that sell bid 1 will be accepted in whole, which results in the following equation:

$$1\,500 + \frac{\lambda_2 - 520}{680 - 520}800 + \frac{\lambda_2 - 540}{640 - 540}500 = 2\,600.$$

Thus, we get the electricity price 640 SEK/MWh in the second hour.

Now it is possible to check if the requirements of the block bids are fulfilled. The average price of the two hours is 605 SEK/MWh, which means that block bid 1 is accepted but not block bid 2. The volumes that will be accepted of the three sell bids then become 1 400 and 2 400 MWh respectively. Redoing the calculations above with this demand results in the electricity price 60 SEK/MWh in the first hour and 620 SEK/MWh in the second hour. The average price of the two hours is then 590 SEK/MWh, which is still higher than the price in block bid 1, and then these prices are final.

Problem 7

The largest transmisison change on the line between Aland and Beland occurs when the load decreases in Aland while it increases in Beland and Celand. This results in a net load increase for the system, which means that there is less load in Aland while at the same time the generation in the power plants participating in primary control have increased; the total excess power must be exported on the transmission line between Aland and Beland. The size of the transmission change is computed below. (Notice that the direction of the load change does not have any importance, as long as the load in Aland is changed in a different direction compared to the load in Beland and Celand.)

$$\Delta D_{tot} = \Delta D_A + \Delta D_B + \Delta D_C = -120 + 180 + 90 = 150 \text{ MW}.$$

$$\Delta G_A = \frac{R_A}{R_{tot}}\Delta D_{tot} = 1\,500/4\,000 \cdot 150 = 56.25 \text{ MW}.$$

$$\Delta P_{AB} = \Delta G_A - \Delta D_A = 56.25 + 120 = 176.25 \text{ MW}.$$

Similarly, the largest change on the interconnection between Celand and Beland occurs when the load in Celand is changing in the opposite direction compared to the load in Aland and Beland:

$$\Delta D_{tot} = \Delta D_A + \Delta D_B + \Delta D_C = 120 + 180 - 90 = 210 \text{ MW}.$$

$$\Delta G_C = \frac{R_C}{R_{tot}}\Delta D_{tot} = 500/4\,000 \cdot 210 = 26.25 \text{ MW}.$$

$$\Delta P_{CB} = \Delta G_C - \Delta D_C = 26.25 + 90 = 116.25 \text{ MW}.$$

Thus, the conclusion is that at least176.25 MW transmission capacity must be reserved between Aland and Beland for the normal operation reserve, and between Beland and Celand, at least 116.25 MW must be reserved.

Problem 8

a) The problem we want to solve is

maximise	*value of sold electricity + value of stored water,*
subject to	*hydrological balance of the reservoirs,*
	limitations in reservoirs, discharge and spillage.

Indices for reservoirs

Vattnet 1, Fallet 2, Strömmen 3.

Indices for power plants

Forsen 1, Fallet 2, Språnget 3, Strömmen 4.

Parameters

The parameters are defined in table 8 in the problem text.

Optimisation variables

$Q_{i,j,t}$ = discharge in power plant k, segment j, during hour t, $k = 1, \ldots, 4$, $j = 1, 2$, $t = 1, \ldots, 24$,

$S_{i,t}$ = spillage from power plant k during hour t, $k = 1, \ldots, 4$, $t = 1, \ldots, 24$,

$M_{i,t}$ = contents of reservoir i at the end of hour t, $i = 1, 2, 3$, $t = 1, \ldots, 24$.

Objective function

$$\text{maximise} \quad \sum_{t=1}^{24}\left(\lambda_{1,t}\sum_{k=1}^{2}\mu_{k,j}Q_{k,j,t} + \lambda_{2,t}\sum_{k=3}^{4}\mu_{k,j}Q_{k,j,t}\right) + \lambda_{f1}\mu_{2,1}M_{2,24} + \lambda_{f2}((\mu_{3,1} + \mu_{4,1})M_{1,24} + \mu_{4,1}M_{4,24})$$

Constraints

Hydrological balance for Vattnet:

$$M_{1,t} = M_{1,t-1} - Q_{1,1,t} - Q_{1,2,t} - S_{1,t} - Q_{3,1,t} - Q_{3,2,t} - S_{3,t} + V_{1,t}, \qquad t = 1, \ldots 24.$$

Hydrological balance for Fallet:

$$M_{2,t} = M_{2,t-1} - Q_{2,1,t} - Q_{2,2,t} - S_{2,t} + Q_{1,1,t} + Q_{1,2,t} + S_{1,t} + V_{2,t}, \qquad t = 1, \ldots 24.$$

Hydrological balance for Strömmen:

$$M_{3,t} = M_{3,t-1} - Q_{4,1,t} - Q_{4,2,t} - S_{4,t} + Q_{3,1,t} + Q_{3,2,t} + S_{3,t} + V_{3,t}, \qquad t = 1, \ldots 24.$$

Variable limits

$0 \le Q_{k,j,t} \le \overline{Q}_{k,j}$, $k = 1, \ldots, 4, j = 1, 2, t = 1, \ldots, 24,$

$0 \le S_{k,t} \le \overline{S}_i$, $k = 1, \ldots, 4, t = 1, \ldots, 24,$

$0 \le M_{i,t} \le \overline{M}_i$, $i = 1, 2, 3, t = 1, \ldots, 24.$

b) To prevent discharge in a specific interval we need to introduce binary variables:

$u_{k,t}$ = unit commitment of power plant k, hour t (1 if water is discharged in the power plant, otherwise 0).

The objective funciton and the hydrological constraints from part a do not need to be modified, but there is a need for new constraints to maintain the relation between unit commitment and discharge. We can notice that for all power plants, the minimal discharge when the power plant is committed results in discharge only in segment 1. Therefore, it should be possible that $Q_{k,2,t}$ equals zeros when a power plant is committed; however, a constraint is necessary for the minimal discharge in the first segment:

$$Q_{k,1,t} \ge 0.2(\overline{Q}_{k,1} + \overline{Q}_{k,2})u_{k,t}, \qquad k = 1, \ldots, 4, t = 1, \ldots, 24.$$

This constraint means that when the power plant is committed the minimal discharge must be larger than or equal to 20% of maximal discharge, whereas the discharge can be larger than or equal to zero when the power plant is not committed.

If the power plant is not committed then the discharge in both segments should be equal to zero. As the lower limit for the discharge is equal to zero, we can express this condition as a requirement that the total discharge is equal to zero when the power plant is not committed and otherwise it should be less than or equal to maximal total discharge. This results in the following constraint:

$$Q_{k,1,t} + Q_{k,2,t} \le (\overline{Q}_{k,1} + \overline{Q}_{k,2})u_{k,t}, \qquad k = 1, \ldots, 4, t = 1, \ldots, 24.$$

Finally, we need to introduce variable limits for the new optimisation variables:

$$u_{k,t} \in \{0, 1\}, \qquad k = 1, \ldots, 4, t = 1, \ldots, 24.$$

Problem 9

a) The variable cost of the family is the sum of the electricity price, the variable grid fee and the energy tax plus 25% VAT, i.e., 1.25· (81 + 19 + 28) = 160 öre/kWh. To compute the expected electricity bill of the family, it is necessary to know how many kWh the family will need to buy for this price. It can be observed in the duration curves that the highest wind power generation, W, is lower than the lowest electricity demand, D. Hence, the family will never need to sell surplus power to the grid—all wind power gneration can be used to reduce the purchase from AB Elleverantören. The expected purchase can then be expressed as

$$E[D - W] = E[D] - E[W].$$

The expected consumption is given by the area below the load duration curve multiplied by the length of the time period, i.e.,

$$E[D] = (31 + 31 + 28) \cdot 24 \int_0^{\infty} \tilde{F}_D(x)dx = 2\,160 \cdot (3 \cdot 1 + 4 \cdot (1 + \pi)/2 + 3\pi/2) =$$
$$= 2\,160 \cdot (5 + 3.5\pi).$$

Similarly, we can determine the expected wind power generation by

$$E[W] = 2\,160 \int_0^{\infty} \tilde{F}_W(x)dx = 2\,160 \cdot (1 \cdot (0.8 + 0.2)/2 + 1 \cdot 0.2/2) = 2\,160 \cdot 0.6.$$

Thus, the expected electricity bill will be

$$1.60 \cdot E[D - W] = 1.60(2\,160 \cdot (5 + 3.5\pi) - 2\,160 \cdot 0.6) = \{\text{use the given formula for } \pi(T)\} =$$

$$\begin{cases} 3\,456(4.4 + 0.35 - 0.035T) = 3\,456(4.750 - 0.035T) & T \ge -2, \\ 3\,456(4.4 + 0.42 - 0.07(T + 2)) = 3\,456(4.68 - 0.07T) & T \le -2. \end{cases}$$

b) The most straightforward choice of variance reduction technique for this problem is to use complementary random numbers. The standardised normal distribution is symmetric around the mean zero, which means that the complementary number of an observation t is equal to $-t$. The ten random numbers in the table then provide in total 20 observations of the temperature. Using the relation from part a we get the following samples:

Original cenario		Complemntary scenario	
Average temperature, t [°C]	Expectged electricity bill, x [SEK]	Average temperature, t [°C]	Expectged electricity bill, x [SEK]
0.5	16 356	–0.5	16 476
–2.3	16 730	2.3	16 138
0.3	16 380	–0.3	16 452
–0.4	16 464	0.4	16 368
3.6	15 981	–3.6	17 045
1.8	16 198	–1.8	16 634
0.9	16 416	–0.9	16 416
–1.3	16 573	1.3	16 259
0.3	16 380	–0.3	16 452
2.8	16 077	–2.8	16 851

The mean of these twenty results is 16 432 SEK.