



KTH Electrical Engineering

## Complementary test in EG2050 System Planning, 13 April 2010, 14:00-16:00, L31

### Instructions

Only the problems indicated on the attached answer sheet have to be answered (the score of the remaining problems is kept from the exam). Motivations and calculations do not have to be presented.

The maximal score of the complementary test is 40 points including the points that are kept from the exam. You are guaranteed to pass if you get at least 33 points.

### Allowed aids

In this complementary test you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten, single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

## Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

**a) (2 p)** The following applies to an up-regulation bid in a regulation market: I) If the bid is activated it means that the player who submitted the bid is selling energy to the system operator, II) An up-regulation bid can be performed by increasing the generation in for example a hydro power plant, III) An up-regulation bid can be performed by decreasing the consumption in for example a large factory.

1. Only I is true.
2. I and II are true but not III.
3. I and III are true but not II.
4. II and III are true but not I.
5. All the statements are true.

**b) (2 p)** Consider a trading period of one hour between 8:00 and 9:00. Post trading for this hour means that I) Balance responsible players who have supplied more energy to the grid than they have extracted must sell balance power to the system operator, II) Balance responsible players who have extracted more energy from the grid than they have supplied must buy balance power of the system operator, III) In many electricity markets (as for example the Nordic) the price of the balance power depends on the up- and down-regulation prices from the real-time trading.

1. None of the statements is true.
2. Only II is true.
3. Only III is true.
4. I and II are true but not III.
5. All the statements are true.

## Problem 2 (6 p)

Assume that the electricity market in Land has perfect competition, perfect information and that there are neither capacity, transmission nor reservoir limitations. Data for the power plants in Land are shown in table 1. The variable operation costs are assumed to be linear within the intervals, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

**Table 1** Data for the electricity producers in Land.

Power source	Production capability [TWh/year]	Variable costs [¤/MWh]
Hydro power	60	5
Nuclear power	50	90–100
Biofuel	20	100–400
Fossil gas	10	200–300
Import from neighbouring countries	10	300–500

- a) (3 p)** What will the electricity price be in Land if the electricity consumption is not price sensitive and amounts to 141.5 TWh/year?
- b) (1 p)** Assume that the nuclear power producers have to pay a waste deposit fee of 20 ¤ for each generated MWh. Which electricity price will there be in Land?
- c) (2 p)** If capacity limitations in the power plants and transmission limits are considered, the electricity prices in Land and the neighbouring countries will vary from hour to hour. Consider an hour when the electricity price in Land is 400 ¤/MWh and the electricity price in Mark is 300 ¤/MWh. Assume that the interconnections between the two countries have a capacity of 400 MW. How much is Land importing from Mark during this hour?

### Problem 3 (6 p)

Consider a power system divided in two areas, A and B. There is only one transmission line between these two areas. This line is a 220 kV AC line with a maximal capacity of 1 000 MW. The line is equipped with a protection system which after a short time delay disconnects the line if the maximal capacity is exceeded.

The frequency of the system is 50.02 Hz at 14:00. The power flow on the transmission line is 300 MW from area A to area B. The total gain in the system is 4 000 MW/Hz and is available in the interval  $50 \pm 1$  Hz. Shortly afterwards, a nuclear power plant (which is not participating in the primary control) is stopped in area B, which results in a loss of 1 000 MW generation. The primary control responds to this outage by increasing the generation by 400 MW in area A and 600 MW in area B.

- a) (1 p) How large is the gain in area A?
- b) (1 p) How large is the gain in area B?
- c) (2 p) How large is the transmission from area A to area B then the primary control has stabilised the frequency in the system? (Answer 0 MW if the connection is disconnected due to overloading.)
- d) (2 p) What is the frequency in area A then the primary control has stabilised the frequency in the system?

## Problem 4 (12 p)

Stads energi AB owns a thermal power plant with three blocks. Moreover, the company owns the hydro power plant Forsen. Assume that the company has formulated their short-term planning problem as a MILP problem and that the following symbols have been introduced:

Indices for the power plants: Block I - 1, Block II - 2, Block III - 3.

- $\beta_{G^g}$  = variable operation cost in power plant  $g$ ,
- $C_g^+$  = start-up cost in power plant  $g$ ,  $g = 1, 2, 3$ ,
- $\gamma$  = expected future production equivalent for water stored in the reservoir of Forsen,
- $D_t$  = contracted load hour  $t$ ,  $t = 1, \dots, 24$ ,
- $G_{g,t}$  = generation in power plant  $g$ , hour  $t$ ,  $g = 1, 2, 3$ ,  $t = 1, \dots, 24$ ,
- $\bar{G}_g$  = installed capacity in power plant  $g$ ,  $g = 1, 2, 3$ ,
- $\underline{G}_g$  = minimal generation when power plant  $g$  is committed,  $g = 1, 2, 3$ ,
- $\lambda_t$  = expected electricity price at ElKräng hour  $t$ ,  $t = 1, \dots, 24$ ,
- $\lambda_{25}$  = expected electricity price at ElKräng after the end of the planning period,
- $M_0$  = contents of the reservoir of Forsen at the beginning of the planning period,
- $M_t$  = contents of the reservoir of Forsen at the end of hour  $t$ ,  $t = 1, \dots, 24$ ,
- $\bar{M}$  = maximal contents of the reservoir of Forsen,
- $p_t$  = purchase from ElKräng hour  $t$ ,  $t = 1, \dots, 24$ ,
- $Q_{j,t}$  = discharge in Forsen, segment  $j$ , hour  $t$ ,  $j = 1, 2$ ,  $t = 1, \dots, 24$ ,
- $\bar{Q}_j$  = maximal discharge in Forsen, segment  $j$ ,  $j = 1, 2$ ,
- $r_t$  = sales to ElKräng hour  $t$ ,  $t = 1, \dots, 24$ ,
- $S_t$  = spillage in Forsen during hour  $t$ ,  $t = 1, \dots, 24$ ,
- $\bar{S}$  = maximal spillage from Forsen,
- $s_{g,t}^+$  = start-up variable for power plant  $g$ , hour  $t$ ,  $g = 1, 2, 3$ ,  $t = 1, \dots, 24$ ,
- $u_{g,0}$  = unit commitment of power plant  $g$  at the beginning of the planning period,  $g = 1, 2, 3$ ,
- $u_{g,t}$  = unit commitment of power plant  $g$ , hour  $t$ ,  $g = 1, 2, 3$ ,  $t = 1, \dots, 24$ .

**a) (4 p)** The best efficiency in the hydro power plant Forsen is obtained for the discharge  $80 \text{ m}^3/\text{s}$  and the electricity generation is then  $64 \text{ MW}$ . The maximal discharge in Forsen is  $120 \text{ m}^3/\text{s}$  and the relative efficiency is then  $95\%$ . Assume that we need a piecewise linear model of electricity generation as function of the discharge in Forsen. The model should have two segments and the breakpoint between them should be located at the best efficiency. Calculate the following parameters:

- $\mu_j$  = marginal production equivalent in Forsen, segment  $j$ ,
- $\bar{Q}_j$  = maximal discharge in Forsen, segment  $j$ .

**b) (4 p)** Assume that Stads energi AB sells power to customers with firm power contracts, and is also trading at the local power exchange ElKräng, where the company has the possibility to both sell and purchase electricity. Formulate the load balance constraint of Stads energi AB. Use the symbols defined above.

**c) (4 p)** Formulate the limits for the optimisation variables defined above for the planning problem of Stads energi AB. To get full score for this problem, you will also have to state the possible index values for each limit.

## Problem 5 (12 p)

Consider an electricity market with a normally distributed load, which is supplied by three power plants having the installed capacities 300, 200 and 100 MW respectively. Table 2 shows some partial results of a probabilistic production cost simulation of this electricity market.

**Table 2** Results from a probabilistic production cost simulation of the electricity market in problem 5.

	$x = 100$	$x = 200$	$x = 300$	$x = 400$	$x = 500$	$x = 600$	$x = \infty$
$F_0(x)$	0.999	0.839	0.156	0.001	0.000	0.000	0.000
$\int_0^x \tilde{F}_0(\xi) d\xi$	100.980	196.673	245.991	249.982	250.000	250.000	250.000
$F_1(x)$	0.999	0.855	0.241	0.101	0.084	0.016	0.000
$\int_0^x \tilde{F}_1(\xi) d\xi$	100.982	197.106	251.492	265.082	274.667	279.599	280.000
$F_2(x)$	0.999	0.870	0.316	0.176	0.100	0.024	0.000
$\int_0^x \tilde{F}_2(\xi) d\xi$	100.983	197.495	256.441	278.284	292.350	298.147	300.000
$F_3(x)$	0.999	0.882	0.372	0.190	0.107	0.032	0.000
$\int_0^x \tilde{F}_3(\xi) d\xi$	100.985	197.844	260.546	286.100	300.943	307.568	310.000

- a) (3 p) What is the *LOLP* of the system?
- b) (2 p) How large is the expected generation per hour in the second power plant?
- c) (1 p) What is the expectation value of the load?  
Hint: Study  $EENS_0$ !
- d) (2 p) Assume that the same system is simulated using Monte Carlo techniques and that the random value 100 MW has been generated for the total load. What is the complementary random number of this value?
- e) (4 p) Assume that the same electricity market is simulated using a multi-area model. The results of 10 000 scenarios are shown in table 3. This simulation is using both control variates and stratified sampling. Which estimate of *LOLP* is obtained from this simulation?

**Table 3** Results from a Monte Carlo simulation of the system in problem 5.

Stratum, $h$	Stratum weight, $\omega_h$	Number of scenarios, $n_h$	Results from multi-area model, $\sum_{i=1}^h x_{i,h}$ (where $x_{i,h}$ is the observed value of <i>LOLO</i> in scenario $i$ , stratum $h$ )	Results from PPC model, $\sum_{i=1}^h z_{i,h}$ (where $z_{i,h}$ is the observed value of <i>LOLO</i> <sub>PPC</sub> in scenario $i$ , stratum $h$ )
1	0.956	5 000	0	0
2	0.012	4 900	980	0
3	0.032	100	100	100



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## Answer sheet

Name: .....

Personal number: .....

### Problem 1

a) Alternative ..... is correct.

b) Alternative ..... is correct.

### Problem 2

a) .....  $\alpha$ /MWh    b) .....  $\alpha$ /MWh

c) ..... MWh

### Problem 3

a) ..... MW/Hz    b) ..... MW/Hz

c) ..... MW    d) ..... Hz

### Problem 4

a)  $\mu_1$  ..... MWh/HE     $\mu_2$  ..... MWh/HE

$\bar{Q}_1$  ..... HE     $\bar{Q}_2$  ..... HE

b) .....

c) .....

.....

.....

### Problem 5

a) ..... %    b) ..... MWh/h

c) ..... MW    d) ..... MW

e) ..... %

Suggested solution for complementary test i EG2050 System Planning, 13 April 2010.

### Problem 1

b) 5, b) 5.

### Problem 2

a) Assume that the electricity price,  $\lambda$ , is in the range 300 to 500  $\text{€}/\text{MWh}$ . Hydro power, nuclear power and fossil gas will generate 120 TWh; thus, the other two power sources must generate 21.5 TWh together. The contribution from biofuel and import can be expressed as

$$\frac{\lambda - 100}{400 - 100} \cdot 20 + \frac{\lambda - 300}{500 - 300} \cdot 8.$$

Setting this expression equal to 21.5 and solving for  $\lambda$  yields the electricity price  $\lambda = 370 \text{ €}/\text{MWh}$ .

b) The variable operation cost of the nuclear power including the fee will be in the interval between 110 and 120  $\text{€}/\text{MWh}$ . Since this is less than the old electricity price in Land, the nuclear power will continue to operate at maximal capacity, and the same power plants as before (i.e. bio fuel and import) will determine the price. Hence, the electricity price will remain 370  $\text{€}/\text{MWh}$ .

c) The import should be maximal, since the electricity price in Mark is lower than the electricity price in Land; hence, the import during this hour is 400 MWh.

### Problem 3

a) As  $\Delta f = \Delta G_{tot}/R_{tot}$  and  $\Delta G_A = R_A \Delta f$  we get that  $R_A/R_{tot} = \Delta G_A/\Delta G_{tot}$  (i.e., the share of area A of the total gain is proportional to the share of area A of the total generation increase due to the failure), which means that  $R_A = 400/1000 \cdot 4000 = 1600 \text{ MW/Hz}$ .

b) The same reasoning as in part a yields  $R_B = 600/1000 \cdot 4000 = 2400 \text{ MW/Hz}$ .

c) Area B has lost 1000 MW generation, out of which 600 MW has been replaced by the primary control in area B. The remaining 400 MW must be covered by increased import from area A. Hence, the new flow from area A to area B is  $300 + 400 = 700 \text{ MW}$ , which is less than the maximal capacity of the line.

d) Since the two areas are still one synchronous grid, the frequency can be computed by  $\Delta f = \Delta G_{tot}/R_{tot} = 1000/4000 = 0.25 \text{ Hz}$ , i.e., the new frequency is  $50.02 - 0.25 = 49.77 \text{ Hz}$ .

### Problem 4

a) The following data are given in the problem text:

$$\begin{aligned} \bar{Q} &= \text{maximal discharge in Forsen} = 120, \\ \hat{Q} &= \text{discharge in Forsen at best efficiency} = 80, \\ \hat{H} &= \text{generation in Forsen at best efficiency} = 64, \\ \eta(\bar{Q}) &= \text{relative efficiency at maximal discharge in Forsen} = 0.95. \end{aligned}$$

To calculate the marginal production equivalents, we need the generation at maximal discharge, which can be calculated using the formula  $H = \gamma_{max} \cdot \eta(\bar{Q}) \cdot \bar{Q}$ . First however, we must calculate the

maximal production equivalent, which is obtained at best efficiency:

$$\gamma_{max} = \text{maximal production equivalent in Forsen} = 64/80 = 0.8 \text{ MWh/HE.}$$

The generation we need is now given by

$$\bar{H}_j = \text{maximal generation in Forsen} = 0.80 \cdot 0.95 \cdot 120 = 91.2 \text{ MW.}$$

The marginal production equivalents can now be calculated according to

$$\mu_1 = \frac{\hat{H}}{\bar{Q}}$$

and

$$\mu_2 = \frac{\bar{H} - \hat{H}}{\bar{Q} - \hat{Q}},$$

which results in the following linear models of the power plant:

$$\begin{aligned} \mu_j &= \text{marginal production equivalent in Forsen, segment } j = \\ &= \begin{cases} 0.80 & j = 1, \\ 0.68 & j = 2, \end{cases} \end{aligned}$$

$$\bar{Q}_j = \text{maximal discharge in Forsen, segment } j = \begin{cases} 80 & j = 1, \\ 40 & j = 2. \end{cases}$$

$$\text{b) } \sum_{g=1}^3 G_{g,t} + \sum_{j=1}^2 \mu_j \bar{Q}_{j,t} + P_t = D_t + r_t$$

c) The minimal and maximal generation for each hour is controlled by special constraints. However, we have the following limits for the hydro power variables, electricity trading and the binary variables:

$$\begin{aligned} 0 \leq M_t &\leq \bar{M}, & t = 1, \dots, 24, \\ 0 \leq Q_{j,t} &\leq \bar{Q}_j, & j = 1, 2, t = 1, \dots, 24, \\ 0 \leq S_j &\leq \bar{S}, & t = 1, \dots, 24, \\ 0 \leq P_t & & t = 1, \dots, 24, \\ 0 \leq r_t & & t = 1, \dots, 24, \\ s_t^+ &\in \{0, 1\}, & t = 1, \dots, 24, \\ u_t &\in \{0, 1\}, & t = 1, \dots, 24. \end{aligned}$$

### Problem 5

$$\begin{aligned} \text{a) } LOLP &= \bar{F}_3(600) = 3.2\%, \\ \text{b) } EG_2 &= EENS_1 - EENS_2 = \int_{300}^{\infty} \bar{F}_1(\xi) d\xi - \int_{500}^{\infty} \bar{F}_2(\xi) d\xi = \end{aligned}$$



$$\begin{aligned}
&= \left( \int_0^{\infty} \tilde{F}_1(\xi) d\xi - \int_0^{300} \tilde{F}_1(\xi) d\xi \right) - \left( \int_0^{\infty} \tilde{F}_2(\xi) d\xi - \int_0^{500} \tilde{F}_2(\xi) d\xi \right) = \\
&= (280.000 - 251.492) - (300.000 - 292.350) = 20.858 \text{ MW/h/h.}
\end{aligned}$$

c)  $E[D] = EENS_0 = \int_0^{\infty} \tilde{F}_0(\xi) d\xi = 250 \text{ MW/h/h.}$

d) The normal distribution is symmetrical, which means that if  $D = \mu_D + X$  then  $D^* = \mu_D - X$ . Hence, the complementary random number must be  $D^* = 400 \text{ MW}$

e) We start by computing the expected difference between the multi-area model and the PPC model in each stratum:

$$\begin{aligned}
m_{(X-Z),h} &= \frac{1}{n_h} \sum_{i=1}^{n_h} (x_{i,h} - z_{i,h}) = \frac{1}{n_h} \left( \sum_{j=1}^{n_h} x_{i,h} - \sum_{i=1}^{n_h} z_{i,h} \right) \\
\Rightarrow m_{(X-Z),1} &= 0, \\
m_{(X-Z),2} &= (980 - 0)/4 \cdot 900 = 0.2, \\
m_{(X-Z),3} &= 0.
\end{aligned}$$

We can now combine the results of each stratum weighted by their stratum weights:

$$m_{(X-Z)} = \sum_{h=1}^3 \omega_h m_{(X-Z),h} = 0 + 0.012 \cdot 0.2 + 0 = 0.0024.$$

LOLP of the multi-area model is given by the expected difference plus the result of the PPC model (which was computed in part a):

$$LOLP = m_{(X-Z)} + LOLP_{PPC} = 0.0024 + 0.032 = 0.0345 = 3.44\%.$$