



KTH Electrical Engineering

**Complementary test in  
EG2050/2C1118 System Planning,  
14 April 2008, 18:00-20:00, the seminar room**

**Instructions**

Only the problems indicated on the attached answer sheet have to be answered (the score of the remaining problems is kept from the exam). Motivations and calculations do not have to be presented.

The maximal score of the complementary test is 40 points including the points that are kept from the exam. You are guaranteed to pass if you get at least 33 points.

**Allowed aids**

In this complementary test you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten, single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

## Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

**a) (2 p)** AB Elbolaget is a balance responsible retailer of electricity. The company has only one customer, namely AB Industri. The electricity trading on the market where AB Elbolaget is active has a trading period of one hour. In the following cases AB Elbolaget are fulfilling their balance responsibility without any imbalance: I) Then AB Elbolaget is producing 1 000 MWh during one hour, while the consumption of AB Industri is varying between 800 MW and 1 250 MW with an average of 1 000 MW during the hour, II) Then AB Elbolaget is producing 750 MW during the first half hour and 1 250 MW during the second half hour, while the consumption of AB Industri amounts to 1 000 MWh, III) Then AB Elbolaget buys 1 000 MWh from the local power exchange, ElKräng, while consumption of AB Industri amounts to 1 000 MWh.

1. None of the statements is true.
2. Only III is true.
3. I and II are true but not III.
4. II and III are true but not I.
5. All the statements are true.

**b) (2 p)** The consumers in a vertically integrated electricity market has the following choices: I) They can choose which system operator they want, II) They can choose which retailer they want, III) They can choose which player should manage their balance responsibility.

1. None of the statements is true.
2. Only I is true.
3. Only III is true.
4. I and II are true but not III.
5. II and III are true but not I.

## Problem 2 (6 p)

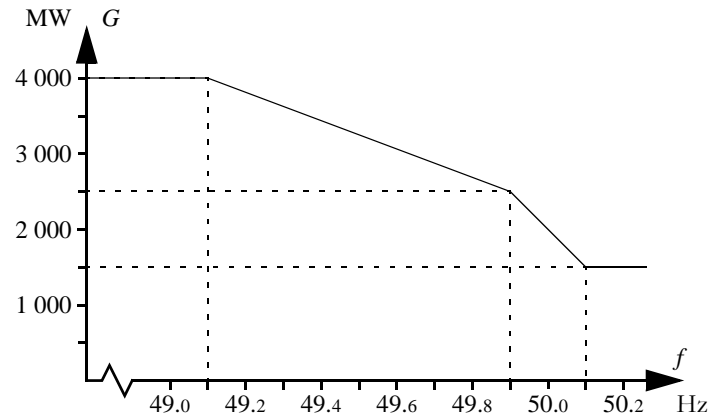
Consider the common electricity market of the two countries Rike and Maa. Assume that there is perfect competition, that all players have perfect information, and that there are neither reservoir nor capacity limitations in the power plants. The power systems of Rike and Maa are interconnected by an HVDC line, which can transfer a certain amount of TWh per year. Assume that the electricity price in Rike is 360 ¤/MWh, whereas the electricity price in Maa is 380 ¤/MWh. The remaining data for the common electricity market are given in table 1. The variable operation costs are assumed to be linear within the intervals, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

**Table 1** Data for the electricity market in Rike and Maa.

Power source	Production capability [TWh/year]		Variable cost [¤/MWh]
	Rike	Maa	
Hydro power	50	10	30–60
Nuclear power	50	20	100–120
Coal condensing	15	15	300–450
Gas turbines	5	5	800–1 000
Electricity consumption [TWh/year]	94	50	

- a) (1 p) How much are the hydro power plants in Rike producing?
- b) (1 p) How much are the gas turbines in Maa producing?
- c) (2 p) How much are the coal condensing power plants in Rike producing?
- d) (2 p) How large is the transmission capacity between Rike and Maa?

### Problem 3 (6 p)



The figure above shows the total generation in the power plants participating in the primary control as a function of the frequency in a certain power system.

- a) (2 p)** How large is the gain of the system when the frequency is 50.0 Hz?
- b) (2 p)** How large is the gain of the system when the frequency is 49.5 Hz?
- c) (2 p)** Assume that the frequency in the system is 49.5 Hz. What happens to the frequency of the system if 900 MW of generation is lost at this point?
1. The system will collapse if the system operator cannot quickly disconnect load, because there is not enough primary control reserves to compensate an outage of 900 MW.
  2. The frequency of the system will be 49.02 Hz when the primary control has compensated the outage.
  3. The frequency of the system will be 49.10 Hz when the primary control has compensated the outage.
  4. The frequency of the system will be 49.68 Hz when the primary control has compensated the outage.
  5. The frequency of the system will be 50.00 Hz when the primary control has compensated the outage.

## Problem 4 (12 p)

Stads energi AB owns a thermal power plant with three blocks. Moreover, the company owns a wind farm. The company sells power to customers with firm power contracts, and is also trading at the local power exchange ElKräng, where the company has the possibility to both sell and purchase electricity. Assume that the company has formulated their short-term planning problem as a MILP problem and that the following symbols have been introduced:

Indices for the power plants: Block I - 1, Block II - 2, Block III - 3.

- $C_g^+$  = start-up cost in power plant  $g$ ,  $g = 1, 2, 3$ ,
- $\bar{D}_t$  = contracted load during hour  $t$ ,  $t = 1, \dots, 24$ ,
- $G_{g,t}$  = generation in power plant  $g$ , hour  $t$ ,  $g = 1, 2, 3$ ,  $t = 1, \dots, 24$ ,
- $\bar{G}_g$  = installed capacity in power plant  $g$ ,  $g = 1, 2, 3$ ,
- $\underline{G}_g$  = minimal generation when power plant  $g$  is committed,  $g = 1, 2, 3$ ,
- $p_t$  = purchase from ElKräng hour  $t$ ,  $t = 1, \dots, 24$ ,
- $r_t$  = sales to ElKräng hour  $t$ ,  $t = 1, \dots, 24$ ,
- $s_{g,t}^+$  = start-up variable for power plant  $g$ , hour  $t$ ,  $g = 1, 2, 3$ ,  $t = 1, \dots, 24$ ,
- $u_{g,0}$  = unit commitment of power plant  $g$  at the beginning of the planning period,  
 $g = 1, 2, 3$ ,
- $u_{g,t}$  = unit commitment of power plant  $g$ , hour  $t$ ,  $g = 1, 2, 3$ ,  $t = 1, \dots, 24$ ,
- $W_t$  = expected wind power generation in hour  $t$ ,  $t = 1, \dots, 24$ ,
- $\beta_{Gg}$  = variable operation cost in power plant  $g$ .

**a) (4 p)** Formulate the load balance constraint of Stads energi AB. Use the symbols defined above.

**b) (4 p)** Formulate the constraint that sets the relation between  $\bar{G}_g$ ,  $G_{g,t}$  and  $u_{g,t}$  for power plant  $g$ , hour  $t$ .

**c) (2 p)** The following symbols in the planning problem of Stads energi AB denote optimisation variables: I)  $D_p$ , II)  $G_{g,p}$ , III)  $W_t$ .

1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. All the statements are true.

**d) (1 p)** Block III is fuelled by biomass which costs 600  $\text{kr}$ /ton. The heat contents of the fuel is 5 MWh/ton and the power plant has an efficiency of 40%. How large is the variable operation cost in block III?

**e) (1 p)** The reservoir of Forsen holds 3 960 000  $\text{m}^3$  water at 18:00. The local inflow as well as discharge and spillage from the power plant upstream amounts to 100 HE between 18:00 and 19:00. During the same time, 140 HE are discharged from Forsen. How much will the reservoir of Forsen hold at 19:00? Notice that the answer should be given in HE!

## Problem 5 (12 p)

Consider an electricity market where the load can be assumed to be normally distributed with the expectation value 800 kW and the standard deviation 100 kW. Selected data of the duration curve  $\tilde{F}_0(x)$  are given in table 2.

**Table 2** The load duration curve in problem 5.

	$x = 700$	$x = 800$	$x = 900$	$x = 1\,000$	$x = 1\,100$	$x = 1\,200$
$\tilde{F}_0(x)$	0.841	0.500	0.159	0.023	0.001	0.000
$\int_x^\infty \tilde{F}_0(\xi) d\xi$	108.33	39.89	8.33	0.85	0.04	0.00

**a) (1 p)** Calculate  $EENS_0$ , i.e., the expected unserved energy if there is no power plant in the system.

**b) (2 p)** Assume that this electricity market is supplied by a 900 kW hydro power plant, and that the installed capacity of this power plant always is available. How large is the expected generation in the hydro power plant?

**c) (3 p)** Assume that this electricity market is supplied by the hydro power plant from the previous question as well as a wind power plant. A model of the available generation capacity is given in table 3. What is the *LOLP* of this system?

*Hint:* The convolution formula for a multi-state model reads

$$\tilde{F}_g(x) = \sum_{i=1}^{N_g} p_{g,i} \tilde{F}_{g-1}(x - x_{g,i}).$$

**Table 3** Model of the wind power plant in problem 5c.

Available generation capacity [MW]	Probability [%]
0	25
100	35
200	20
300	20

**d) (2 p)** In reality there is a weak correlation between the load and the available generation capacity of the wind power plant. Assume that this correlation is to be included and that the system therefore is simulated using Monte Carlo methods. In total, 2 500 scenarios are generated and there is power deficit appears in 112 of these scenarios. Which estimate of *LOLP* is obtained from this simulation?

**e) (2 p)** Assume that a value of the load has been randomised and that the result was 700 kW. What is the complementary random number of this value?

**f) (2 p)** The expectation value  $E[X]$  is to be determined using a combination of a control variate and stratified sampling. Assume that  $L$  strata have been defined and let  $\omega_h$  denote the stratum weight of stratum  $h$ . Introduce the symbol  $x_{h,i}$  for the  $i$ :th observation of  $X$  from stratum  $h$  and let  $z_{h,i}$  denote the  $i$ :th observation from stratum  $h$  of the control variate,  $Z$ . The total number of observations is  $n$ , and we use the symbol  $n_h$  to denote the number of observations from  $h$ . How is the estimate  $m_X$  calculated?

1.  $m_X = \frac{1}{n} \sum_{h=1}^L \sum_{i=1}^{n_h} \omega_h x_{h,i} + E[Z].$
2.  $m_X = \frac{1}{n} \sum_{h=1}^L \sum_{i=1}^{n_h} \omega_h z_{h,i} + E[Z].$
3.  $m_X = \sum_{h=1}^L \frac{\omega_h}{n_h} \sum_{i=1}^{n_h} z_{h,i} + E[Z].$
4.  $m_X = \sum_{h=1}^L \frac{\omega_h}{n_h} \sum_{i=1}^{n_h} (x_{h,i} - z_{h,i}) + E[Z].$
5.  $m_X = \sum_{h=1}^L \frac{1}{n_h} \sum_{i=1}^{n_h} \omega_h x_{h,i} z_{h,i} + E[Z].$



KTH Electrical Engineering

## Answer sheet

Name: .....

Personal number: .....

### Problem 1

a) Alternative ..... is correct.

b) Alternative ..... is correct.

### Problem 2

a) ..... TWh/year      b) ..... TWh/year

c) ..... TWh/year      d) ..... TWh/year

### Problem 3

a) ..... MW/Hz

b) ..... MW/Hz

c) Alternative ..... is correct.

### Problem 4

a) .....

b) .....

c) Alternative ..... is correct.

d) .....  $\text{m}^3/\text{MWh}$       e) ..... HE

### Problem 5

a) ..... kWh/h      b) ..... kWh/h

c) ..... %      d) ..... %

e) ..... kW

c) Alternative ..... is correct.



### Problem 1

- a) 5, b) 1.

### Problem 2

- a) As the electricity price is higher than the variable operation cost in the most expensive hydro power plant, all hydro power in Rike will be utilised, i.e., 50 TWh/year.
- b) As the electricity price is lower than the variable operation cost in the least expensive gas turbine, no gas turbines in Maa will be used.
- c) The part of the coal condensing which has a lower variable operation cost than the electricity price 360 ¢/MWh will be used, i.e.,  $(360 - 300)/(450 - 300) \cdot 15 = 6$  TWh/year.
- d) The electricity price is lower in Rike than in Maa; hence, the interconnection between Rike and Maa will be fully utilised (if there was no transmission limitation, the price would be the same in both countries). The total generation in Rike is 106 TWh/year, whereas the consumption is 94 TWh/year. The surplus is exported to Maa. Thus, we can conclude that the maximal transmission capacity is 12 TWh/year.

### Problem 3

- a) In the interval 49,9–50,1 Hz we have  $R = \Delta G/\Delta f = 1\,000/0,2 = 5\,000$  MW/Hz.
- b) In the interval 49,1–49,9 Hz we have  $R = \Delta G/\Delta f = 1\,500/0,8 = 1\,875$  MW/Hz.
- c) 1.

### Problem 4

- a)  $\sum_{g=1}^3 G_{g,t} + W_t + P_t = D_t + r_t$
- b)  $G_{g,t} \leq u_{g,t} \bar{G}_g$
- c) 3.
- d) One tonne fuel yields  $0,4 \cdot 5 = 2$  MWh and costs 600 ¢; hence, the variable operation cost is  $600/2 = 300$  ¢/MWh.
- e)  $3\,960\,000 \text{ m}^3$  water corresponds to  $3\,960\,000/3\,600 = 1\,100$  HE. As the inflow to the reservoir is 100 HE and the flow out from the reservoir is 140 HE, the content at the end of the hour must be 1 060 HE.

### Problem 5

- a)  $EENS_0 = E[D] = 800$  kWh/h.

- b) Since the hydro power plant is 100% we get  $\tilde{F}_1(x) = \tilde{F}_0(x)$ . The expected generation is then

$$EG_1 = EENS_0 - EENS_1 = 800 - \int_{900}^{\infty} \tilde{F}_1(x) dx = 800 - 8,33 = 791,67 \text{ kWh/h.}$$

$$\text{c) } LOLP = \tilde{F}_2(1\,200) = 0,20\tilde{F}_1(1\,200) + 0,20\tilde{F}_1(1\,100) + 0,35\tilde{F}_1(1\,000) + 0,25\tilde{F}_1(900) = 0,2 \cdot 0 + 0,2 \cdot 0,001 + 0,35 \cdot 0,023 + 0,25 \cdot 0,159 = 4,8\%$$

$$\text{d) } LOLP \text{ is estimated by } m_{LOLO} = 112/2\,500 = 4,48\%$$

- e) The normal distribution is symmetrical, which means that if  $D = \mu_D + X$  then  $D^* = \mu_D - X$ . Hence, the complementary random number of  $D = 700$  kW must be  $D^* = 900$  kW.
- f) 4.