



KTH Electrical Engineering

**Complementary test in  
EG2050 System Planning,  
14 October 2008, 17:00-19:00, the seminar room**

**Instructions**

Only the problems indicated on the attached answer sheet have to be answered (the score of the remaining problems is kept from the exam). Motivations and calculations do not have to be presented.

The maximal score of the complementary test is 40 points including the points that are kept from the exam. You are guaranteed to pass if you get at least 33 points.

**Allowed aids**

In this complementary test you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten, single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

## Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

**a) (2 p)** A balance responsible player has the following responsibilities: I) Economical responsibility that the system during each trading period (for example one hour) is supplied as much energy as consumed by the customers of the player, II) Physical responsibility that the system during each trading period (for example one hour) is supplied as much energy as consumed by the customers of the player, III) Physical responsibility that the system continuously is supplied as much power as consumed by the customers of the player.

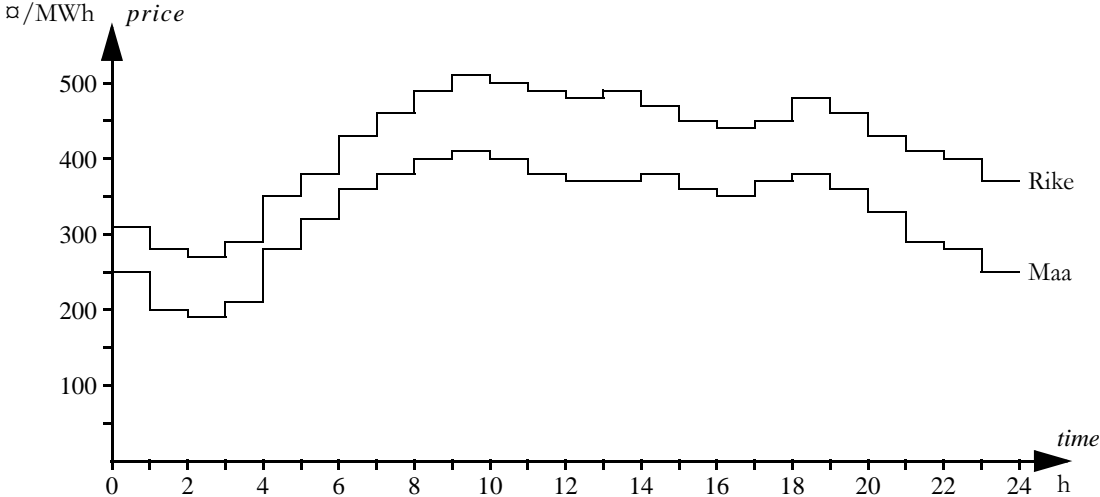
1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. I and II are true but not III.

**b) (2 p)** The following applies to a down-regulation bid in a regulation market: I) If the bid is activated it means that the player who submitted the bid is buying energy from the system operator, II) A down-regulation bid can be performed by decreasing the generation in for example a hydro power plant, III) A down-regulation bid can be performed by decreasing the consumption in for example a large factory.

1. Only I is true.
2. I and II are true but not III.
3. I and III are true but not II.
4. II and III are true but not I.
5. All the statements are true.

### Problem 2 (6 p)

a) (3 p) Consider the common electricity market of the two countries Rike and Maa. Assume that there is perfect competition, that all players have perfect information and that there are neither reservoir nor capacity limitations in the power plants. The power systems of Rike and Maa are connected by an HVDC line, which can transfer at most 1 000 MW. The figure below shows the electricity prices in Rike and Maa during a day. How much is traded between the two countries, i.e., which country will export and how large is the total export during these 24 hours?



b) (2 p) Assume perfect competition in the electricity market of Rike, that all players have perfect information and that there are neither capacity, transmission nor reservoir limitations within Rike. Data for the electricity producers of Rike are given in table 1. The variable costs of the coal condensing are assumed to be linear in the given interval, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level. In addition to the domestic generation, Rike is importing 6 TWh from Maa. How large is the electricity consumption in Rike if the electricity price during a certain year is 220 €/MWh.

Table 1 Data for the electricity producers in Rike.

Power source	Production capability [TWh/year]	Variable costs [€/MWh]
Hydro	53	5
Nuclear	49	90–100
Bio-fuel	20	100–300
Fossil fuel	20	200–400

c) (1 p) Assume that the nuclear power producers have to pay a waste deposit fee of 10 € for each generated MWh. Which electricity price will there be in Rike if the electricity consumption is the same as in part b)?

### Problem 3 (6 p)

The power system in Land is divided in two areas (A and B) which are connected by an AC transmission line. This line has a maximal capacity of 1 000 MW and is equipped with a protection system which after a short time delay disconnects the line if the power flow exceeds the maximal capacity of the line.

At 8:45 a fire breaks out in a substation in Stad (which is located in area A). Due to the fire, the entire regional grid of Stad must immediately be disconnected from the national grid in Land, which means that the national grid loses 200 MW generation and 500 MW load. After the disconnection of Stad, the gain in Land is Land 5 000 MW/Hz in area A and 5 000 MW/Hz in area B.

Just before the disconnection of Stad, the frequency of the system was 50.02 Hz and 750 MW was transmitted from area A to area B.

**a) (1 p)** What happens when the regional grid of Stad is disconnected?

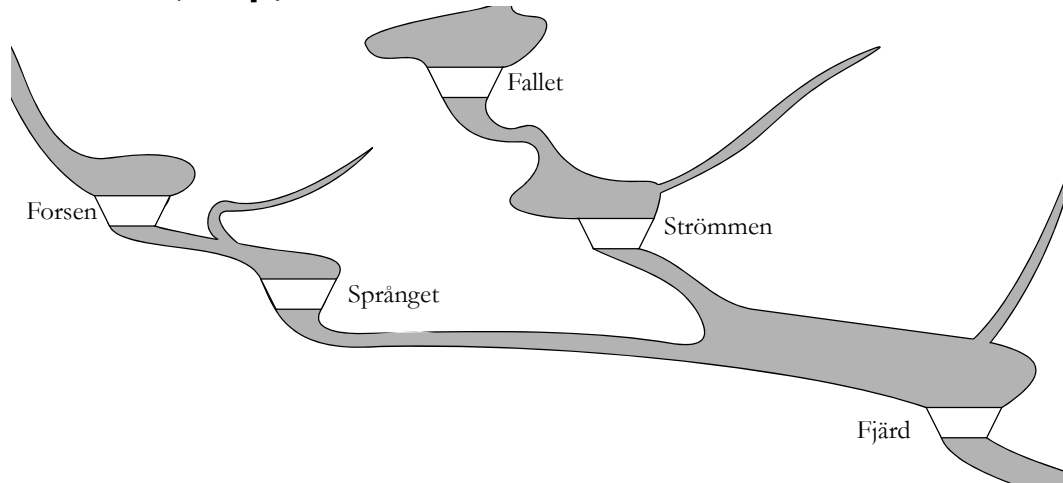
1. There is a surplus of energy, which results in a voltage increase in the grid. The control systems of the power plants participating in the primary control responds to the voltage increase by reducing the electricity generation.
2. There is a surplus of energy, which is stored as kinetic energy in all synchronous generators; hence, the frequency of the system increases. The control systems of the power plants participating in the primary control responds to the frequency increase by reducing the electricity generation.
3. There is a deficit of energy, which is compensated using kinetic energy from all synchronous generators; hence, the frequency of the system increases. The control systems of the power plants participating in the primary control responds to the frequency increase by increasing the electricity generation.

**b) (1 p)** Will the transmission line between area A and B be disconnected due to overloading?

**c) (2 p)** Which frequency will there be in area A and B respectively, once the primary control has restored the balance between generation and consumption?

**d) (2 p)** The power plant Språnget is located in area B. The installed capacity of the power plant is 300 MW and the gain is set to 200 MW/Hz. The base generation (i.e., the generation when the frequency is exactly 50 Hz) is 200 MW. How much does Språnget generate when the frequency is 50.02 Hz?

## Problem 4 (12 p)



AB Vattenkraft owns five hydro power plant located as in the figure above. The following symbols have been introduced in a short-term planning problem for these hydro power plants:

Indices for the power plants: Forsen 1, Språnget 2, Fallet 3, Strömmen 4, Fjärd 5.

$\gamma_i$  = expected future production equivalent for water stored in reservoir  $i$ ,  
 $i = 1, \dots, 5$ ,

$\lambda_t$  = expected electricity price at ElKräng hour  $t$ ,  $t = 1, \dots, 24$ ,

$\lambda_{25}$  = expected electricity price at ElKräng after the end of the planning period,

$M_{i,0}$  = contents of reservoir  $i$  at the beginning of the planning period,  $i = 1, \dots, 5$ ,

$M_{i,t}$  = contents of reservoir  $i$  at the end of hour  $t$ ,  $i = 1, \dots, 5$ ,  $t = 1, \dots, 24$ ,

$\mu_{i,j}$  = marginal production equivalent in power plant  $i$ , segment  $j$ ,  
 $i = 1, \dots, 5$ ,  $j = 1, 2$ ,

$Q_{i,j,t}$  = discharge in power plant  $i$ , segment  $j$ , during hour  $t$ ,  
 $i = 1, \dots, 5$ ,  $j = 1, 2$ ,  $t = 1, \dots, 24$ .

**a) (4 p)** Formulate the objective function if the purpose of the planning is to maximise the value of stored water. Use the symbols defined above.

**b) (2 p)** The maximal production equivalent in Fjärd is 0.8 MWh/TE. Assume that the power plant should generate 15.68 MWh between 12:00 and 13:00. How large is the discharge in Fjärd during this hour if the relative efficiency is 98%? The answer should be given in HE.

**c) (2 p)** The reservoir of Fjärd holds 432 000 m<sup>3</sup> water at 12:00 and the total inflow (i.e., both local inflow as well as discharge and spillage from the power plant upstream) between a 12:00 and 13:00 amounts to 32 400 m<sup>3</sup>. Assume that AB Vattenkraft in addition to the discharge calculated in part b also has to spill 1 m<sup>3</sup>/s through a fish ladder at Fjärd. How large is the contents of the reservoir at 13:00? The answer should be given in HE.

**d) (2 p)** Consider a short-term planning problem concerning  $G$  thermal power plants and  $T$  time periods. The following constraints for maximal and minimal generation have been introduced in this problem:

$$G_{g,t} \leq u_{g,t} \bar{G}_g, \quad g = 1, 2, \dots, G, t = 1, 2, \dots, T$$

$$G_{g,t} \geq u_{g,t} \underline{G}_g, \quad g = 1, 2, \dots, G, t = 1, 2, \dots, T$$

Which of the symbols above represent parameters and variables respectively?

1. All symbols are parameters.
2.  $\bar{G}_g$ ,  $\underline{G}_g$ , and  $u_{g,t}$  are parameters,  $G_{g,t}$  are optimisation variables.
3.  $\bar{G}_g$  and  $\underline{G}_g$  are parameters,  $u_{g,t}$  and  $G_{g,t}$  are optimisation variables.
4. Only  $u_{g,t}$  are parameters,  $\bar{G}_g$ ,  $\underline{G}_g$ , and  $G_{g,t}$  are optimisation variables.
5. All symbols are optimisation variables.

**e) (2 p)** Assume that it has been decided that a thermal power plant should not be shut down for a shorter time than four hours, i.e., if the power plant is stopped at 12:00 then it may not be started again before 16:00. Introduce the following symbols:

$s_t^+$  = start-up variable for hour  $t$  (1 if the power plant is starting generation at the beginning of hour  $t$ , otherwise 0),

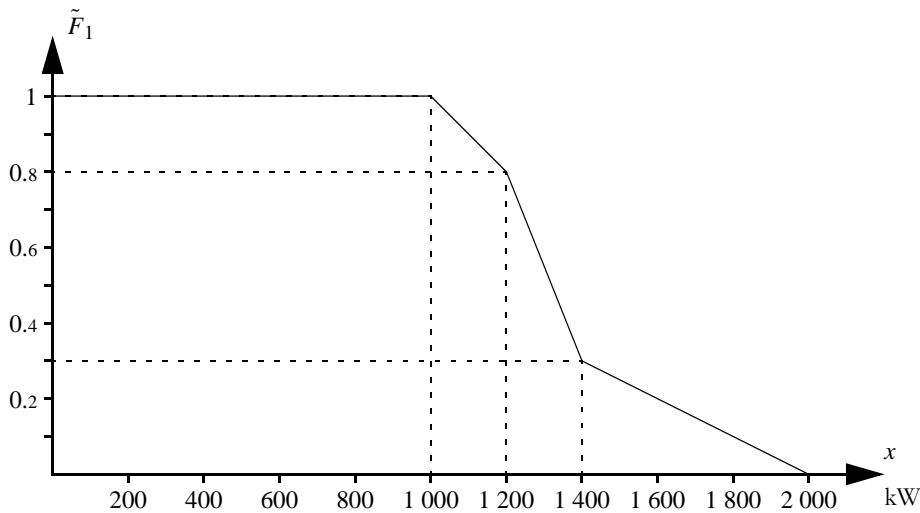
$s_t^-$  = stop variable for hour  $t$  (1 if the power plant is stopping generation at the beginning of hour  $t$ , otherwise 0).

How should a linear constraint be formulated in order to describe the relation between  $s_t^-$ ,  $s_{t+1}^+$ ,  $s_{t+2}^+$  and  $s_{t+3}^+$ ?

1.  $s_t^- - s_{t+1}^+ - s_{t+2}^+ - s_{t+3}^+ = 0$ .
2.  $s_t^- - s_{t+1}^+ - s_{t+2}^+ - s_{t+3}^+ \leq 1$ .
3.  $s_t^- - s_{t+1}^+ - s_{t+2}^+ - s_{t+3}^+ = 1$ .
4.  $s_t^- + s_{t+1}^+ + s_{t+2}^+ + s_{t+3}^+ \leq 1$ .
5.  $s_t^- + s_{t+1}^+ + s_{t+2}^+ + s_{t+3}^+ = 1$ .

## Problem 5 (12 p)

Mji is a town in East Africa. The town is not connected to a national grid, but has a local system of its own. The local grid is supplied by a hydro power plant and two diesel generator sets. The hydro power plant does not have a reservoir, but the water flow is always sufficient to generate the installed capacity (1 400 kW) and the risk for outages in the power plant is negligible. The diesel generator sets have a capacity of 200 kW each, the availability is 80% and the operation cost is 1 ¢/kWh.



**a) (2 p)** What is the expectation value of the load?

*Hint: Study  $EENS_0$ !*

**b) (2 p)** Using probabilistic production cost simulation it is calculated that the expected hydro power generation of this system is 1 290 kWh/h and that the unserved energy is 22.8 kWh/h. How large is the expected operation cost?

**c) (3 p)** Use probabilistic production cost simulation to calculate the risk of power deficit in the system.

**d) (2 p)** Assume that complementary random numbers are used to improve the simulation of Rike. What is the value of the complementary random number,  $D^*$ , if the total load of the system is randomised to  $D = 1\,100$  MW?

**e) (3 p)** To consider the losses in the grid, a Monte Carlo simulation has been performed of the power system in Mji. The simulation is using control variates. The simplified model corresponds to the model used in probabilistic production cost simulation, whereas the detailed model considers such factors as how the losses depends on which power plants that are operated, and how the load is distributed within the system. The results are shown in table 2. Which estimate of  $ETOC$  is obtained using the detailed model?

**Table 2** Results from a Monte Carlo simulation of the power system in Mji.

Number of scenarios, $n$	Results from the detailed model, $\sum_{i=1}^n toc_i$	Results from the simplified model, $\sum_{i=1}^n \tilde{toc}_i$
1 000	68 100	66 500



KTH Electrical Engineering

## Answer sheet

Name: .....

Personal number: .....

### Problem 1

a) Alternative ..... is correct.

b) Alternative ..... is correct.

### Problem 2

a) ..... will export ..... MWh

b) ..... TWh/year c) .....  $\alpha$ /MWh

### Problem 3

a) Alternative ..... is correct b) .....

c) Area A: ..... Hz Area B: ..... Hz

d) ..... MW

### Problem 4

a) .....  
.....

b) ..... HE c) ..... HE

d) Alternative ..... is correct.

e) Alternative ..... is correct.

### Problem 5

a) ..... kWh/h b) .....  $\alpha$ /h

c) ..... % d) ..... MW

e) .....  $\alpha$ /h



### Problem 1

- a) 2, b) 2.

### Problem 2

- a) Since the electricity price is higher in Rike than in Maa during each hour, Maa will continuously export 1 000 MW, which yields a total export of 24 000 MWh during this day.
- b) At the electricity price 220 ¢/MWh all hydro and nuclear power plants are generating as much as possible, i.e.,  $53 + 49 = 102$  TWh. The contribution from the bio-fuelled power plants is  $(220 - 100)/(300 - 100) \cdot 20 = 12$  TWh and the fossil-fuelled power plants deliver  $(220 - 200)/(400 - 200) \cdot 20 = 2$  TWh. Thus, the total generation is 116 TWh/year. In addition to that, 6 TWh are imported which yields a total consumption of 122 TWh/year.
- c) The variable operation cost of the nuclear power including the fee will be in the interval between 100 and 110 ¢/MWh. Since this is less than the old electricity price in Land, the nuclear power will continue to operate at maximal capacity, and the same power plants as before (i.e. bio-fuel and fossil fuel) will determine the price. Hence, the electricity price will remain 220 ¢/MWh.

### Problem 3

- a) 2.
- b) After the disconnection of Stad, the primary control has to reduce the generation by 300 MW. Area B has half the gain in the system and will consequently contribute to half of the generation decrease. Since the load and the other generation in area B remains the same, the generation decrease will have to be compensated by increased import from area A. The transmission on the line will therefore increase to 900 MW, which is less than the capacity of the line. Hence, the line will not be disconnected.
- c) Since the two areas are still connected by an AC line, the frequency will be the same in both areas. If the generation is to decrease by 300 MW then it is necessary that the frequency increases by  $\Delta f = \Delta G/R = 300/10\,000$  Hz, i.e., the new frequency will be  $50.02 + 0.03 = 50.05$  Hz.
- d) When the frequency is 50.02 Hz, Språngstet is generating

$$G = G_0 - R(f - f_0) = 200 - 200(50.02 - 50) = 196 \text{ MW.}$$

### Problem 4

- a) maximise  $\lambda_{25}(\gamma_1 + \gamma_2 + \gamma_5)M_{1,24} + (\gamma_2 + \gamma_5)M_{2,24} + (\gamma_3 + \gamma_4 + \gamma_5)M_{3,24} + (\gamma_4 + \gamma_5)M_{4,24} + \gamma_5 M_{5,24}$ .
- b) The generation as a function of discharge can be expressed as  $H(Q) = \eta(Q)X_{\max}Q \Rightarrow Q = H(Q)/\eta(Q)X_{\max} = 15.68 \text{ MWh}/(0.98 \cdot 0.8 \text{ MWh}/\text{HE}) = 20 \text{ HE}$ .

- c) The contents of the reservoir is  $432\,000/3\,600 = 120$  HE at 12:00. The total inflow amounts to  $32\,400/3\,600 = 9$  HE and the total outflow is  $25 + 1 = 26$  HE. Thus, the reservoir holds  $120 + 9 -$

26 = 103 HE at 13:00.

- d) 3.  
e) 4.

### Problem 5

- a)  $EENS_0 = \int_0^{\infty} \tilde{F}_0(x) dx = 1\,000 \cdot 1 + 200 \cdot (1 + 0.8)/2 + 200 \cdot (0.8 + 0.3)/2 + 600 \cdot 0.3/2 = 1\,380 \text{ MWh}/\text{h}$ .
- b) The expected load is 1 380 kWh/h, which means that  $EG_1 + EG_2 + EG_3 + EENS_3 = 1\,380$ . As  $EG_1 = 1\,290 \text{ kWh}/\text{h}$  and  $EENS_3 = 22.8 \text{ kWh}/\text{h}$  it can be concluded that the expected generation in the diesel generator sets is  $67.2 \text{ kWh}/\text{h}$ . Hence, the expected operation cost is  $ETOC = 1 \cdot (EG_2 + EG_3) = 67.2 \text{ ¢}/\text{h}$ .
- c) The risk of power deficit is given by

$$\begin{aligned} \tilde{F}_3(1\,800) &= 0.8\tilde{F}_2(1\,800) + 0.2\tilde{F}_2(1\,600) = \\ &= 0.8(0.8\tilde{F}_1(1\,800) + 0.2\tilde{F}_1(1\,600)) + 0.2(0.8\tilde{F}_1(1\,600) + 0.2\tilde{F}_1(1\,400)). \end{aligned}$$

Since the hydro power plant has 100% availability we get  $\tilde{F}_1(x) = \tilde{F}_0(x)$ , hence,  $LOLP = 0.8(0.8 \cdot 0.1 + 0.2 \cdot 0.2) + 0.2(0.8 \cdot 0.2 + 0.2 \cdot 0.3) = 14\%$ .

- d) The inverse transform method states that  $D = F_D^{-1}(U)$ , where  $U$  is a  $U(0, 1)$ -distributed random number. Since it is the duration curve that is given in the problem, we may as well use the transform  $D = \tilde{F}_D^{-1}(U)$ . The original random number must then have been  $U = \tilde{F}_D(1\,100) = 0.9$ . Hence,  $U^* = 1 - U = 0.1$ , which results in  $D^* = \tilde{F}_D^{-1}(U^*) = 1\,800 \text{ MW}$ .

$$\begin{aligned} e) \quad m_{TOC} &= m_{TOC} - \tilde{TOC} + \mu_{TOC} = \frac{1}{n} \left( \sum_{i=1}^n toc_i - \sum_{i=1}^n \tilde{toc}_i \right) + 67.2 = \\ &= \frac{1}{1\,000} (68\,100 - 66\,500) + 67.2 = 68.8 \text{ ¢}/\text{h}. \end{aligned}$$