Basic Queuing Theory Formulas

Poisson distribution

$$P[X=k]=\frac{\lambda^k}{k!}e^{-\lambda}, \quad k=0,1,2,...$$

Exponential distribution

$$f(x)=\begin{cases}\lambda e^{-\lambda x} & x\ge 0\\ 0 & x<0\end{cases}$$

$$F(x)=\begin{cases}1-e^{-\lambda x} & x\ge 0\\ 0 & x<0\end{cases}$$

z-transform

$$F(z)=\sum_{n=0}^{\infty} f_n z^n$$

Laplace-transform

$$F(s)=\int_0^{\infty} f(t)e^{-st}dt$$

Little's theorem

$$\bar{N}=\lambda_{eff}T$$

M/M/1

$$\rho=\lambda/\mu$$

$$P_k=(1-\rho)\rho^k$$

$$\bar{N}=\rho/(1-\rho)$$

$$W=\rho/(\mu-\lambda)$$

$$F_W(t)=1-\rho e^{-\mu(1-\rho)t}$$

M/M/c

$$\rho=\lambda/\mu$$

$$P_k=\begin{cases}P_0\frac{\rho^k}{k!} & k\le c\\ P_0\frac{\rho^k}{c!c^{k-c}} & k>c\end{cases}$$

Erlang-C

$$D_c(\rho)=C(\rho,c)=P(wait)=\frac{cE_c(\rho)}{c-\rho(1-E_c(\rho))}$$

$$\bar{N}_q=\frac{\lambda}{c\mu-\lambda}P(wait)$$

$$W=\frac{1}{c\mu-\lambda}P(wait)$$

$$F_W(t)=1-D_c(\rho)e^{-\mu(c-\rho)t}$$

M/M/1/L

$$\rho=\lambda/\mu$$

$$P_k=\frac{(1-\rho)\rho^k}{1-\rho^{L+1}}$$

$$P_L=P(blocking)=\frac{(1-\rho)\rho^L}{1-\rho^{L+1}}$$

$$\bar{N}=\frac{\rho}{1-\rho}(1-(L+1)P_L)$$

M/M/c/c

$$\rho=\lambda/\mu$$

$$P_k=\frac{\rho^k/k!}{\sum_{i=0}^{c}\frac{\rho^i}{i!}}$$

Erlang-B

$$E_c(\rho) = B(\rho, c) = P(blocking) = \frac{\rho^c / c!}{\sum_{i=0}^{c} \frac{\rho^i}{i!}}$$

$$\bar{N} = \rho(1 - P(blocking))$$

M/M/c/c/M

$$\rho = \lambda / \mu$$

$$P_k = \frac{\binom{M}{k}\rho^k}{\sum_{i=0}^{c}\binom{M}{i}\rho^i}$$

M/E$_r$/1

$$B^*(s) = \prod_{i=1}^{r} \frac{\mu_i}{s + \mu_i}$$

M/H$_r$/1

$$B^*(s) = \sum_{i=1}^{r} \alpha_i \frac{\mu_i}{s + \mu_i}$$

M/G/1

$$\bar{N} = \rho + \rho^2 \frac{1 + C_b^2}{2(1-\rho)}, \; C_b^2 = \frac{\sigma_b^2}{\bar{x}^2}$$

$$W = \frac{\lambda \overline{x^2}}{2(1-\rho)}$$

$$Q(z) = B^*(\lambda - \lambda z) \frac{(1-\rho)(1-z)}{B^*(\lambda - \lambda z) - z}$$

$$W^*(s) = \frac{s(1-\rho)}{s - \lambda + \lambda B^*(s)}$$

Laplace Transforms

Function	Laplace Transform
$\delta(t)$ *unit impulse*	1
$\delta(t-a)$	e^{-as}
1 *unit step*	$1/s$
t	$1/s^2$
$t^{n-1}/(n-1)!$	$1/s^n$
Ae^{at}	$A/(s-a)$
te^{at}	$1/(s-a)^2$
$t^{n-1}e^{at}/(n-1)!$	$1/(s-a)^n, \quad n=1,2,\ldots L$

Random variable	Laplace Transform
$Uniform \quad a<x<b$	$F(s)=e^{-sa}-e^{-sb} \big/ s(b-a)$
$Exponent$	$F(s)=\lambda/(s+\lambda)$
$Gamme$	$F(s)=\lambda^{\alpha}/(s+\lambda)^{\alpha}$
$Erlang-k$	$F(s)=\lambda^{k}/(s+\lambda)^{k}$

z-Transforms

Sequence	z-transform
$u_k = 1,\ k = 1, 2, \ldots$	$1/(1 - z^{-1})$
u_{k-a}	$z^{-a}/(1 - z^{-1})$
Aa^k	$A/(1 - az^{-1})$
ka^k	$az^{-1}/(1 - az^{-1})^2$
$(k+1)a^k$	$1/(1 - az^{-1})^2$
$a^k/k!$	$e^{a/z}$

Random variable	z-transform
Bernoulli	$G(z) = q + pz$
Binomial	$G(z) = (q + pz)^n$
Geometric	$G(z) = p/(1 - qz)$
Poisson	$G(z) = e^{-\lambda(1-z)}$