



KTH Electrical Engineering

**Complementary test in  
EG2050 System Planning,  
17 September 2009, 9:00-11:00, large conference room**

**Instructions**

Only the problems indicated on the attached answer sheet have to be answered (the score of the remaining problems is kept from the exam). Motivations and calculations do not have to be presented.

The maximal score of the complementary test is 40 points including the points that are kept from the exam. You are guaranteed to pass if you get at least 33 points.

**Allowed aids**

In this complementary test you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten, single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

## Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

**a) (2 p)** A balance responsible player has the following responsibilities: I) Economical responsibility that the system during each trading period (for example one hour) is supplied as much energy as consumed by the customers of the player, II) Physical responsibility that the system during each trading period (for example one hour) is supplied as much energy as consumed by the customers of the player, III) Physical responsibility that the system continuously is supplied as much power as consumed by the customers of the player.

1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. I and II are true but not III.

**b) (2 p)** The following applies to a down-regulation bid in a regulation market: I) If the bid is activated it means that the player who submitted the bid is buying energy from the system operator, II) A down-regulation bid can be performed by decreasing the generation in for example a hydro power plant, III) A down-regulation bid can be performed by decreasing the consumption in for example a large factory.

1. Only I is true.
2. I and II are true but not III.
3. I and III are true but not II.
4. II and III are true but not I.
5. All the statements are true.

## Problem 2 (6 p)

**a) (2 p)** The electricity market in Land has perfect competition, all players have perfect information and there are neither capacity, transmission nor reservoir limitations in the system. Data for the electricity producers of Land are given in table 1. The variable costs of the coal condensing are assumed to be linear in the given interval, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level. Assume that the consumers in the electricity market of Land are not price sensitive. What will the electricity price be if the consumption is 175 TWh/year?

**Table 1** Data for the electricity producers in Land.

Power source	Production capability [TWh/year]	Variable costs [ $\text{€}/\text{MWh}$ ]
Hydro	65	10
Nuclear	60	100
Coal condensing	25	300–500

**b) (2 p)** Consider an electricity market where there is perfect competition, where all players have perfect information and where there are neither capacity, transmission nor reservoir limitations. During a certain day, the electricity price in this electricity market will exceed 380  $\text{€}/\text{MWh}$  during 20 hours. How much will a power plant with the variable operation cost 380  $\text{€}/\text{MWh}$  generate during this day, if the installed capacity in the power plant is 200 MW?

**c) (2 p)** Strålinge AB owns a nuclear power plant with a production capability of 8 TWh per year. The variable costs of the power plant are 100  $\text{€}/\text{MWh}$  and the company has fixed costs of 2 600 M $\text{€}/\text{year}$ . How high must the electricity price at least become if the company is not going to make a loss?

### Problem 3 (6 p)

The power system in Land is divided in two areas (A and B) which are connected by an AC transmission line. This line has a maximal capacity of 1 000 MW and is equipped with a protection system which after a short time delay disconnects the line if the power flow exceeds the maximal capacity of the line.

At 8:45 a fire breaks out in a substation in Stad (which is located in area A). Due to the fire, the entire regional grid of Stad must immediately be disconnected from the national grid in Land, which means that the national grid loses 200 MW generation and 500 MW load. After the disconnection of Stad, the gain in Land is Land 5 000 MW/Hz in area A and 5 000 MW/Hz in area B.

Just before the disconnection of Stad, the frequency of the system was 50.02 Hz and 750 MW was transmitted from area A to area B.

**a) (1 p)** What happens when the regional grid of Stad is disconnected?

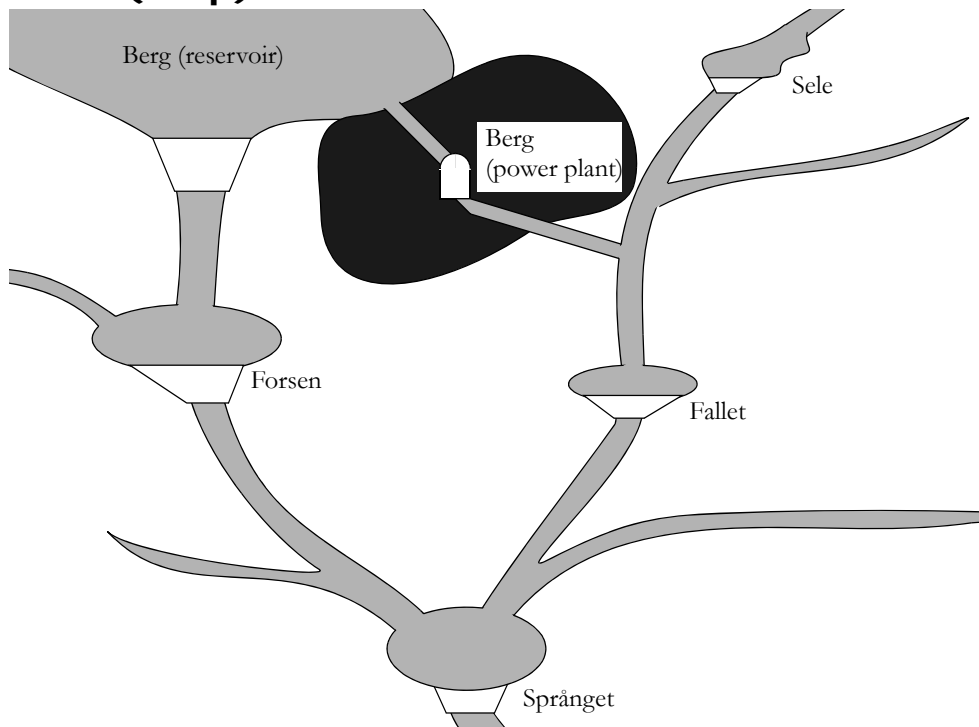
1. There is a surplus of energy, which results in a voltage increase in the grid. The control systems of the power plants participating in the primary control responds to the voltage increase by reducing the electricity generation.
2. There is a surplus of energy, which is stored as kinetic energy in all synchronous generators; hence, the frequency of the system increases. The control systems of the power plants participating in the primary control responds to the frequency increase by reducing the electricity generation.
3. There is a deficit of energy, which is compensated using kinetic energy from all synchronous generators; hence, the frequency of the system increases. The control systems of the power plants participating in the primary control responds to the frequency increase by increasing the electricity generation.

**b) (1 p)** Will the transmission line between area A and B be disconnected due to overloading?

**c) (2 p)** Which frequency will there be in area A and B respectively, once the primary control has restored the balance between generation and consumption?

**d) (2 p)** The power plant Språnget is located in area B. The installed capacity of the power plant is 300 MW and the gain is set to 200 MW/Hz. The base generation (i.e., the generation when the frequency is exactly 50 Hz) is 200 MW. How much does Språnget generate when the frequency is 50.02 Hz?

### Problem 4 (12 p)



AB Vattenkraft owns three hydro power plant located as in the figure above. Notice that Berg is an underground power plant and that water that is discharged through the turbine is flowing to Fallet, whereas spillage ends up in Forsen. The following symbols have been introduced in a short-term planning problem for these hydro power plants:

Indices for the power plants: Berg - 1, Sele - 2, Forsen - 3, Fallet - 4, Språnget - 5.

$\gamma_i$  = expected future production equivalent for water stored in reservoir  $i$ ,  
 $i = 1, \dots, 5$ ,

$D_t$  = contracted load hour  $t$ ,  $t = 1, \dots, 24$ ,

$\lambda_{25}$  = expected electricity price at Elkkrång after the end of the planning period,

$M_{i,0}$  = contents of reservoir  $i$  at the beginning of the planning period,  $i = 1, \dots, 5$ ,

$M_{i,t}$  = contents of reservoir  $i$  at the end of hour  $t$ ,  $i = 1, \dots, 5$ ,  $t = 1, \dots, 24$ ,

$\mu_{i,j}$  = marginal production equivalent in power plant  $i$ , segment  $j$ ,  
 $i = 1, \dots, 5$ ,  $j = 1, 2$ ,

$Q_{i,j,t}$  = discharge in power plant  $i$ , segment  $j$ , during hour  $t$ ,  
 $i = 1, \dots, 5$ ,  $j = 1, 2$ ,  $t = 1, \dots, 24$ ,

$S_{i,t}$  = spillage from reservoir  $i$  during hour  $t$ ,  $i = 1, \dots, 5$ ,  $t = 1, \dots, 24$ ,

$V_{i,t}$  = local inflow to reservoir  $i$  during hour  $t$ ,  $i = 1, \dots, 5$ ,  $t = 1, \dots, 24$ .

**a) (3 p)** Which of the symbols above represent optimisation variables and parameters respectively?

**b) (4 p)** Formulate the objective function if the purpose of the planning is to maximise the value of stored water. Use the symbols defined above.

**c) (1 p)** The reservoir of Språnget holds 1 000 HE at 9:00. The local inflow as well as discharge and spillage from the power plant upstream amounts to  $120 \text{ m}^3/\text{s}$  between 9:00 and 10:00. During the same time, 75 HE are discharged from Forsen. How much will the reservoir of Forsen hold at 10:00? Notice that the answer should be given in  $\text{m}^3$ !

**d) (2 p)** Consider a short-term planning problem concerning  $G$  thermal power plants and  $T$  time periods. The following constraints for maximal and minimal generation have been introduced in this problem:

$$G_{g,t} \leq u_{g,t} \bar{G}_g, \quad g = 1, 2, \dots, G, t = 1, 2, \dots, T$$

$$G_{g,t} \geq u_{g,t} \underline{G}_g, \quad g = 1, 2, \dots, G, t = 1, 2, \dots, T$$

Which of the symbols above represent parameters and variables respectively?

1. All symbols are parameters.
2.  $\bar{G}_g$ ,  $\underline{G}_g$ , and  $u_{g,t}$  are parameters,  $G_{g,t}$  are optimisation variables.
3.  $\bar{G}_g$  and  $\underline{G}_g$  are parameters,  $u_{g,t}$  and  $G_{g,t}$  are optimisation variables.
4. Only  $u_{g,t}$  are parameters,  $\bar{G}_g$ ,  $\underline{G}_g$ , and  $G_{g,t}$  are optimisation variables.
5. All symbols are optimisation variables.

**e) (2 p)** Assume that it has been decided that a thermal power plant should not be shut down for a shorter time than four hours, i.e., if the power plant is stopped at 12:00 then it may not be started again before 16:00. Introduce the following symbols:

$s_t^+$  = start-up variable for hour  $t$  (1 if the power plant is starting generation at the beginning of hour  $t$ , otherwise 0),

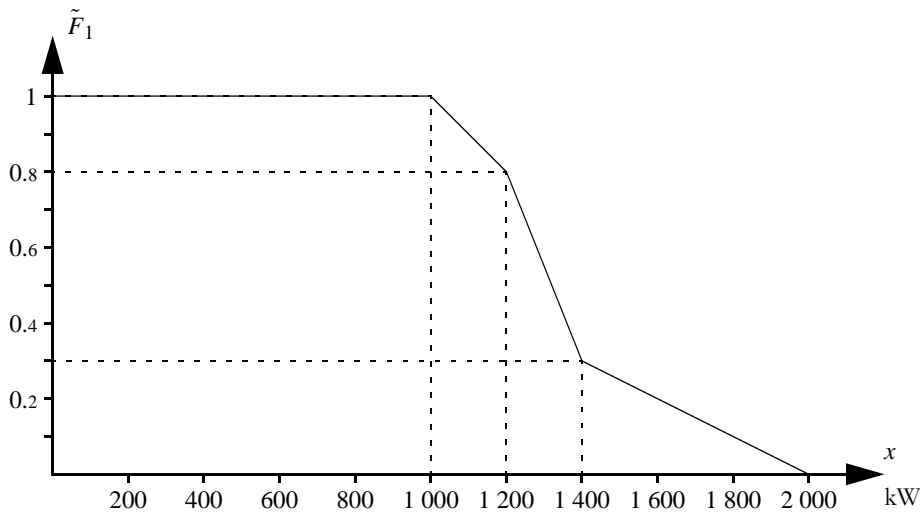
$s_t^-$  = stop variable for hour  $t$  (1 if the power plant is stopping generation at the beginning of hour  $t$ , otherwise 0).

How should a linear constraint be formulated in order to describe the relation between  $s_t^-$ ,  $s_{t+1}^+$ ,  $s_{t+2}^+$  and  $s_{t+3}^+$ ?

1.  $s_t^- - s_{t+1}^+ - s_{t+2}^+ - s_{t+3}^+ = 0$ .
2.  $s_t^- - s_{t+1}^+ - s_{t+2}^+ - s_{t+3}^+ \leq 1$ .
3.  $s_t^- - s_{t+1}^+ - s_{t+2}^+ - s_{t+3}^+ = 1$ .
4.  $s_t^- + s_{t+1}^+ + s_{t+2}^+ + s_{t+3}^+ \leq 1$ .
5.  $s_t^- + s_{t+1}^+ + s_{t+2}^+ + s_{t+3}^+ = 1$ .

## Problem 5 (12 p)

Mji is a town in East Africa. The town is not connected to a national grid, but has a local system of its own. The local grid is supplied by a hydro power plant and two diesel generator sets. The hydro power plant does not have a reservoir, but the water flow is always sufficient to generate the installed capacity (1 400 kW) and the risk for outages in the power plant is negligible. The diesel generator sets have a capacity of 200 kW each, the availability is 80% and the operation cost is 1 ¢/kWh.



**a) (2 p)** What is the expectation value of the load?

*Hint: Study  $EENS_0$ !*

**b) (2 p)** Using probabilistic production cost simulation it is calculated that the expected hydro power generation of this system is 1 290 kWh/h and that the unserved energy is 22.8 kWh/h. How large is the expected operation cost?

**c) (3 p)** Use probabilistic production cost simulation to calculate the risk of power deficit in the system.

**d) (2 p)** Assume that complementary random numbers are used to improve the simulation of Rike. What is the value of the complementary random number,  $D^*$ , if the total load of the system is randomised to  $D = 1\,100$  MW?

**e) (3 p)** To consider the losses in the grid, a Monte Carlo simulation has been performed of the power system in Mji. The simulation is using control variates. The simplified model corresponds to the model used in probabilistic production cost simulation, whereas the detailed model considers such factors as how the losses depends on which power plants that are operated, and how the load is distributed within the system. The results are shown in table 2. Which estimate of  $ETOC$  is obtained using the detailed model?

**Table 2** Results from a Monte Carlo simulation of the power system in Mji.

Number of scenarios, $n$	Results from the detailed model, $\sum_{i=1}^n toc_i$	Results from the simplified model, $\sum_{i=1}^n \tilde{toc}_i$
1 000	68 100	66 500



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## Answer sheet

Name: .....

Personal number: .....

### Problem 1

a) Alternative ..... is correct.

b) Alternative ..... is correct.

### Problem 2

a) .....  $\alpha$ /MWh    b) ..... MWh

c) The price must be higher than .....  $\alpha$ /MWh

### Problem 3

a) Alternative ..... is correct    b) .....

c) Area A: ..... Hz    Area B: ..... Hz

d) ..... MW

### Problem 4

a) Parameters: .....

Optimisation variables: .....

b) .....

c) .....  $m^3$

d) Alternative ..... is correct.

e) Alternative ..... is correct.

### Problem 5

a) ..... kWh/h    b) .....  $\alpha$ /h

c) ..... %    d) ..... MW

e) .....  $\alpha$ /h



### Problem 1

- a) 2, b) 2.

### Problem 2

- a) Hydro power and nuclear power can generate in total 125 TWh, which means that 20 TWh is needed, i.e., 80% of the coal condensing potential. The electricity price must therefore use 80% of the price interval for coal condensing, which means that the electricity price has to be 460  $\text{€}/\text{MWh}$ .
- b) The power plant will generate its installed capacity during those hours when the electricity price is higher than the variable operation cost. 200 MW during 20 hours equals a total generation of 4 000 MWh.
- c) For each  $\text{€}/\text{MWh}$  the electricity price exceeds the variable costs, the company earns 1  $\text{€}/\text{MWh}$  · 8 TWh/year = 8 M $\text{€}/\text{year}$ . This surplus must be sufficient to cover the fixed costs, which means that the electricity price must be  $100 + 2 \cdot 600/8 = 425 \text{ €}/\text{MWh}$ .

### Problem 3

- a) 2.
- b) After the disconnection of Stad, the primary control has to reduce the generation by 300 MW. Area B has half the gain in the system and will consequently contribute to half of the generation decrease. Since the load and the other generation in area B remains the same, the generation decrease will have to be compensated by increased import from area A. The transmission on the line will therefore increase to 900 MW, which is less than the capacity of the line. Hence, the line will not be disconnected.
- c) Since the two areas are still connected by an AC line, the frequency will be the same in both areas. If the generation is to decrease by 300 MW then it is necessary that the frequency increases by  $\Delta f = \Delta G/R = 300/10\,000 \text{ Hz}$ , i.e., the new frequency will be  $50.02 + 0.03 = 50.05 \text{ Hz}$ .
- d) When the frequency is 50.02 Hz, Språnglet is generating

$$G = G_0 - R(f - f_0) = 200 - 200(50.02 - 50) = 196 \text{ MW.}$$

### Problem 4

- a) Parameters:  $\gamma_p, D, \lambda_{25}, M_{i, \phi}, H_{i, j}$  and  $V_{i, r}$ . Optimisation variables:  $M_{i, p}, S_{i, r}$  and  $Q_{i, j, r}$
- b) maximise  $\lambda_{25}(\gamma_1 + \gamma_4 + \gamma_5)M_{1, 24} + \lambda_{25}(\gamma_2 + \gamma_4 + \gamma_5)M_{2, 24} + \lambda_{25}(\gamma_3 + \gamma_5)M_{3, 24} + \lambda_{25}(\gamma_4 + \gamma_5)M_{4, 24} + \lambda_{25}^2/\gamma_5 M_{5, 24}$
- c) As the flow into the reservoir is 120 HE and the flow out from the reservoir is 75 HE, it must hold  $1\,045 \text{ HE} = 3\,762\,000 \text{ m}^3$  water at the end of the hour.
- d) 3.
- e) 4.

### Problem 5

- a)  $EENS_0 = \int_0^\infty \tilde{F}_0(x) dx = 1\,000 \cdot 1 + 200 \cdot (1 + 0.8)/2 + 200 \cdot (0.8 + 0.3)/2 + 600 \cdot 0.3/2 = 1\,380 \text{ MWh/h}$ .
- b) The expected load is 1 380 kWh/h, which means that  $EG_1 + EG_2 + EG_3 + EENS_3 = 1\,380$ . As  $EG_1 = 1\,290 \text{ kWh/h}$  and  $EENS_3 = 22.8 \text{ kWh/h}$  it can be concluded that the expected generation in the diesel generator sets is 67.2 kWh/h. Hence, the expected operation cost is  $ETOC = 1 \cdot (EG_2 + EG_3) = 67.2 \text{ €}/\text{h}$ .
- c) The risk of power deficit is given by
- $$\begin{aligned} \tilde{F}_3(1\,800) &= 0.8\tilde{F}_2(1\,800) + 0.2\tilde{F}_2(1\,600) = \\ &= 0.8(0.8\tilde{F}_1(1\,800) + 0.2\tilde{F}_1(1\,600)) + 0.2(0.8\tilde{F}_1(1\,600) + 0.2\tilde{F}_1(1\,400)). \end{aligned}$$
- Since the hydro power plant has 100% availability we get  $\tilde{F}_1(x) = \tilde{F}_0(x)$ ; hence,  $LOLP = 0.8(0.8 \cdot 0.1 + 0.2 \cdot 0.2) + 0.2(0.8 \cdot 0.2 + 0.2 \cdot 0.3) = 14\%$ .
- d) The inverse transform method states that  $D = F_D^{-1}(U)$ , where  $U$  is a  $U(0, 1)$ -distributed random number. Since it is the duration curve that is given in the problem, we may as well use the transform  $D = \tilde{F}_D^{-1}(U)$ . The original random number must then have been  $U = \tilde{F}_D(1\,100) = 0.9$ . Hence,  $U^* = 1 - U = 0.1$ , which results in  $D^* = \tilde{F}_D^{-1}(U^*) = 1\,800 \text{ MW}$ .
- e)  $m_{TROC} = m_{TROC} - \tilde{r}OC + \mu \tilde{r}OC = \frac{1}{n} \left( \sum_{i=1}^n \text{toc}_i - \sum_{i=1}^n \tilde{\text{toc}}_i \right) + 67.2 =$   
 $= \frac{1}{1\,000} (68\,100 - 66\,500) + 67.2 = 68.8 \text{ €}/\text{h}$ .