KTH Electrical Engineering

Exam in EG2050 System Planning, 15 March 2012, 14:00–19:00, L21-22, L31, L41-44, Q21

Allowed aids

In this exam you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten**, **single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

PART I (MANDATORY)

Write all answers on the answer sheet provided. Motivations and calculations do not have to be presented.

Part I can yield 40 points in total. The examinee is guaranteed to pass if the score is at least 33 points. If the result in part I is at least 31 points, then there will be a possibility to complement for passing the exam with the grade E.

Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

a) (1 p) Which players have the responsibility to maintain the frequency of the power system within nominal values (for example 49.9–50.1 Hz in the Nordel system)?

1. The system operator.
2. The balance responsible players.
3. The producers.

b) (2 p) The following applies to a vertically integrated electricity market I) The power companies are free to sell to any other power company, II) All electricity trading must be performed via a power exchange, III) The consumers are free to buy from which producer or retailer they want.

1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. I and III are true but not II.

c) (1 p) What does an up-regulation bid mean?

1. A power company is selling electricity to a customer and the customer must in advance notify the power company about how much the customer will consume during each trading period.
2. A player offers to increase the generation (alternatively decrease the consumption) at the request of the system operator.
3. A player offers to decrease the generation (alternatively increase the consumption) at the request of the system operator.

Problem 2 (6 p)

Consider the common electricity market of the two countries Rike and Maa. Assume that there is perfect competition, that all players have perfect information, and that there are neither reservoir nor capacity limitations in the power plants. The power systems of Rike and Maa are interconnected by an HVDC line, which can transfer a certain amount of TWh per year. Data for the common electricity market are shown in table 1. The variable production costs are assumed to be linear in the intervals, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

Table 1 Data for the electricity market in Rike and Maa.

Power source	Production capability [TWh/year]		Variable cost [¤/MWh]
	Rike	Maa	
Hydro power	50	25	30–60
Nuclear power	32	28	100–120
Coal condensing	20	20	300–500
Gas turbines	5	5	800–1 000
Electricity consumption [TWh/year]	80	75	

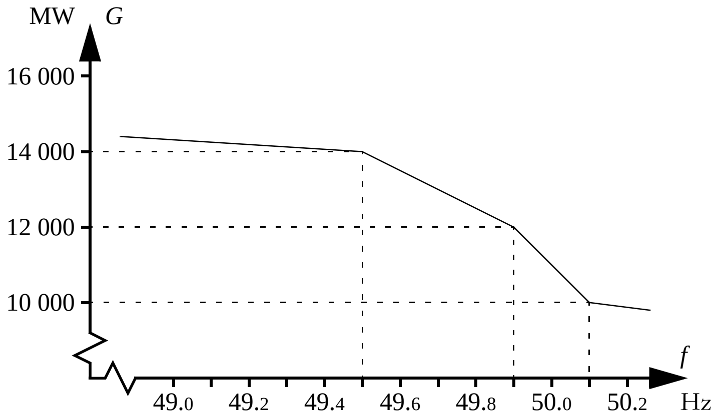
a) (2 p) Assume that the electricity price in Rike is 380 ¤/MWh. How much are the coal condensing power plants in Rike producing?

b) (2 p) Assume that the electricity price in Rike is 380 ¤/MWh and that the electricity price in Maa is higher than 380 ¤/MWh. How large is the transmission capacity between Rike and Maa?

c) (2 p) Which electricity price will there be in Maa?

Problem 3 (6 p)

The figure below shows the total generation (except for wind power) as a function of the frequency in a certain power system.



a) (2 p) At 11:04 there is balance between production and consumption in the system and the frequency is 50.04 Hz. At this time the load of the system is decreased by 100 MW. What will the frequency be when the primary control has restored the balance between generation and consumption?

b) (2 p) At 11:06 there is balance between production and consumption in the system and the frequency is 50.04 Hz. At this time the wind power generation in the system increases by 100 MW. What will the frequency be when the primary control has restored the balance between generation and consumption?

c) (2 p) At 11:08 there is balance between production and consumption in the system and the frequency is 49.86 Hz. At this time a lightning strikes in a substation disconnects the regional grid in Stad, which means that 1 100 MW load is lost. What will the frequency be when the primary control has restored the balance between generation and consumption?

Problem 4 (12 p)

Stads energi AB owns a thermal power plant with three blocks. Moreover, the company owns a wind farm. Assume that the company has formulated their short-term planning problem as a MILP problem and that the following symbols have been introduced:

Indices for the power plants: Block I - 1, Block II - 2, Block III - 3.

β_{Gg} = variable operation cost in power plant g,
C_g^+ = start-up cost in power plant g, g = 1, 2, 3,
C_g^- = stop cost in power plant g, g = 1, 2, 3,
D_t = contracted load during hour t, t = 1, …, 24,
$G_{g,t}$ = generation in power plant g, hour t, g = 1, 2, 3, t = 1, …, 24,
$\overline{G}_g$ = installed capacity in power plant g, g = 1, 2, 3,
$\underline{G}_g$ = minimal generation when power plant g is committed, g = 1, 2, 3,
λ_t = expected electricity price at ElKräng hour t, t = 1, …, 24,
p_t = purchase from ElKräng hour t, t = 1, …, 24,
r_t = sales to ElKräng hour t, t = 1, …, 24,
$s_{g,t}^+$ = start-up variable for power plant g, hour t, g = 1, 2, 3, t = 1, …, 24,
$s_{g,t}^-$ = stop variable for power plant g, hour t, g = 1, 2, 3, t = 1, …, 24,
$u_{g,t}$ = unit commitment of power plant g, hour t, g = 1, 2, 3, t = 1, …, 24,
W_t = expected wind power generation in hour t, t = 1, …, 24.

a) (5 p) Stads energi AB sells power to customers with firm power contracts, but the company also has the possibility to trade at the local power exchange ElKräng.. Formulate the objective function if the aim of the planning problem is to maximise the income of sold electricity at ElKräng minus the costs of purchasing electricity from ElKräng and minus the costs of the thermal power plant. Use the symbols defined above.

b) (4 p) Assume that Stads energi AB sells power to customers with firm power contracts, and is also trading at the local power exchange ElKräng, where the company has the possibility to both sell and purchase electricity. Formulate the load balance constraint of the company. Use the symbols defined above.

c) (3 p) At installed capacity the hydro power plant Fallet generates 200 MW and the production equivalent is then 0.8 MWh/HE. The reservoir can hold 1 800 000 m^3. Assume that the power plants up-stream neither discharge or spill any water and that the local inflow is negligible. How many hours can Fallet generate its installed capacity before the reservoir is empty if we start with a full reservoir?

Problem 5 (12 p)

Akabuga is a small town in Eastern Africa. The town is not connected to a national grid, but has a local system of its own. The local grid is supplied by a hydro power plant and two diesel generator sets. The hydro power plant is a run-of-the-river station and it has 400 kW capacity and the risk of failure is negligible. The natural flow in the river passing by the power plant is always sufficient to generate the installed capacity. The diesel generator sets have an installed capacity of 200 kW each, the availability is 85% and the operation cost is 10 ¤/kWh. Table 2 shows some partial results of a probabilistic production cost simulation of the power system in Akabuga.

a) (2 p) Use probabilistic production cost simulation to calculate the expected unserved energy per hour in Akabuga.

Table 2 Results from a probabilistic production cost simulation of the power system in Akabuga.

	x = 200	x = 400	x = 600	x = 800	x = 1 000	x = 1 200	x = ∞
$\tilde{F}_0(x)$	1.000	0.996	0.500	0.004	0.000	0.000	0.000
$\int_0^x \tilde{F}_0(\xi)d\xi$	200.0	399.9	570.1	599.9	600.0	600.0	600.0
$\tilde{F}_1(x)$	1.000	0.996	0.500	0.004	0.000	0.000	0.000
$\int_0^x \tilde{F}_1(\xi)d\xi$	200.0	399.9	570.1	599.9	600.0	600.0	600.0
$\tilde{F}_2(x)$	1.000	0.997	0.574	0.078	0.001	0.000	0.000
$\int_0^x \tilde{F}_2(\xi)d\xi$	200.0	399.9	574.6	625.4	630.0	630.0	630.0
$\tilde{F}_3(x)$	1.000	0.997	0.638	0.153	0.012	0.000	0.000
$\int_0^x \tilde{F}_3(\xi)d\xi$	200.0	399.9	578.4	647.8	659.3	660.0	660.0

b) (2 p) Use probabilistic production cost simulation to calculate the *total* expected generation energy per hour in the two diesel generator sets.

c) (1 p) Use probabilistic production cost simulation to calculate the expected operation cost per hour in Akabuga.

d) (2 p) Use probabilistic production cost simulation to compute the risk of power deficit in Akabuga.

e) (2 p) The total load in Akabuga is normally distributed. Assume that the system is simulated using Monte Carlo techniques and that the value 400 kW has been randomised for the total load. What is the complementary random number of this value.

f) (3 p) Assume that the power system in Akabuga is simulated using stratified sampling. Three strata have been defined, and the stratum weights are displayed in table 3. Load shedding does never occur in the strata 1. In stratum 2, load shedding occurs in 46% of the studied scenarios. In stratum 3, all scenarios result in load shedding. Which estimate of *LOLP* is obtained for the system?

Table 3 Strata in the simulation of Akabuga.

Stratum, h	1	2	3
Stratum weight, ω_h	0.8	0.05	0.15

PART II (FOR HIGHER GRADES)

All introduced symbols must be defined. Solutions should include sufficient detail that the argument and calculations can be easily followed.

The answer to each problem must begin on a new sheet, but answers to different parts of the same problem (a, b, c, etc.) can be written on the same sheet. The fields *Namn* (Name), *Blad nr* (Sheet number) and *Uppgift nr* (Problem number) must be filled out on every sheet.

Part II gives a total of 60 points, but this part will only be marked if the candidate has obtained at least 33 points in part I. Then the results of parts I and II and the bonus points will be added together to determine the examination grade (A, B, C, D, E).

Problem 6 (10 p)

Electricity prices in Land are based on long-term forecasts considering the next 12 months. In these long-term forecasts, the maximal generation in different time periods are estimated, as shown in table 4. At the end of April, the hydro reservoirs have the normal contents for this time of the year, and the players in the electricity market expect that the next year will be a normal year. However, during the annual maintenance shutdown of the nuclear power plants in Land, numerous safety issues are detected and must be taken care of; hence, from August the players expect lower nuclear generation until the end of the year. Then, the autumn turns out to be unusually wet and windy, which means that the hydro reservoirs are unusually filled and that the wind power generates more than expected.

Besides the above described incomplete information (which is the same for all players) it can be assumed that there is perfect competition and that there are neither capacity, transmission nor reservoir limitations in the electricity market in Land. The variable operation costs are shown in table 4 and are assumed to be linear within the intervals, i.e., the production is zero if the price is on the lower price level and the production is maximal at the higher price level. The load in Land is not price sensitive and can be assumed to be 150 TWh for the next year regardless of when the forecast is made.

Table 4 Forecasts for electricity generation in Land during the next twelve months.

Power source	Production capability during the next twelve months [TWh]			Variable costs [¤/MWh]
	May	August	October	
Wind power	6	6	7	5
Hydro power	65	65	75	5
Nuclear power	70	62	65	80–120
Fossil fuels	20	20	20	300–500

a) (6 p) Compute how the electricity price in Land is varying during the period May to November.

b) (4 p) Strålinge Kraftverksgrupp AB owns two nuclear power plants with a total installed capacity of 1 200 MW. The reactors in Strålinge were thoroughly revised already last year and are not affected by the safety issues mentioned above. According to the planning of the company, Strålinge 2 should be shut down from 1 July to 31 August. Would it be profitable for the company to deliberately delay the maintenance works in Strålinge 2 during one week in September? Such a delay would mean that the generation capacity of the company for August and September would decrease from 1.1 TWh to 1.0 TWh. The variable cost in Strålinge 2 can be assumed to 100 ¤/MWh and it can be assumed that all players in the electricity market would receive information about the additional reduction in generation capacity in their August forecast (that the reduction is intentional would however be a well kept company secret).

Problem 7 (10 p)

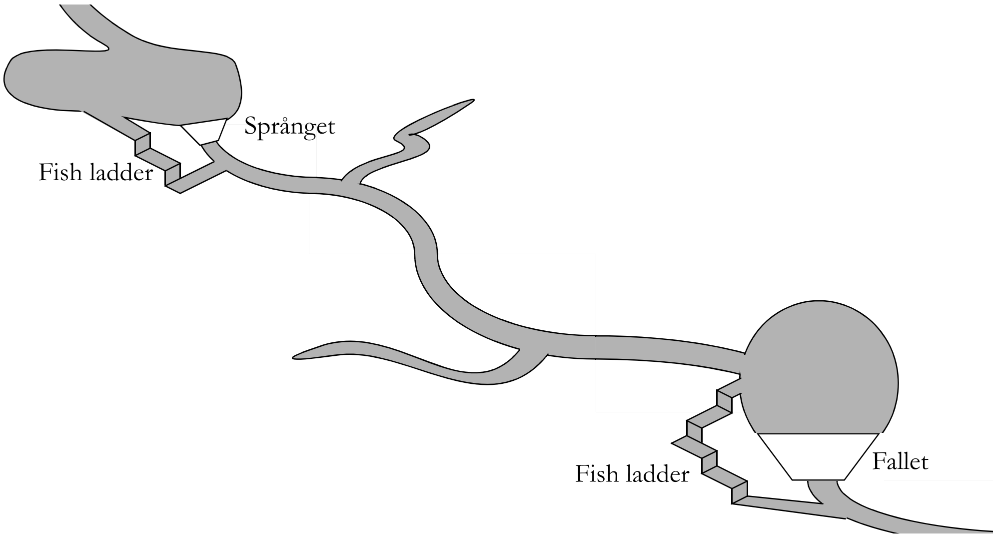
The power system in Rike is divided in two price areas. There is a lot of hydro power in the northern part of the system, but most of the load is located in the southern part. The primary control of Rike is divided in a normal operation reserve and a disturbance reserve. The normal operation reserve is available in the frequency range 49.9–50.1 Hz and has a total gain of 3 000 MW/Hz, where 2 500 MW/Hz is provided by power plants in northern Rike. The disturbance reserve is available in the frequency range 49.5–49.9 Hz and has a total gain of 2 500 MW/Hz, where 2 000 MW/Hz is provided by power plants in northern Rike.

a) (6 p) There are eight parallel AC transmission lines between the two areas, where each line has a maximal transmission capacity of 500 MW. Moreover, there is an HVDC line, which also can transfer up to 500 MW. The power flow on the HVDC line is controlled manually by the system operator, Riksnät, and is not affected neither by the frequency of the system or how generation and load is divided.

Hence, the total transmission capacity between the northern and southern parts is 4 500 MW, but the entire capacity may not be utilised by the players of the electricity market, because Riksnät must have sufficient margins for primary control as well as to manage outages in transmission lines. The requirement is that the system should be able to survive that one transmission line is disconnected, while the normal operation reserve and disturbance reserve are fully utilised to cover load increase and generation outages in southern Rike. (The load and generation that is not part of the primary control can thus be assumed to be constant in northern Rike.) How large transmission capacity may the players of the electricity market use (i.e., what is the largest acceptable transmission when the frequency is exactly 50 Hz) in order to fulfil this requirement?

b) (4 p) Assume that the HVDC line is equipped with a frequency sensitive control system, making the transmission on this line a linear function of the frequency in the system: $P = a + bf$, where a and b are two constants that can be chosen freely. Can such a control system be used to increase the part of the transmission capacity from north to south that is available to the market? How should then the constants a and b be set?

Problem 8 (20 p)



AB Vattenkraft owns two hydro power plants, located as shown in the figure above. Data for the power plants are given in table 5. The company is planning to sell their electricity generation at the local power pool, ElKräng. It is assumed that the company can sell as much as they want to the prices listed in table 6. After this period the average electricity price is estimated to 400 SEK/MWh and stored water is assumed to be used for generation at best efficiency. The water delay time between the power plants can be neglected.

Since the river where the power plants are located is an important breeding area for salmon, the Environment Court has judged that the company must build fish ladders which allow fish to pass the dams. The minimum water flow in each fish ladder must be 1 m^3/s; this water can thus not be used for power generation. Moreover, the Environment Court has decided that for tourism reasons the average daily flow in the river sections downstream each power plant may not be less than 10 m^3/s.

a) (10 p) Formulate the planning problem of AB Vattenkraft as an LP problem. Use the notation in table 7 for the parameters (it is however permitted to add further symbols if you consider it necessary).

NOTICE! The following is required to get full score for this problem:

- The symbols for the optimisation variables must be clearly defined.
- The optimisation problem should be formulated so that it is easy to determine what the objective function is, which constraints there are and which limits there are.
- The possible values for all indices should be clearly stated for each equation.

b) (10 p) When a hydro power plant is started, some water must be discharged through the turbines without generating electricity. Let ξ_i denote the amount of water that is lost when starting power plant i. How must the planning problem from part a be reformulated in order to consider these losses? Do not forget to define all new variables and parameters that you introduce!

Table 5 Data for the power plants of AB Vattenkraft.

Power plant	Start contents of reservoir [HE]	Maximal contents of reservoir [HE]	Marginal production equivalents [MWh/HE]		Maximal discharge [HE]		Local inflow [HE]
			Segment 1	Segment 2	Segment 1	Segment 2	
Språnget	1 000	3 600	0.64	0.56	85	30	112
Fallet	1 200	3 900	0.72	0.64	125	40	2

Table 6 Expected prices at ElKräng.

Hour	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Price at ElKräng [SEK/MWh]	325	265	255	255	245	300	370	395	420	425	425	425
Hour	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24
Price at ElKräng [SEK/MWh]	420	425	415	415	410	415	415	410	395	385	395	365

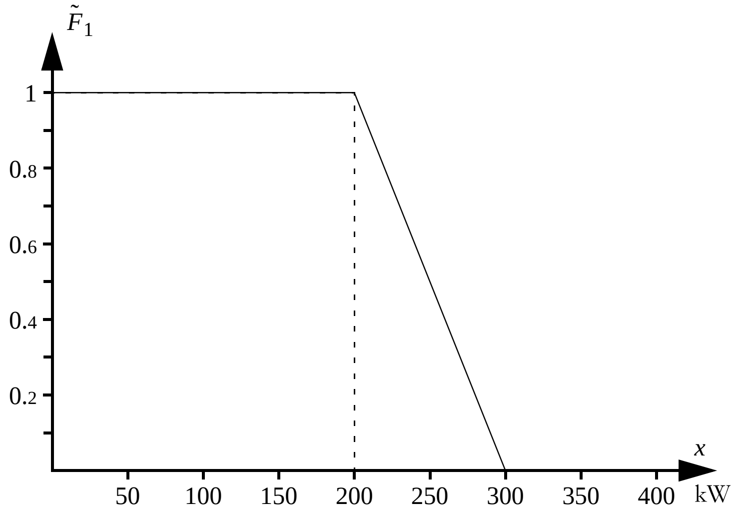
Table 7 Notation for the planning problem of AB Vattenkraft.

Symbol	Explanation	Value
$\underline{S}_i$	Minimal flow through the fish ladder at reservoir *i*	1
$\underline{W}_i$	Minimal daily average flow downstream of power plant *i*	10
$M_{i,0}$	Start contents of reservoir *i*	See table 5
$\overline{M}_i$	Maximal contents of reservoir *i*	See table 5
$\mu_{i,j}$	Marginal production equivalent in power plant *i*, segment *j*	See table 5
$\overline{Q}_{i,j}$	Maximal discharge in power plant *i*, segment *j*	See table 5
V_i	Local inflow to reservoir *i*	See table 5
λ_t	Expected price at ElKräng hour *t*	See table 6
λ_f	Expected future electricity price	400

Problem 9 (20 p)

Ekyaro is a small town in Eastern Africa. The town is not connected to the national grid, but has a local grid of its own, which is supplied by a hydro power plant, a wind power plant and a diesel generator set. The hydro power plant has the installed capacity 200 kW and there is always enough water to run the plant at installed capacity. The risk of outages in the hydro power plant is negligible. The variable costs of hydro power and wind power are negligible. The electricity generation in the wind power plant is uniformly distributed between 0 and 100 kW. The diesel generator set has a capacity of 100 kW, the availability is 90% and the variable cost is 6 ¤/kWh.

The figure below shows the load duration curve for Ekyaro. The load is not price sensitive and the distribution losses in the local grid can be neglected.



a) (5 p) How large is the risk of power deficit when the diesel generator set is unavailable?

b) (1 p) How large is the risk of power deficit when the diesel generator set is available?

c) (1 p) What is the *LOLP* of the system?

d) (5 p) What is the *ETOC* of the system?

e) (8 p) How accurate are your calculations in parts a–d? Describe which sources of error can be found and how you could improve your calculations if you had more time and larger computational capacity at your disposal.

NOTICE! To get full score for this problem you must provide motivations as well as details. Answering for example "The calculations are not very accurate, but could have been improved by using a variance reduction technique." will not yield any points.

Table 8 Some random numbers from a $U(0, 1)$-distribution.

0.76	0.39	0.17	0.03	0.05	0.82	0.32	0.03	0.38	0.80
0.74	0.66	0.71	0.28	0.10	0.69	0.95	0.44	0.77	0.19

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Answer sheet for part I

Name: ....

Personal number: ....

Problem 1

b) Alternative .... is correct.

b) Alternative .... is correct.

c) Alternative .... is correct.

Problem 2

a) .... TWh/year **b)** .... TWh/year

c) .... ¤/MWh

Problem 3

a) .... Hz **b)** .... Hz

c) .... Hz

Problem 4

a) ....

....

b) ....

c) .... hours

Problem 5

a) .... kWh/h **b)** .... kWh/h

c) .... ¤/h **d)** .... %

e) .... kW **f)** .... %

Problem 1

a) 1, **b)** 2, **c)** 2.

Problem 2

a) The part of the coal condensing which has a lower variable operation cost than the electricity price 380 ¤/MWh will be used, i.e., (380 – 300)/(500 – 300) · 20 = 8 TWh/year.

b) The electricity price is lower in Rike than in Maa; hence, the interconnection between Rike and Maa will be fully utilised (if there was no transmission limitation, the price would be the same in both countries). The total generation at this electricity price is 50 (hydro) + 32 (nuclear) + 8 (coal condensing) = 90 TWh, whereas the consumption is 80 TWh/year. The surplus is exported to Maa. Thus, we can conclude that the maximal transmission capacity is 10 TWh/year.

c) First of all, Maa is importing 10 TWh from Rike. Hydro power and nuclear power contributes to another 53 TWh. The coal condensing must then produce 12 TWh. Hence, 60% of the price interval of the coal condensing is used, i.e., the electricity price must then 420 ¤/MWh.

Problem 3

a) The total generation is 11 400 MW when the frequency is 50.04 Hz (according to the figure). These power plants have to decrease their generation to 11 300 MW when the load decreases by 100 MW, which means that the frequency must increase to 50.05 Hz.

b) The total generation is 11 400 MW when the frequency is 50.04 Hz (according to the figure). These power plants have to decrease their generation to 11 300 MW when the wind power generation increases by 100 MW, which means that the frequency must increase to 50.05 Hz.

c) The total generation is 12 200 MW when the frequency is 49.86 Hz (according to the figure). These power plants have to decrease their generation to 11 100 MW when the load decreases by 1 100 MW, which means that the frequency must increase to 49.99 Hz.

Problem 4

a) maximise $\sum_{t=1}^{24}\left(\lambda_t(r_t - p_t) - \sum_{g=1}^{3}(C_g^+ s_{g,t}^+ + C_g^- s_{g,t}^- + \beta_{Gg} G_{g,t})\right)$.

b) $\sum_{g=1}^{3} G_{g,t} + W_t + p_t = D_t + r_t$.

c) The discharge at installed capacity is given by the relation $Q = H/\gamma(Q)$ = 200/0,8 = 250 HE. As the reservoir can store 1 800 000/3 600 = 500 HE, a full reservoir will be enough for two hours maximal generation.

Problem 5

a) The expected energy not served is given by

$$EENS_3 = \int_{800}^{\infty} \tilde{F}_3(x)dx = \int_{0}^{\infty} \tilde{F}_3(x)dx - \int_{0}^{800} \tilde{F}_3(x)dx = 660.0 - 647.8 = 12.2 \text{ kWh/h}.$$

b) The total generation is given by

$$EG_{23} = EENS_1 - EENS_3 = \int_{400}^{\infty} \tilde{F}_1(x)dx - EENS_3 = \int_{0}^{\infty} \tilde{F}_1(x)dx - \int_{0}^{400} \tilde{F}_1(x)dx - EENS_3 =$$
$$= 600.0 - 399.9 - 12.2 = 187.9 \text{ kWh/h}.$$

c) The expected total operation cost is given by

$$ETOC = 10EG_{23} = 10 \cdot 187.9 = 1\,879 \text{ ¤/h}.$$

d) The risk of power deficit is given by

$$LOLP = \tilde{F}_3(800) = 15.3\%.$$

e) The normal distribution is symmetrical, which means that if $D = \mu_D + X$ then $D^* = \mu_D - X$. In this case, the normal distribution has the mean 600 (which we can conclude either from observing that $\tilde{F}_0(600) = 0.5$ or because $EENS_0 = 600$). Hence, the complementary random number must be $D^* = 800$ kW.

f) According to the problem text, we have the following estimates for the strata: $m_{LOLO1} = 0$, $m_{LOLO2} = 0.46$ and $m_{LOLO3} = 1$. Hence, we get

$$m_{LOLO} = \sum_{h=1}^{3} \omega_h m_{LOLOh} = 0 + 0.05 \cdot 0.46 + 0.15 \cdot 1 = 17.3\%.$$

Problem 6

a) From May to July, it is expected that wind, hydro and nuclear can generate in total 141 TWh the next twelve months, whereas the demand is 150 TWh. It would then be necessary to use 9 TWh generation from fossil fuels, which corresponds to 45% of the price interval—thus the electricity price is 390 ¤/MWh for this period.

In August and September, it is expected that wind, hydro and nuclear can generate in total 133 TWh the next twelve month. It would then be necessary to use 17 TWh generation from fossil fuels, which corresponds to 85% of the price interval—thus the electricity price is 470 ¤/MWh for this period.

In October and November, it is expected that wind, hydro and nuclear can generate in total 147 TWh the next twelve month. It would then be necessary to use 3 TWh generation from fossil fuels, which corresponds to 15% of the price interval—thus the electricity price is 330 ¤/MWh for this period.

b) By deliberately reducing the generation capacity in Strålinge 2, the company can increase the electricity price, but at the same time they do of course loose some income. The question is if the price increase is large enough to compensate the lost income.

In the August forecast, the players of the electricity market would now account for 61.9 TWh nuclear, which means that 17,1 TWh fossil fuel generation is needed. This corresponds to 85.5%

of the price interval and thus the new electricity price for August and September would be a 471 ¤/MWh (the forecast for October and November is not changed and therefore the price from October remains the same as before). With the increased electricity price, the income of the company will increase by 1.0 TWh · 1 ¤/MWh = 1 M¤. The reduced generation in Strålinge results that the income from selling 0,1 TWh for the earlier price 470 ¤/MWh is lost. At the same time, there is a saving in variable costs of 100 ¤/MWh. The lost income is then 0.1 TWh · (470 – 100) ¤/MWh = 37 M¤. Hence, it would not be profitable for the company to attempt to manipulate the electricity price in this manner.

Problem 7

a) If an outage occurs on the HVDC line then the flow will be redirected to the AC lines. If an outage occurs on one of the AC lines then the flow will be divided among the other seven AC lines (the flow on the HVDC line is not changed in this case). Either way, there must be enough reserves on the AC lines to accommodate another 500 MW. To manage an outage in one line, the transmission may not exceed 7 · 500 (AC lines) + 500 (HVDC line) = 4 000 MW.

A part of these 4 000 MW must be reserved for primary control. If the entire primary control reserve in Rike is to be exported southwards then the reserve must be in total 2 500 MW/Hz · 0.1 Hz (normal operation reserve) + 2 000 MW/Hz · 0,4 Hz (disturbance reserve) = 1 050 MW. The players in the electricity market may therefore at most have access to 2 950 MW transmission capacity.

b) The suggested control system can govern how the power flow is distributed between the AC lines and the HVDC line for different frequencies. However, this distribution is not important in this case; it must still be possible to transfer 1 050 MW at the frequency 49.5 Hz without getting a total flow that exceeds 4 000 MW.

Problem 8

a) In words the planning problem can be formulated as

maximise *value of sold electricity + value of stored water*,
subject to *hydrological balance for Språnget and Fallet*,
minimal flow in the river.

Indices for power plants

Språnget 1, Fallet 2.

Parameters

The parameters are defined in table 7 of the problem text.

Optimisation variables

$Q_{i,j,t}$ = discharge in power plant i, segment j, during hour t, i = 1, 2, j = 1, 2, t = 1, …, 24,
$S_{i,t}$ = spillage from reservoir i during hour t, i = 1, 2, t = 1, …, 24,
$M_{i,t}$ = contents of reservoir i at the end of hour t, i = 1, 2, t = 1, …, 24.

Objective function

$$\text{maximise} \quad \sum_{t=1}^{24} \lambda_t \sum_{i=1}^{2} \sum_{j=1}^{2} \mu_{i,j} Q_{i,j,t} + \lambda_f((\mu_{1,1} + \mu_{2,1})M_{1,24} + \mu_{2,1}M_{2,24}).$$

Constraints

Hydrological balance for Språnget:

$$M_{1,t} = M_{1,t-1} - Q_{1,1,t} - Q_{1,2,t} - S_{1,t} + V_{1,t}, \qquad t = 1, \dots 24.$$

Hydrological balance for Fallet:

$$M_{2,t} = M_{2,t-1} - Q_{2,1,t} - Q_{2,2,t} - S_{2,t} + Q_{1,1,t} + Q_{1,2,t} + S_{1,t} + V_{2,t}, \qquad t = 1, \dots 24.$$

Minimal daily average flow in the river:

$$\frac{1}{24} \sum_{t=1}^{24} (Q_{i,1,t} + Q_{i,2,t} + S_{i,t}) \geq \underline{W}_i, \qquad i = 1, 2, t = 1, \dots 24.$$

Variable limits

$$0 \leq Q_{i,j,t} \leq \overline{Q}_{i,j}, \qquad i = 1, 2, j = 1, 2, t = 1, \dots, 24,$$

$$\underline{S}_i \leq S_{i,t}, \qquad i = 1, 2, t = 1, \dots, 24,$$

$$0 \leq M_{i,t} \leq \overline{M}_i, \qquad i = 1, 2, t = 1, \dots, 24.$$

b) First we must introduce new variables for unit commitment and start of the hydro power plants:

$s^+_{i,t}$ = start-up variable for power plant i at the beginning of hour t, t = 1, …, 24,
$u_{i,t}$ = unit commitment of power plant i during hour t, t = 1, …, 24.

The objective function does not have to be changed, but we need not reformulate the hydrological constraints to account for the water that is lost when starting:

$$M_{1,t} = M_{1,t-1} - Q_{1,1,t} - Q_{1,2,t} - S_{1,t} - \xi_i s^+_{1,t} + V_{1,t}, \qquad t = 1, \dots 24,$$

$$M_{2,t} = M_{2,t-1} - Q_{2,1,t} - Q_{2,2,t} - S_{2,t} - \xi_i s^+_{2,t} + Q_{1,1,t} + Q_{1,2,t} + S_{1,t} + \xi_i s^+_{1,t} + V_{2,t}, \qquad t = 1, \dots 24.$$

The water lost when starting should also be accounted for in the average daily flow:

$$\frac{1}{24} \sum_{t=1}^{24} (Q_{i,1,t} + Q_{i,2,t} + S_{i,t} + \xi_i s^+_{i,t}) \geq \underline{W}_i, \qquad i = 1, 2, t = 1, \dots 24.$$

We also need to introduce constraints for the relation between unit commitment and start-up of the hydro power plants:

$$s^+_{i,t} \geq u_{i,t} - u_{i,t-1}, \qquad i = 1, 2, t = 1, \dots, 24.$$

It is also necessary to introduce new constraints that prohibit discharge in the power plants when they are not committed:

$$Q_{i,j,t} \le u_{i,t}\overline{Q}_{i,j}, \qquad i = 1, 2, j = 1, 2, t = 1, \ldots, 24.$$

Finally, we need to state the variable limits of the new variables:

$$s^{+}_{i,t} \in \{0, 1\}, \qquad i = 1, 2, t = 1, \ldots, 24,$$
$$u_{i,t} \in \{0, 1\}, \qquad i = 1, 2, t = 1, \ldots, 24.$$

Problem 9

a) We can notice that without the diesel generator set, both available generation capacity and load are $U(200, 300)$-distributed. The probability that one is larger than the other is then 50%, which means that the risk of power deficit is 50% without the diesel generator set.

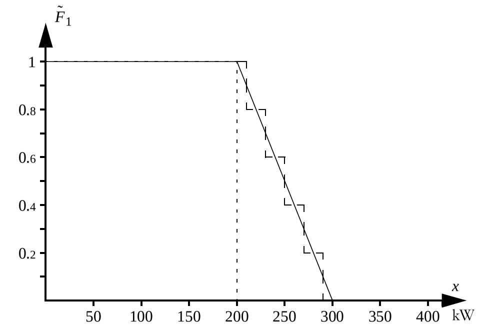
b) The hydro power plant and diesel generator set can supply **300** kW—which is always sufficient to cover the load—when the diesel generator set is operational. The risk of power deficit is thus 0% in this case.

c) The probability that the diesel generator set is unavailable is 10% and it is then 50% probability that there is a power deficit. The system *LOLP* is thus 5%.

d) To calculate the expected operation cost of the system, we need the expected generation in the diesel generator set, which can be computed according to

$$EG_g = T \cdot p_g \int_{\hat{G}^{tot}_{g-1}}^{\hat{G}^{tot}_{g}} \tilde{F}_{g-1}(x)dx.$$

In this case do we want the operation cost per hour (i.e., $T = 1$). The availability of the diesel generator set is 0.9 according to the problem text. Thus, we are only missing the equivalent load duration curve including outages in the hydro power and wind power for the interval 300 (installed capacity of the system excluding the diesel generator set) to 400 (installed capacity including the diesel generator set). To compute this curve we choose to use discrete approximations of the continuous probability distributions for load and wind power generation. A simple approximation is to use five equally probable states for each distribution. The load duration curve is then divided in segments with a step size of 20 kW and in order to avoid changing the surface below the duration curve, the limits between these segments should be placed at 210, 230, 250, 270 and 290 kW, as shown in the figure below.



Similarly, the outage in the wind power plant is approximated by five states (10, 30, 50, 70 and 90 kW) each with 20% probability.

The expected total operation cost can now be calculated as follows:

$$\tilde{F}_0(x) = \begin{cases} 1 & x < 210, \\ 0.8 & 210 \le x < 230, \\ 0.6 & 230 \le x < 250, \\ 0.4 & 250 \le x < 270, \\ 0.2 & 270 \le x < 290, \\ 0 & 290 \le x, \end{cases} \quad \tilde{F}_1(x) = 1 \cdot \tilde{F}_0(x),$$

$$\tilde{F}_2(x) = 0.2\tilde{F}_1(x-10) + 0.2\tilde{F}_1(x-30) + 0.2\tilde{F}_1(x-50) + 0.2\tilde{F}_1(x-70) + 0.2\tilde{F}_1(x-90) =$$

$$= \begin{cases} \ldots & \ldots \\ 0.2 \cdot 0 + 0.2 \cdot 0.2 + 0.2 \cdot 0.4 + 0.2 \cdot 0.6 + 0.2 \cdot 0.8 = 0.4 & 300 \le x < 320, \\ 0 + 0.2 \cdot 0 + 0.2 \cdot 0.2 + 0.2 \cdot 0.4 + 0.2 \cdot 0.6 = 0.24 & 320 \le x < 340, \\ 0 + 0 + 0.2 \cdot 0 + 0.2 \cdot 0.2 + 0.2 \cdot 0.4 = 0.12 & 340 \le x < 360, \\ 0 + 0 + 0 + 0.2 \cdot 0 + 0.2 \cdot 0.2 = 0.04 & 360 \le x < 380, \\ 0 & 380 \le x, \end{cases}$$

$$ETOC = 6EG_3 = 6 \cdot 0.9 \int_{300}^{400} \tilde{F}_2(x)dx = 6 \cdot 0.9 \cdot 20 \cdot (0.4 + 0.24 + 0.12 + 0.04) = 86.4 ¤/\text{h}.$$

e) The solutions above for parts a–c are based on analytical calculations. In this case the accuracy of the answer only depends on the quality of the inputs of the model. In part d we used discrete approximations of continuous random variables, which means that we are introducing a numerical error in the calculations. This error can be reduced by using more states in the discrete approximations (i.e, the shorter segment size we use).