



KTH Electrical Engineering

## Complementary test in EG2050 System Planning, 20 March 2009, 15:00-17:00, the seminar room

### Instructions

Only the problems indicated on the attached answer sheet have to be answered (the score of the remaining problems is kept from the exam). Motivations and calculations do not have to be presented.

The maximal score of the complementary test is 40 points including the points that are kept from the exam. You are guaranteed to pass if you get at least 33 points.

### Allowed aids

In this complementary test you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten, single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

## Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

**a) (2 p)** A balance responsible player has the following responsibilities: I) Physical responsibility that the system continuously is supplied as much power as consumed by the customers of the player, II) Economical responsibility that the system continuously is supplied as much power as consumed by the customers of the player, III) Economical responsibility that the system during each trading period (for example one hour) is supplied as much energy as consumed by the customers of the player.

1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. I and II are true but not III.

**b) (1 p)** We use the notion “post trading” to describe all the trading which occurs after the hour of delivery (or any other trading period). Which of the following contracts can be traded in a post market?

1. Balance power, i.e., when a balance responsible player is selling any surplus in their balance to the system operator, or when a balance responsible player is buying from the system operator to cover for any deficit in their balance.
2. Firm power, i.e., the customer buys the same amount of energy in each trading period as long as the contract is valid.
3. Regulation power, i.e., when a player at request from the system operator is supplying more power to the system (up-regulation) or when a player at request from the system operator is supplying less power to the system (down-regulation).

**c) (1 p)** What does a take-and-pay contract mean?

1. The customer must in advance notify the supplier about how much the customer will consume during each trading period.
2. The customer buys the same amount of energy in each trading period as long as the contract is valid.
3. During the time the contract is valid, the customer is allowed to consume as much energy they want each trading period, provided that the maximal power is not exceeded.

## Problem 2 (6 p)

Assume that the electricity market in Land has perfect competition, all players have perfect information, and there are neither transmission nor capacity limitations. However the hydro reservoirs of Land has a limited storage capacity. The variable operation cost in the hydro power is negligible. On 1 January the reservoirs holds in total 20 TWh and according to the long-term forecast for the electricity market (which as already mentioned is assumed to be faultless), the reservoirs should hold 25 TWh on 31 December. The inflow and other data for the electricity market in Land are given in table 1 below. The variable costs are assumed to be linear in the given interval, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

Assume that the electricity price is 350  $\text{€}/\text{MWh}$  between 1 January and 30 June, and that it is 400  $\text{€}/\text{MWh}$  between 1 July and 31 December.

**Table 1** Data for the electricity market in Land.

Power source	Production capability [TWh/year]		Variable cost [ $\text{€}/\text{MWh}$ ]
	1 January to 30 June	1 July to 31 December	
Nuclear	35	35	100–120
Coal condensing	15	15	300–450
Gas turbines	5	5	800–1 000
Inflow to the hydro reservoirs [TWh]	150	55	
Electricity consumption [TWh]	145	140	

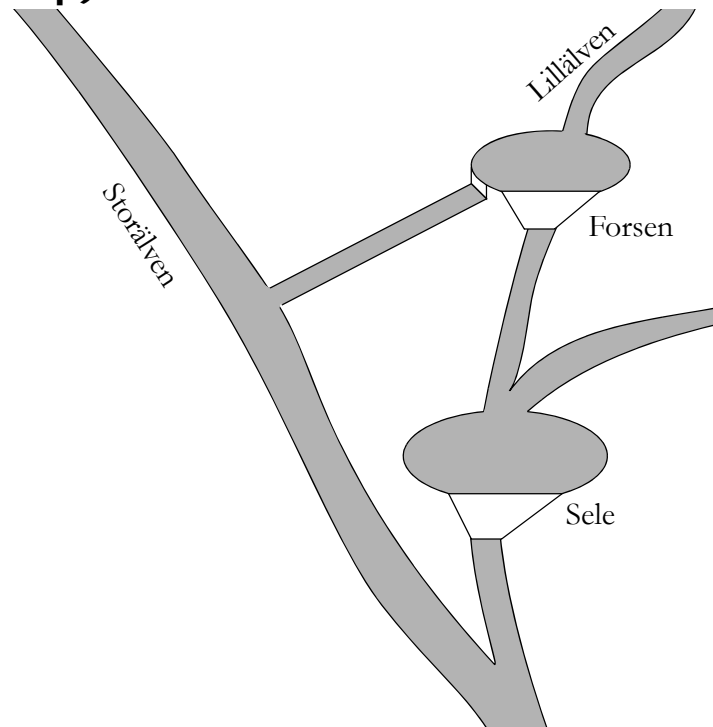
- a) (1 p) How large is the total nuclear generation in Land between 1 January and 30 June?
- b) (1 p) How large is the total gas turbine generation in Land between 1 July and 31 December?
- c) (2 p) How large is the total coal condensing generation in Land between 1 January and 30 June?
- d) (2 p) How large is the total storage capacity of the hydro reservoirs in Land?

## Problem 3 (6 p)

Consider a power plant that generates 194 MW when the frequency of the system is 49.92 Hz and 179 MW when the frequency is 50.02 Hz. The installed capacity of the power plant is 200 MW.

- a) (2 p) How large is the gain of the power plant?
- b) (2 p) Which base generation (i.e., the generation when the frequency is exactly 50 Hz) is set in the power plant?
- c) (2 p) How much is the power plant generating when the frequency of the system is 49.8 Hz?

### Problem 4 (12 p)



**a) (4 p)** The maximal discharge in the hydro power plant Forsen is  $120 \text{ m}^3/\text{s}$  and the best efficiency is obtained for the discharge  $90 \text{ m}^3/\text{s}$ . The maximal production equivalent of the power plant is  $0.25 \text{ MWh/HE}$  and the production equivalent at maximal discharge is  $0.24 \text{ MWh/HE}$ . Assume that we need a piecewise linear model of electricity generation as function of the discharge in Forsen. The model should have two segments and the breakpoint between them should be located at the best efficiency. Calculate the following parameters:

$\mu_j$  = marginal production equivalent in Forsen, segment  $j$ ,  
 $\bar{Q}_j$  = maximal discharge in Forsen, segment  $j$ .

**b) (4 p)** AB Vattenkraft owns two hydro power plant located as in the figure above. To enable salmon to migrate from Storälven to the exploited part of Lillälven upstream of Forsen, the environment court has decided that AB Vattenkraft must release a flow of  $10 \text{ m}^3/\text{s}$  in the channel between the reservoir of Forsen and Storälven. The following symbols have been introduced in a short-term planning problem for these hydro power plants:

Indices for the power plants: Forsen 1, Sele 2.

$M_{i,0}$  = contents of reservoir  $i$  at the beginning of the planning period,  $i = 1, 2$ ,  
 $M_{i,t}$  = contents of reservoir  $i$  at the end of hour  $t$ ,  $i = 1, 2$ ,  $t = 1, \dots, 24$ ,  
 $p_t$  = purchase from ElKräng hour  $t$ ,  $t = 1, \dots, 24$ ,  
 $Q_{i,j,t}$  = discharge in power plant  $i$ , segment  $j$ , during hour  $t$ ,  
 $i = 1, 2$ ,  $j = 1, 2$ ,  $t = 1, \dots, 24$ .  
 $r_t$  = sales to ElKräng hour  $t$ ,  $t = 1, \dots, 24$ ,  
 $S_{i,t}$  = spillage from reservoir  $i$  during hour  $t$ ,  $i = 1, 2$ ,  $t = 1, \dots, 24$ ,  
 $S_{\text{channel}}$  = water flow through the channel between Forsen and Storälven,  
 $V_{i,t}$  = local inflow to reservoir  $i$  during hour  $t$ ,  $i = 1, 2$ ,  $t = 1, \dots, 24$ .

Formulate the hydrological constraint of Forsen, hour  $t$ . The water delay time between the power plants can be neglected. Use the symbols above.

**c) (2 p)** In the following cases it is necessary to use integer variables to model the electricity generation in a thermal power plant: I) When the power plant has a start-up cost stated in SEK/start, II) When the power plant has a stop cost which is stated in SEK/stop, III) When the power plant has a maximum generation level,  $\bar{G}$ , when committed.

1. None of the statements is true.
2. I and II are true but not III.
3. I and III are true but not II.
4. II and III are true but not I.
5. All the statements are true.

**d) (2 p)** The following variables and parameters have been introduced in a short-term planning problem for a thermal power plant:

- $C^+$  = start-up cost of the power plant,
- $C^-$  = stop cost of the power plant,
- $G_t$  = generation in the power plant during hour  $t$ ,
- $s_t^+$  = start-up variable for hour  $t$  (1 if the power plants starts generating at the beginning of hour  $t$ , otherwise 0),
- $s_t^-$  = stop variable for hour  $t$  (1 if the power plant stops generating at the beginning of hour  $t$ , otherwise 0).
- $u_t$  = unit commitment during hour  $t$  (1 if the power plant is committed, otherwise 0),
- $\beta$  = variable generation cost.

How can the unit commitment constraint be formulated in this problem?

$$\text{minimise} \quad \sum_{t \in \mathcal{T}} (\beta G_t + C^+ s_t^+ + C^- s_t^-).$$

Which of the following constraints can be used to control the relation between start-up, stop and unit commitment?

- I)  $u_t - u_{t-1} - s_t^+ = 0$ ,
- II)  $u_t - u_{t-1} - s_t^+ \leq 0$ ,
- III)  $u_t - u_{t-1} - s_t^+ - s_t^- = 0$ .

1. None of the alternatives is correct.
2. Only alternative I is correct.
3. Only alternative II is correct.
4. Only alternative III is correct.
5. It is possible to choose between using alternative I and alternative II.

## Problem 5 (12 p)

Consider an electricity market where there are three power plants: a 400 MW thermal power plant (variable cost 15  $\text{€}/\text{MWh}$ ), a 600 MW hydro power plant (negligible operation cost) and an 800 MW thermal power plant (variable cost 10  $\text{€}/\text{MWh}$ ). The equivalent load duration curves when adding these power plants are shown in table 2.

**Table 2** Equivalent load duration curves for the electricity market in problem 5a–d.

Interval	$\tilde{F}_1(x)$	$\tilde{F}_2(x)$	$\tilde{F}_3(x)$
$0 \leq x < 400$	1	1	1
$400 \leq x < 600$	0.9900	0.9910	0.9919
$600 \leq x < 800$	0.9000	0.9100	0.9190
$800 \leq x < 1\,000$	0.5000	0.5500	0.5941
$1\,000 \leq x < 1\,200$	0.1000	0.1900	0.2620
$1\,200 \leq x < 1\,400$	0.0100	0.1080	0.1522
$1\,400 \leq x < 1\,600$	0	0.0900	0.1000
$1\,600 \leq x < 1\,800$	0	0.0500	0.0558
$1\,800 \leq x < 2\,000$	0	0.0100	0.0180
$2\,000 \leq x < 2\,200$	0	0.0010	0.0059
$2\,200 \leq x < 2\,400$	0	0	0.0010
$2\,400 \leq x < 2\,600$	0	0	0.0001
$2\,600 \leq x$	0	0	0

- a) (2 p) How large is the expected generation per hour in the smaller thermal power plant?
- b) (2 p) How large is the expected generation per hour in the larger thermal power plant?
- c) (1 p) How large is the expected operation cost of this system?
- d) (1 p) How large is the risk of power deficit in this system?
- e) (4 p) Assume that an electricity market has been simulated using Monte Carlo methods. To obtain an accurate result from the Monte Carlo simulation, it has been decided to use stratified sampling.

The results of the fifteen first scenarios of the Monte Carlo-simulation are compiled in table 3. Which estimates of *ETOC* and *LOLP* are obtained from these results?

**Table 3** Results from a Monte Carlo simulation of an electricity market.

Stratum	Stratum weight	Observations of <i>TOC</i> [ $\text{€}/\text{h}$ ]	Observations of <i>LOLO</i>
1	0.9	3 500, 5 000, 4 500, 3 000, 4 000	0, 0, 0, 0, 0
2	0.09	5 250, 5 500, 7 500, 5 500, 6 250	0, 0, 1, 0, 0
3	0.01	7 500, 7 500, 7 500, 5 000, 7 500	1, 1, 1, 1, 1

**f) (2 p)** The expectation value  $E[X]$  is to be determined using a control variate. Let  $x_i$  denote the  $i$ :th observation of  $X$  and let  $z_i$  denote the  $i$ :th observation of the control variate,  $Z$ . The total number of observations is  $n$ . How is the estimate  $m_X$  calculated?

$$1. m_X = \frac{1}{n} \sum_{i=1}^n x_i + E[Z].$$

$$2. m_X = \frac{1}{n} \sum_{i=1}^n z_i - E[Z]$$

$$3. m_X = \frac{1}{n} \sum_{i=1}^n (x_i - z_i) + E[Z].$$

$$4. m_X = \frac{1}{n} \sum_{i=1}^n (x_i - z_i) - E[Z].$$

$$5. m_X = \frac{1}{n} \sum_{i=1}^n (z_i - x_i) + E[Z].$$



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## Answer sheet

Name: .....

Personal number: .....

### Problem 1

a) Alternative ..... is correct.

b) Alternative ..... is correct.

c) Alternative ..... is correct.

### Problem 2

a) ..... TWh      b) ..... TWh

c) ..... TWh      d) ..... TWh

### Problem 3

a) ..... MW/Hz      b) ..... MW

c) ..... MW

### Problem 4

a)  $\mu_1$  ..... MWh/HE       $\mu_2$  ..... MWh/HE

$\bar{Q}_1$  ..... HE       $\bar{Q}_2$  ..... HE

b) .....

c) Alternative ..... is correct.

d) Alternative ..... is correct.

### Problem 5

a) ..... MWh/h      b) ..... MWh/h

c) .....  $\alpha$ /h      d) ..... %

e) *ETOC* .....  $\alpha$ /h      *LOLP* ..... %

f) Alternative ..... is correct.



### Problem 1

- a) 4, b) 1, c) 3.

### Problem 2

- a) As the electricity price is higher than the variable operation cost in the most expensive nuclear power plant, all nuclear power in Land will be utilised, i.e., 35 TWh/year.  
 b) As the electricity price is lower than the variable operation cost in the least expensive gas turbine, no gas turbines will be used.  
 c) The part of the coal condensing which has a lower variable operation cost than the electricity price 350  $\square$ /MWh will be used, i.e.,  $(350 - 300)/(450 - 300) \cdot 15 = 5$  TWh/year.  
 d) As the price is lower during the first half of the year, the reservoir will be filled between 30 June and 1 July (if there had not been a reservoir limitation then the electricity price would have been the same for the entire year). The hydro power can generate in total 20 TWh (start contents) + 150 TWh (inflow) = 170 TWh during the first half of the year. The electricity consumption during this period is 145 TWh, out of which 40 TWh will be covered by nuclear power and coal condensing; hence, the hydro generation must be 105 TWh. Consequently, the reservoirs must store 65 TWh.

### Problem 3

- a) For a frequency change of 0.1 Hz the generation increases by 15 MW; thus, the gain is  $R = \Delta G/\Delta f = 25/0.1 = 150$  MW/Hz.  
 b) If we for example consider the electricity generation at the frequency 49.92 Hz then the formula  $G = G_0 - R(f - f_0)$  yields that  $G_0 = 194 + 150(49.92 - 50) = 182$  MW.  
 c) The generation at the frequency 49.8 Hz should be  $G = G_0 - R(f - f_0) = 182 - 100(49.8 - 50) = 212$  MW. This generation is however larger than the installed capacity of the power plant. The generation at this frequency must therefore be equal to installed capacity, i.e., 200 MW.

### Problem 4

- a) The following data are given in the problem text:

$$\bar{Q} = \text{discharge in Forsen at best efficiency} = 90,$$

$$\hat{Q} = \text{maximal discharge in Forsen} = 120,$$

$$\chi(\hat{Q}) = \text{production equivalent at best efficiency in Forsen} = 0.24,$$

$$\chi(\bar{Q}) = \text{production equivalent at maximal discharge in Forsen} = 0.25.$$

To calculate the marginal production equivalents, we need the generation at best efficiency as well as maximal discharge. According to the definition we have

$$H(Q) = \chi(Q) \cdot Q = \begin{cases} 22.5 & Q = 90, \\ 28.8 & Q = 120. \end{cases}$$

The marginal production equivalents can now be calculated according to

$$\mu_1 = \frac{\hat{H}}{\hat{Q}}$$

and

$$\mu_2 = \frac{\bar{H} - \hat{H}}{\bar{Q} - \hat{Q}},$$

which results in the following linear models of the power plant:

$\mu_j$  = marginal production equivalent in Forsen, segment  $j$  =

$$= \begin{cases} 0.25 & j = 1, \\ 0.21 & j = 2, \end{cases}$$

$$\bar{Q}_j = \text{maximal discharge in Forsen, segment } j = \begin{cases} 90 & j = 1, \\ 30 & j = 2. \end{cases}$$

- b)  $M_{1,r} = M_{1,r-1} + V_{1,r} - Q_{1,1,r} - Q_{1,2,r} - S_{1,r} - S_{\text{channel}}$   
 c) 2.  
 d) 1.

### Problem 5

- a) The smaller thermal power plant is added as the third unit, because it is the most expensive power plant in the system. The expected generation is then  

$$EG_3 = EENS_2 - EENS_3 = \int_{1400}^{\infty} \bar{F}_2(x) dx - \int_{1800}^{\infty} \bar{F}_3(x) dx =$$

$$= 200(0.09 + 0.05 + 0.01 + 0.001) - 200(0.018 + 0.0059 + 0.001 + 0.0001) = 25.2 \text{ MWh/h.}$$
 b) The smaller thermal power plant is added as the second unit, because it is the second least expensive power plant in the system. The expected generation is then

$$EG_2 = EENS_1 - EENS_2 = \int_{600}^{\infty} \bar{F}_1(x) dx - \int_{1400}^{\infty} \bar{F}_2(x) dx =$$

$$= 200(0.9 + 0.5 + 0.1 + 0.01) - 200(0.09 + 0.05 + 0.01 + 0.001) = 271.8 \text{ MWh/h.}$$

- c)  $ETOC = 10EG_2 + 15EG_3 = 3096 \square$  /h.

- d)  $LOLP = \bar{F}_3(1800) = 1.8\%$ .

- e) The following estimates are obtained of the expectation value in each stratum:

$$m_{TOC1} = 20000/5 = 4000$$

$$m_{LOLO1} = 0$$

$$m_{TOC2} = 30000/5 = 6000$$

$$m_{LOLO2} = 1/5 = 0.2$$

$$m_{TOC3} = 35000/5 = 7000$$

$$m_{LOLO3} = 5/5 = 1$$

Thus, we get

$$m_{TOC} = \sum_{h=1}^3 \omega_h m_{TOCh} = 0.9 \cdot 4.000 + 0.09 \cdot 6.000 + 0.01 \cdot 7.000 = 4.210 \text{ €}/h,$$

$$m_{LOLO} = \sum_{h=1}^3 \omega_h m_{LOLoh} = 0 + 0.09 \cdot 0.2 + 0.01 \cdot 1 = 2.8\%.$$

☐ 3.