



ROYAL INSTITUTE  
OF TECHNOLOGY

# Electricity Pricing

Lectures 3-4 in EG2050 System Planning

Lars Abrahamsson

# Course objectives

To pass the course, the students should show that they are able to

- perform rough estimations of electricity prices.

To receive a higher grade (A, B, C, D) the students should also show that they are able to

- identify factors that have a large importance for the electricity pricing, and to indicate how these factors affect for example producers and consumers.
-

# Ideal Pricing

What price would we have in an **ideal** market?

- Assume a set,  $\mathbf{G}$ , of producers, where each producer,  $g$ , has to decide its production,  $G_g$ .
  - Assume a set,  $\mathbf{C}$ , of consumers, where each consumer,  $c$ , has to decide its consumption,  $D_c$ .
  - Ignore transaction costs.
-

# Price-taking producer (1/2)

Definition: A price-taking producer has such a small market share that the market price,  $\lambda$ , is not affected by the choice of production level,  $G_g$ .

The profit is equal to

$$PS_g = \lambda G_g - C_{G_g}(G_g),$$

where

$PS_g$  represents the surplus of producer  $g$

$C_{G_g}(G_g)$  represents the cost to produce  $G_g$

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# Price-taking producer (2/2)

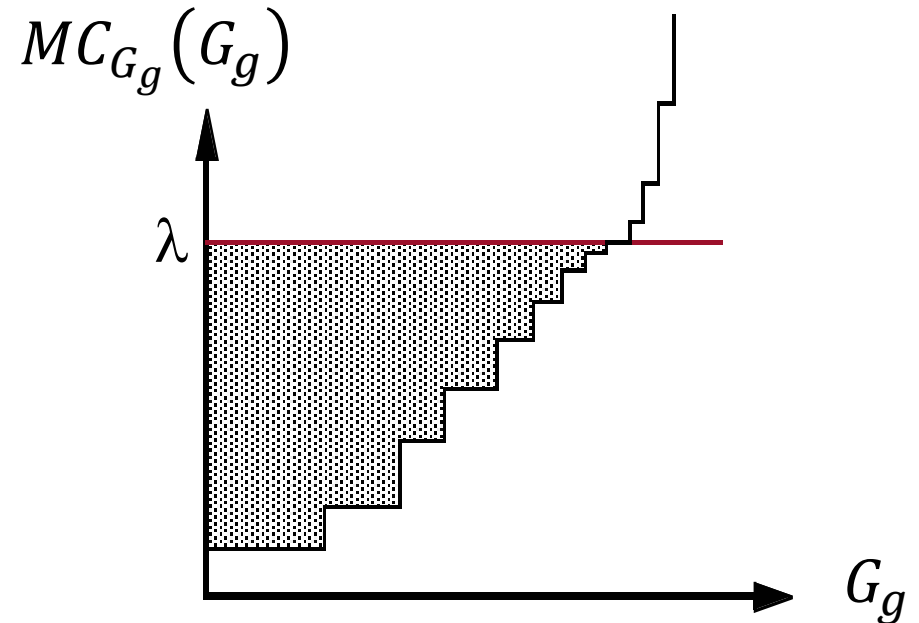
Which production level maximizes profits?

Study the marginal cost curve, i.e., the variable costs

$$MC_{G_g}(G_g) = \frac{dC_{G_g}(G_g)}{dG_g}$$

=> choose  $G_g$  such that

$$MC_{G_g}(G_g) = \lambda.$$



# Price-taking Consumer (1/2)

Definition: A price-taking consumer has such a small market share that the market price,  $\lambda$ , is not affected by the choice of consumption level,  $D_c$ .

The profit is equal to

$$CS_c = B_{D_c}(D_c) - \lambda D_c,$$

where

$CS_c$  represents the surplus of customer  $c$ ,

$B_{D_c}(D_c)$  represents the benefit of consuming  $D_c$ .

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# Price-taking Consumer (2/2)

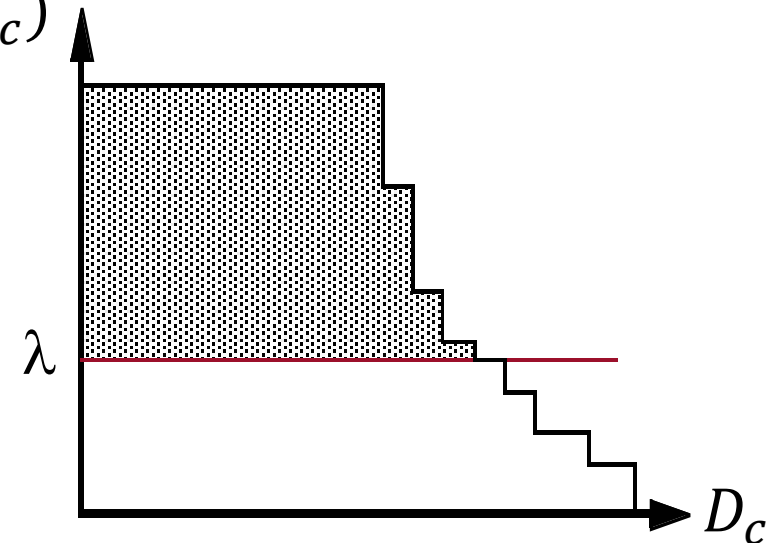
Which consumption level maximizes profits?

Study the marginal benefit curve, i.e., the willingness to pay

$$MB_{D_c}(D_c) = \frac{dB_{D_c}(D_c)}{dD_c}$$

=> choose  $D_c$  such that

$$MB_{D_c}(D_c) = \lambda.$$



# Benefit to the society

- Producers will increase their production until the marginal production cost is equal to the market price.
- Consumers will increase their consumption until the marginal benefit is equal to the market price.

Is this behavior beneficial to the society?

=> Study the total surplus.

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# Total surplus (1/2)

Definition: The total surplus,  $TS$  is given by

$$TS = \sum_c CS_c + \sum_g PS_g = \dots = \sum_c B_{D_c}(D_c) + \sum_g C_{G_g}(G_g)$$

**Note!** The total surplus is not a perfect measure of the benefit to the society.

It presumes that all benefits and costs can be assigned monetary values (externalities)

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## Total surplus (2/2)

- Combine all consumers marginal benefit functions into a **demand curve**,  $MB$ .
- Combine all producers marginal cost functions into a **supply curve**,  $MC$ .

The total surplus is maximized if

$$MB = MC = \lambda.$$

- $MB$  denotes Marginal Benefit, whereas
  - $MC$  denotes Marginal Cost.
-

# Market price

- In an ideal market (perfect competition, perfect information, etc.) there will be a market price which maximizes both the total surplus and each individual surplus of all the producers and consumers.
  - The market price is set by the intersection of the supply curve and the demand curve, i.e., marginal production costs and willingness to pay.
-

# Simple Price Model

Assume:

- Perfect competition
- Perfect information
- No capacity limitations
- No transmission limitations
- No reservoir limitations
- Price insensitive load

=> Mean electricity price can be estimated by studying the supply curve on an **annual** basis.

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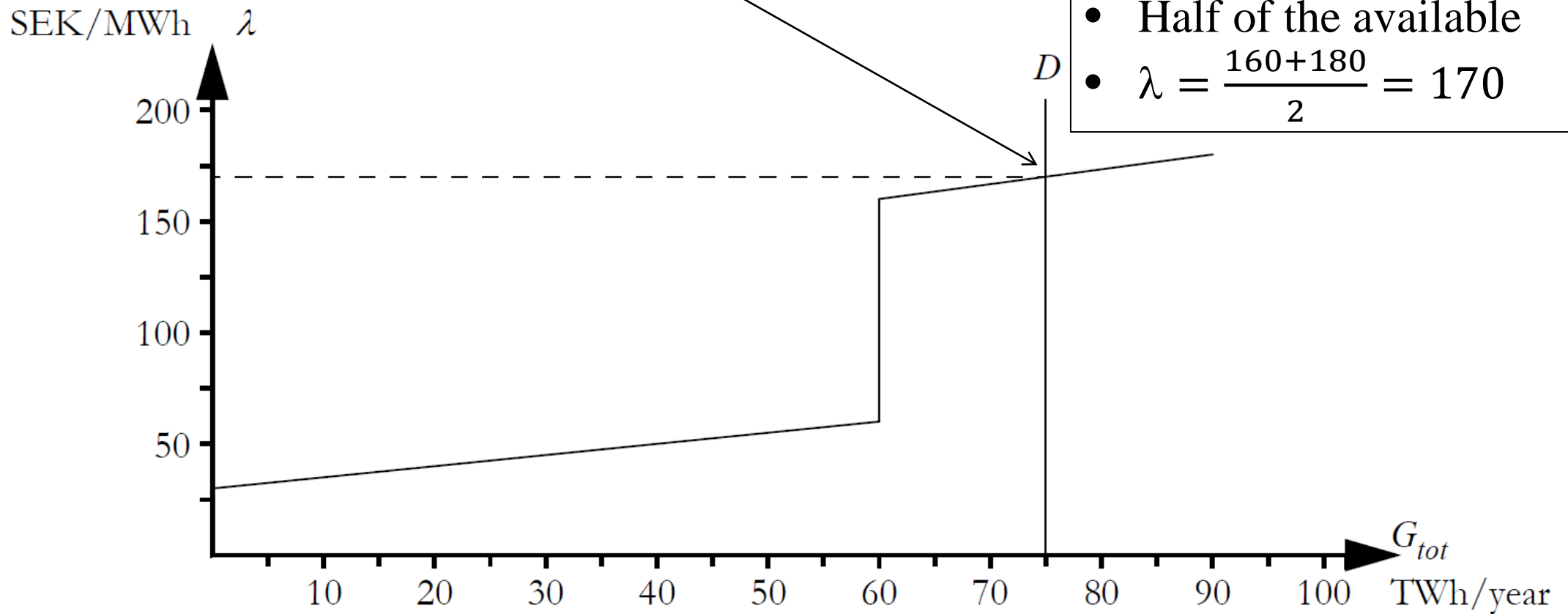
## Example 3.1 – Problem

- Load 75 TWh/year
  - Hydro power
    - 60 TWh/year
    - 30-60 SEK/MWh
  - Coal condensing
    - 30 TWh/year
    - 160-180 SEK/MWh
  - What will the electricity price be?
  - Calculate the difference between variable operation cost and total income of the power producers! (total producers' surplus)
-

# Example 3.1 – Solution (1/2)

$$\lambda = 170 \text{ SEK/MWh}$$

- $75 > 60 \Rightarrow$  thermal
- 15 TWh thermal needed
- Half of the available
- $\lambda = \frac{160+180}{2} = 170$



## Example 3.1 – Solution (2/2)

- Total variable operation cost
  - $PC = 60 \frac{30+60}{2} + 15 \frac{160+170}{2} = 5175$  MSEK
  - whereas the income
  - $PV = 75 \cdot 170 = 12750$  MSEK
  - and, thus the total producers' surplus is
  - $12750 - 5175 = 7575$  MSEK
-

## Exercise 3.6 (p 32) – Problem (1/5)

- Table 3.2 shows
    - the available generation resources,
    - marginal generation costs,
    - and annual consumption in the Nordic countries during year 2000.
  - The operation costs are assumed to be linear within the stated intervals.
  - Assume
    - Perfect competition
    - Perfect information
    - *Continues ...*
-



## Exercise 3.6 (p 32) – Problem (2/5)

- Assume
    - *Continued ...*
    - That there are neither any
      - Capacity
      - Transmission, nor
      - Reservoir limitations
  - Trade
    - Finland imports 4 TWh from Russia
    - Denmark exports 5 TWh to Germany
    - Sweden exports
      - 0.5 TWh to Germany and
      - 0.5 TWh to Poland
-

## Exercise 3.6 (p 32) – Problem (3/5)

**Table 3.2** Available generation resources and annual consumption.

Power source	Production capability [TWh/year]				Cost [SEK/MWh]
	Sweden	Norway	Finland	Denmark	
Hydro power	78	142	14	-	40
Wind power	0.5	0	0	4	20
Nuclear power	55	-	21.5	-	50–75
Industrial backpressure	5	-	13	2	40–100
Comb. heat and power	5	-	13	24	60–140
Coal condensing	-	-	13	24	120–140
Consumption	146	124	79	35	



## Exercise 3.6 (p 32) – Problem (5/5)

- Estimate the electricity price
  - Estimate the trading between the Nordic countries
  - How would the electricity price be affected if the reservoir sizes are reduced such that they are filled on July 31?
    - Everything else is kept constant in the problem
    - Describe the trend orally, no numerical values or computations needed
-

## Exercise 3.6 (p 32) – Solution (1/3)

- Net export:  $1 - 4 + 5 = 2$
  - Total consumption in the Nordic countries:  $146 + 124 + 79 + 35 + 2$  (net export) = 386.
  - Assume that all hydro, wind, nuclear, and industrial back-pressure is utilized =>
  - Total production:  $78 + 142 + 14 + 0.5 + 4 + 55 + 21.5 + 5 + 13 + 2 = 335$ , which is not enough,
  - 51 TWh more needed.
  - Assume a price between 100 and 120 SEK/MWh:
    - Combined heat and power (CHP), capacity  $5 + 13 + 24 = 42 < 51$
    - Coal condensing needed!
-

## Exercise 3.6 (p 32) – Solution (2/3)

- Assume a price between 100 and 140 SEK/MWh:
  - $\frac{\lambda-60}{140-60} \cdot 42 + \frac{\lambda-120}{140-120} \cdot 37 = 51 \Rightarrow \lambda = \frac{2436}{19} \approx 128.21$  SEK/MWh
  - **Error-check:** Is the price within the assumed range?
  - **Answer:** 128.21 SEK/MWh
  - Shares of total possible production:
    - CHP:  $\frac{\lambda-60}{140-60} = \frac{51}{79} \approx 0.853$
    - Coal:  $\frac{\lambda-120}{140-120} = \frac{39}{95} \approx 0.411$
  - **Swedish** net production:  $78 + 0.5 + 55 + 5 + 5 \cdot 0.853 - 146 - 1 \approx -4.2 \Rightarrow$  **Imports 4.2 TWh**
-

## Exercise 3.6 (p 32) – Solution (3/3)

- **Norwegian** net production:  $142 - 124 = 18 \Rightarrow$  **Exports 18 TWh**
  - **Finnish** net production:  $14 + 21.5 + 13 + 13 \cdot 0.853 + 13 \cdot 0.411 - 79 + 4 \approx -10.1 \Rightarrow$  **Imports 10.1 TWh**
  - **Danish** net production:  $4 + 2 + 24 \cdot 0.853 + 24 \cdot 0.411 - 35 - 5 \approx -3.7 \Rightarrow$  **Imports 3.7 TWh**
  - **Check:** Sums to zero!, CHP preferred!
  - **Reservoir sizes such that they are filled on July 31**
    - It is better to use the water rather than to spill it
    - This reduces the prices before July 31
    - And increases the prices after July 31, since there will be a shortage of water then
-

## Example 3.3 – Problem (1/2)

- Load:
    - 1 January - 30 June: 35 TWh
    - 1 July - 31 December: 40 TWh
  - Coal condensing 30 TWh/year.
    - evenly distributed over the year
    - 160-180 SEK/MWh
  - Hydro power
    - Similar to Example 3.1
    - 30-60 SEK/MWh
    - *Continues ...*
-



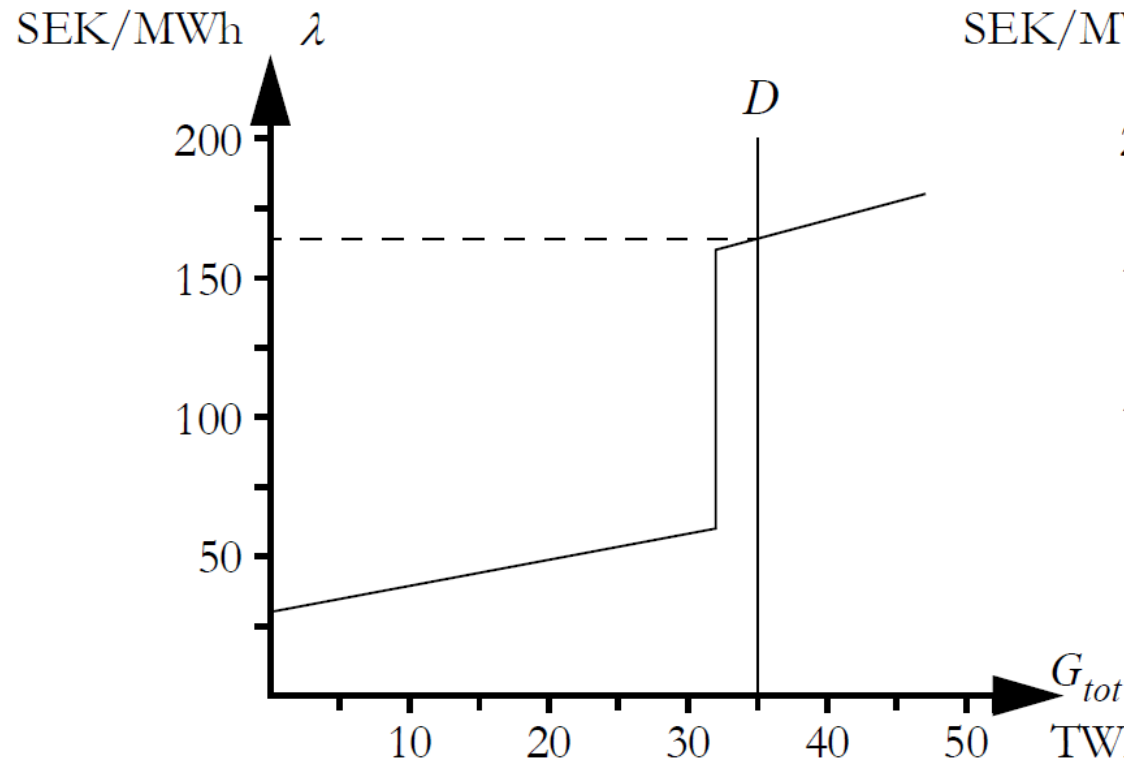
## Example 3.3 – Problem (2/2)

- Hydro power
    - *Continued ...*
    - Reservoir capacity: 18 TWh
    - Reservoir contents:
      - 1 January, 00:00: 0 TWh
      - 1 July, 00:00: 18 TWh
      - 31 December, 23:59: 0 TWh
    - Inflow:
      - 1 January - 30 June: 50 TWh
      - 1 July - 31 December: 10 TWh
  - Determine the electricity price during the year
-

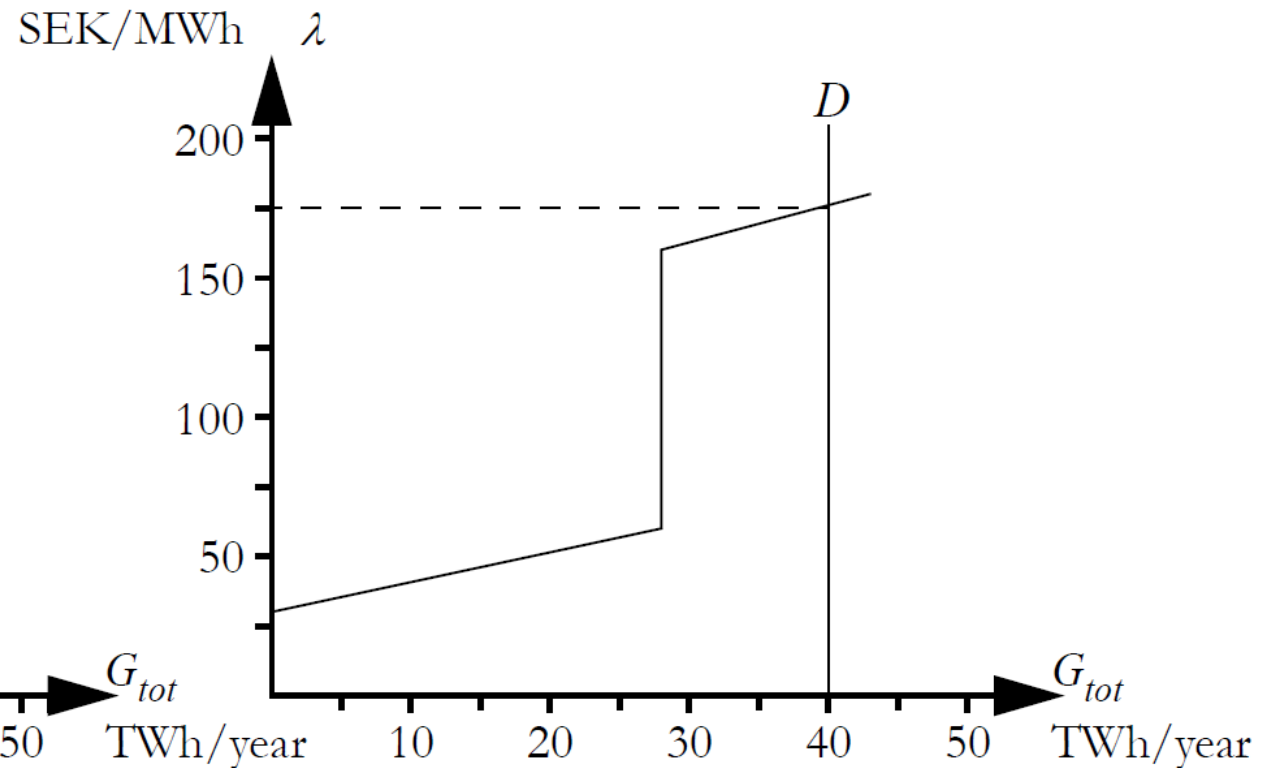
## Example 3.3 – Solution #1 (1/3)

- First six months:
    - Hydro potential:  $50 - 18 = 32$  TWh.
    - Load: 35 TWh  $\Rightarrow$  Fully utilized  $\Rightarrow \lambda > 60$  SEK/MWh.
    - Coal condensing potential: 15 TWh.
    - 3 TWh coal energy needed  $\Rightarrow$  20% utilized  $\Rightarrow \lambda = 0.8 \cdot 160 + 0.2 \cdot 180 = 164$  SEK/MWh.
  - Last six months:
    - Hydro potential:  $10 + 18 = 28$  TWh.
    - Load: 40 TWh  $\Rightarrow$  Fully utilized  $\Rightarrow \lambda > 60$  SEK/MWh.
    - Coal condensing potential: 15 TWh.
    - 12 TWh coal energy needed  $\Rightarrow$  80% utilized  $\Rightarrow \lambda = 0.2 \cdot 160 + 0.8 \cdot 180 = 176$  SEK/MWh.
-

# Example 3.3 – Solution #2 (2/3)



**Figure 3.4** Supply and demand during the first six months.



**Figure 3.5** Supply and demand during the last six months.

## Example 3.3 – Solution #2 (3/3)

- First six months:

$$- 32 + 15 \left( \frac{\lambda - 160}{180 - 160} \right) = 35$$

$$- \lambda = 164$$

- Last six months:

$$- 28 + 15 \left( \frac{\lambda - 160}{180 - 160} \right) = 40$$

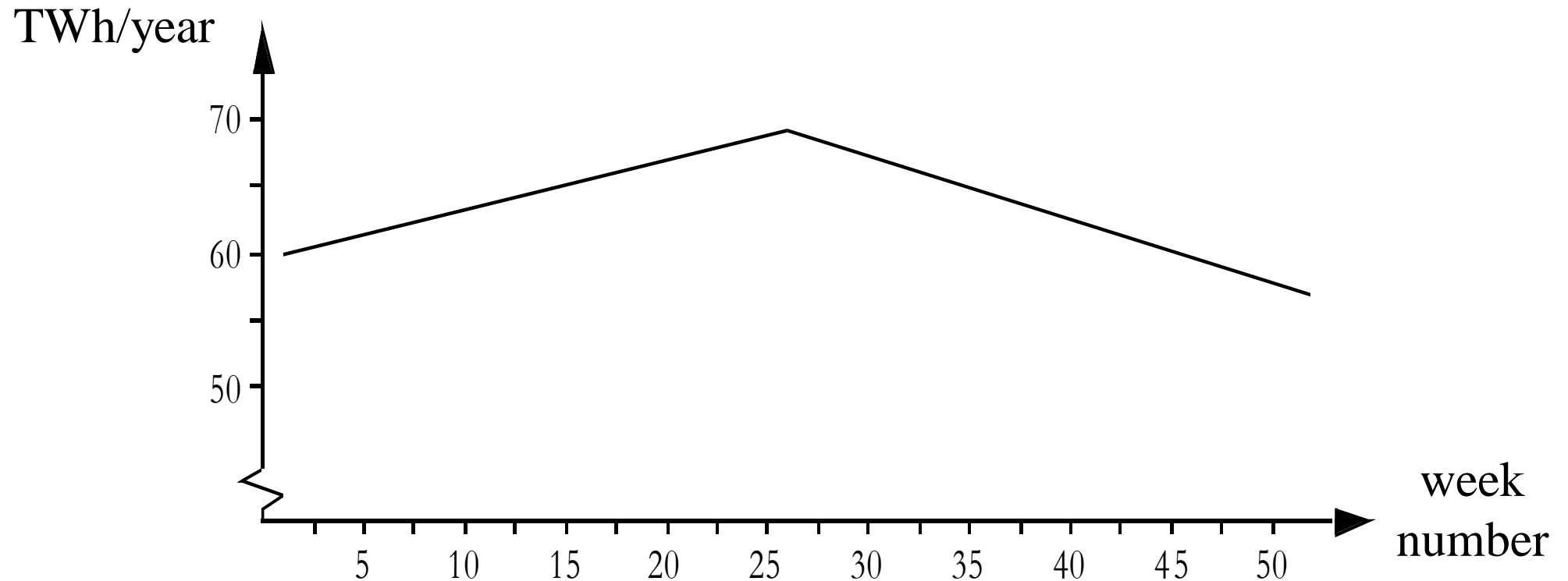
$$- \lambda = 176$$

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## Example 3.4 - Problem

How will the price develop during the studied year?

The same system as in Example 3.1, but the inflow forecast for the next 12 months varies as follows:



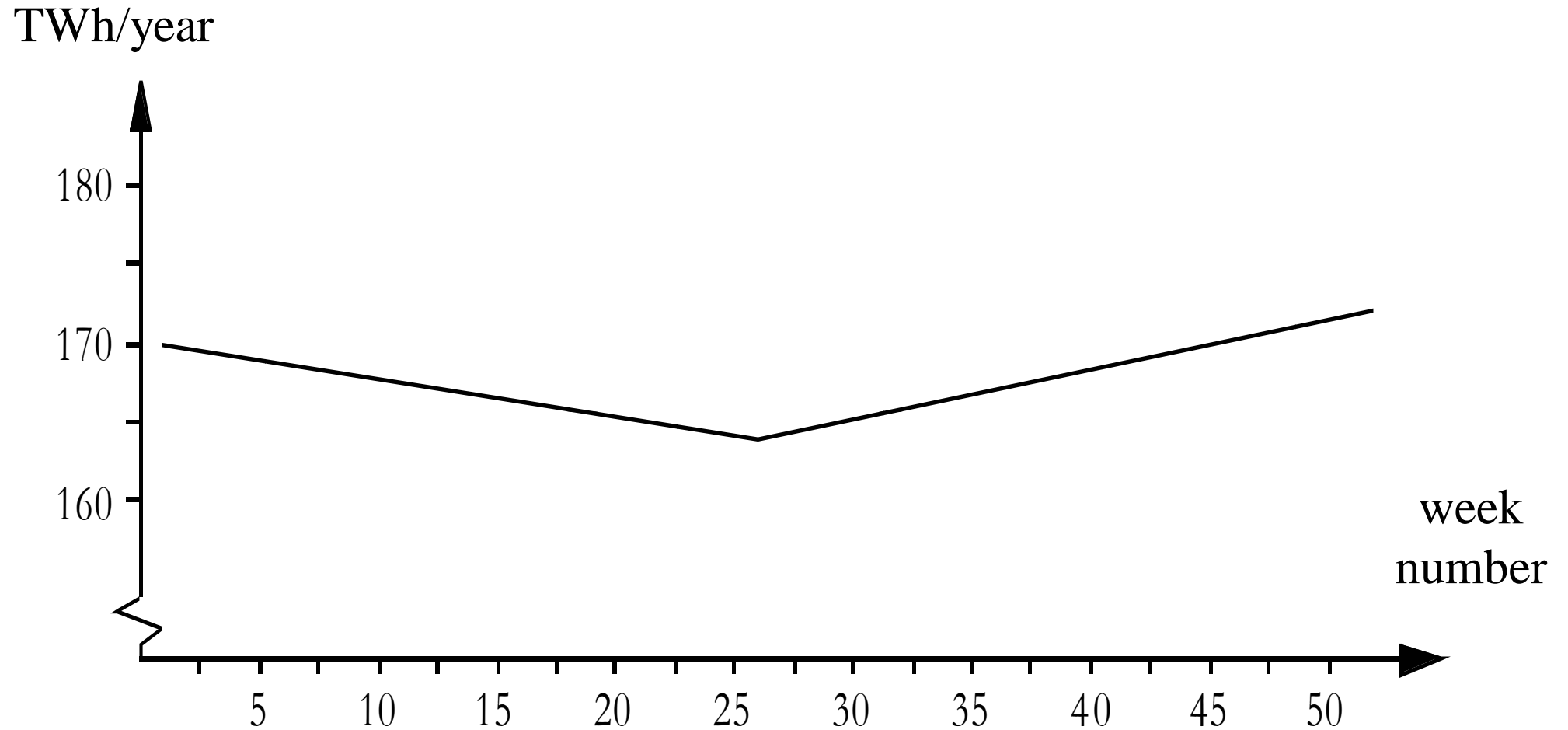
## Example 3.4 – Solution #1 (1/4)

- Strategy:
    - Find the brake point of the piecewise linear function
    - Draw the lines between these points
  - Week 1: Same as in Example 3.1
    - $\lambda = 170$  SEK/MWh
  - Week 26:
    - Hydro potential: 69 TWh.
    - Load is till 75 TWh
    - Fully utilized  $\Rightarrow \lambda > 60$  SEK/MWh.
    - Coal condensing potential: 30 TWh.
    - 6 TWh coal energy needed
    - 20% utilized  $\lambda = 0.8 \cdot 160 + 0.2 \cdot 180 = 164$  SEK/MWh.
-

## Example 3.4 – Solution #1 (2/4)

- Week 52:
    - Hydro potential: 57 TWh.
    - Load is till 75 TWh
    - Fully utilized  $\Rightarrow \lambda > 60$  SEK/MWh.
    - Coal condensing potential: 30 TWh.
    - 18 TWh coal energy needed
    - 60% utilized  $\Rightarrow \lambda = 0.4 \cdot 160 + 0.6 \cdot 180 = 172$  SEK/MWh.
-

## Example 3.4 – Solution #1 (3/4)



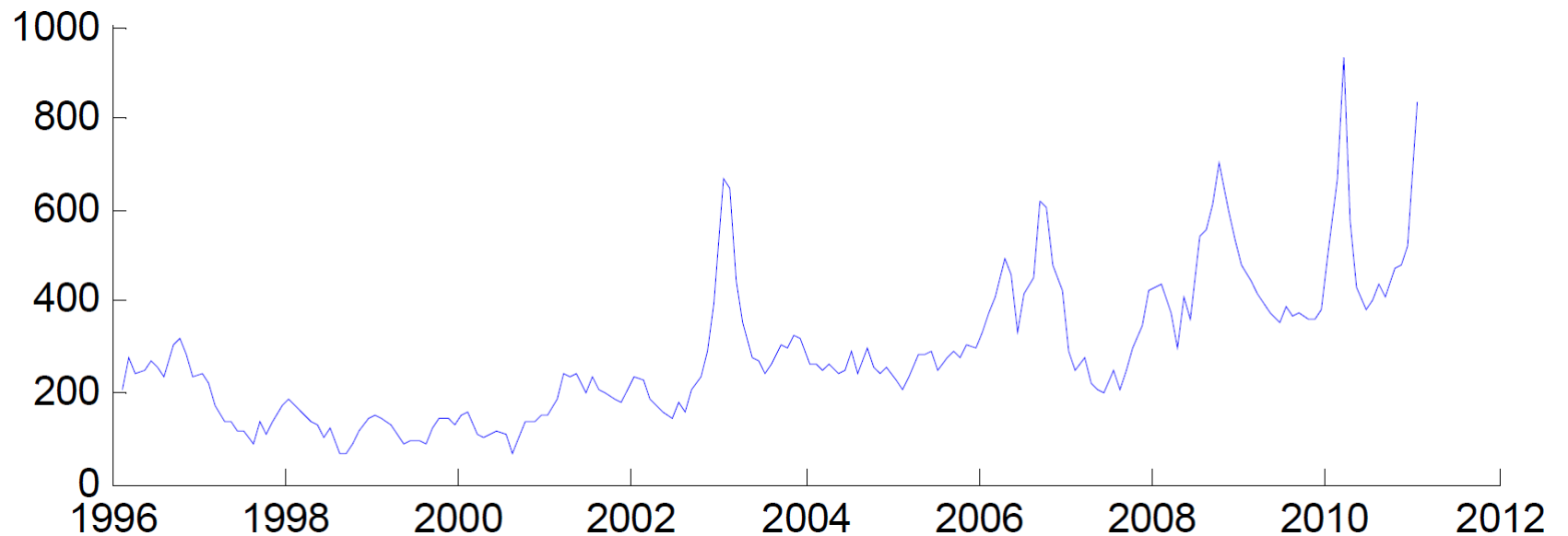


## Example 3.4 – Solution #2 (4/4)

- Strategy:
    - Demand of 75 TWh
    - Hydro power will never suffice by its own
    - Coal needed, the amount needed is a **linear** function of forecasted hydro potential
    - Price varies **linearly** with marginal coal price
  - $\lambda = 160 + \left(\frac{180-160}{30}\right) (75 - V)$ 
    - Where,  $(75 - V)$  represents the remaining demand on coal
    - $75 - V \leq 30 \Rightarrow V \geq 45$ , where 30 denotes the coal capacity
    - $75 - V \geq 0 \Rightarrow V \leq 75$ , which ensures that the needed coal energy is nonnegative
    - Thus: holds as long  $45 \leq V \leq 75$
-

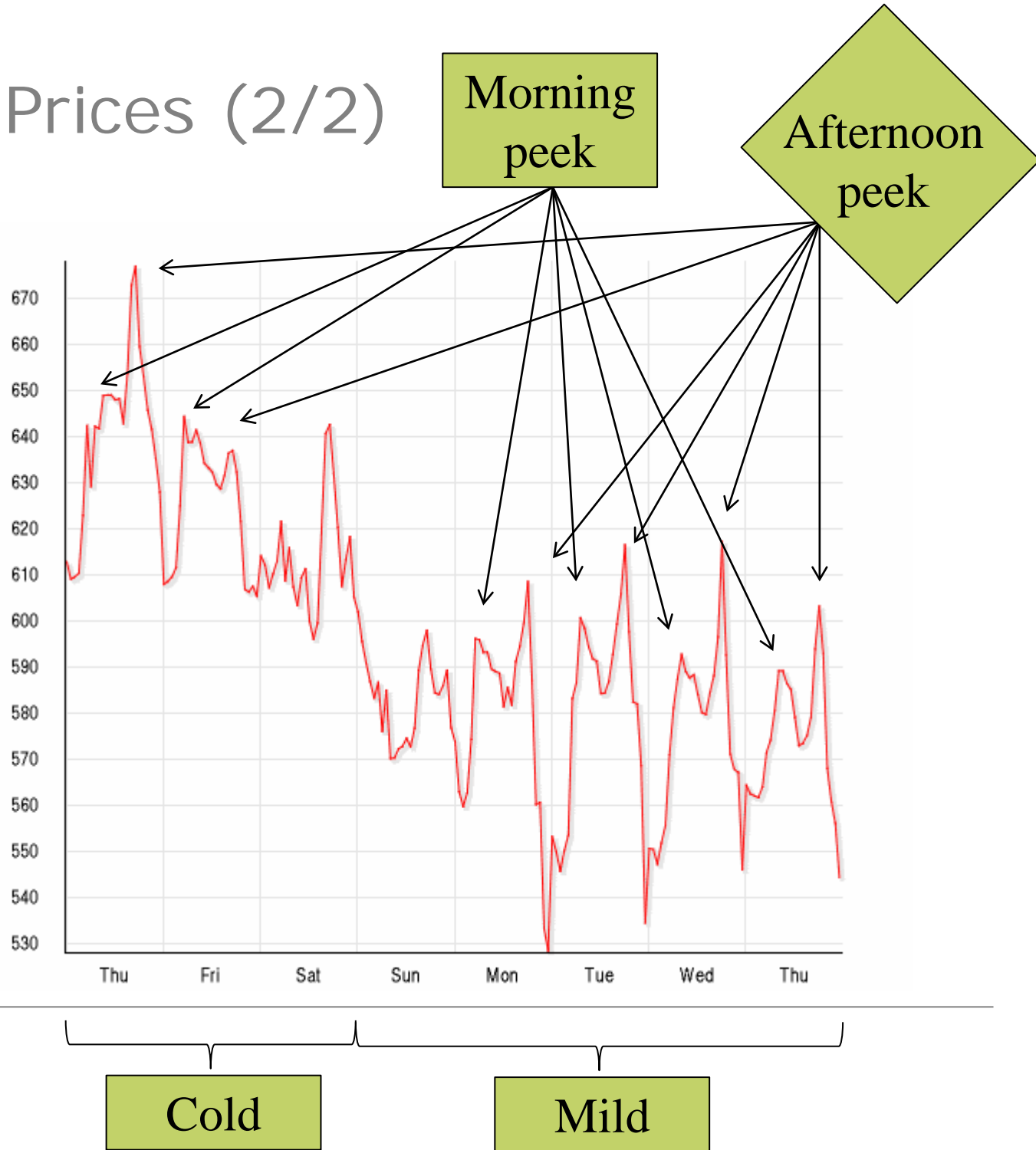
# Nord Pool Prices (1/2)

Average electricity price [SEK/MWh] per month in Nord Pool price area Stockholm



# Nord Pool Prices (2/2)

Stockholm 13-20  
January 2011

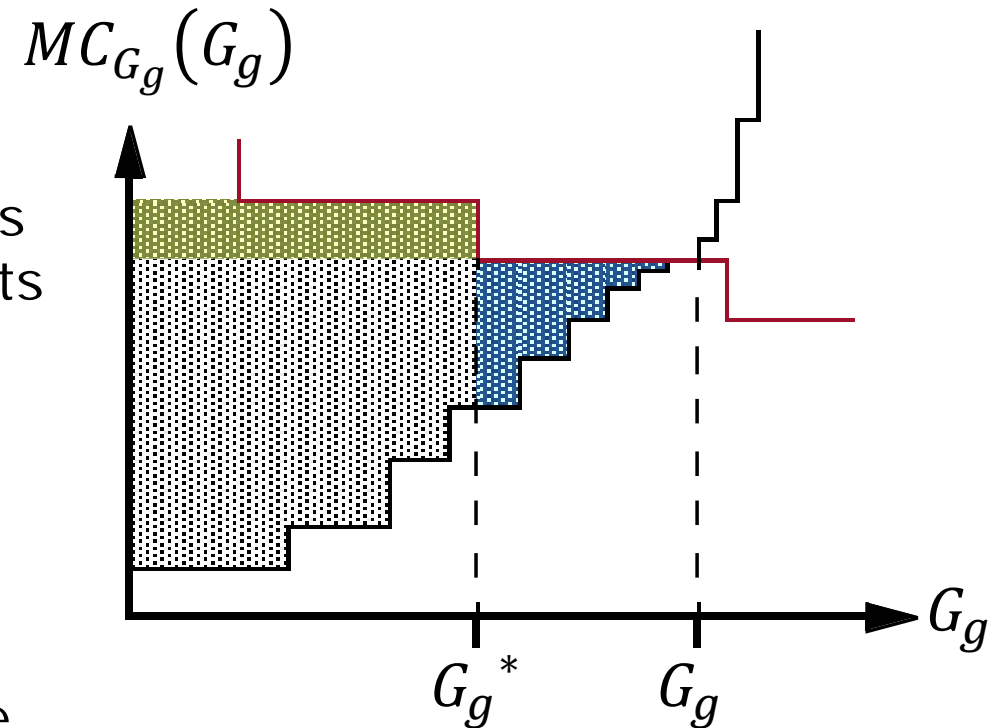


# Market Power

- Market power arises
    - when a player has such a large market share
    - that the actions of that individual player
    - will affect the market price,
    - i.e. the player is a **price setter**.
  - A price setter can increase its own profits
    - compared to the ideal market ...
    - ... this will decrease the total surplus.
  - It is illegal to exercise market power
    - (but it is hard to prove that a player actually is using market power).
-

# Price-setting producer

- In the ideal market,
  - the producer would choose the production level  $G_g$ ,
  - where the marginal costs of the company intersects the **demand curve**.
- However, reducing the production to  $G_g^* < G_g$  is profitable if
  - the lost earnings (blue area) is smaller than the increased earnings (green area).



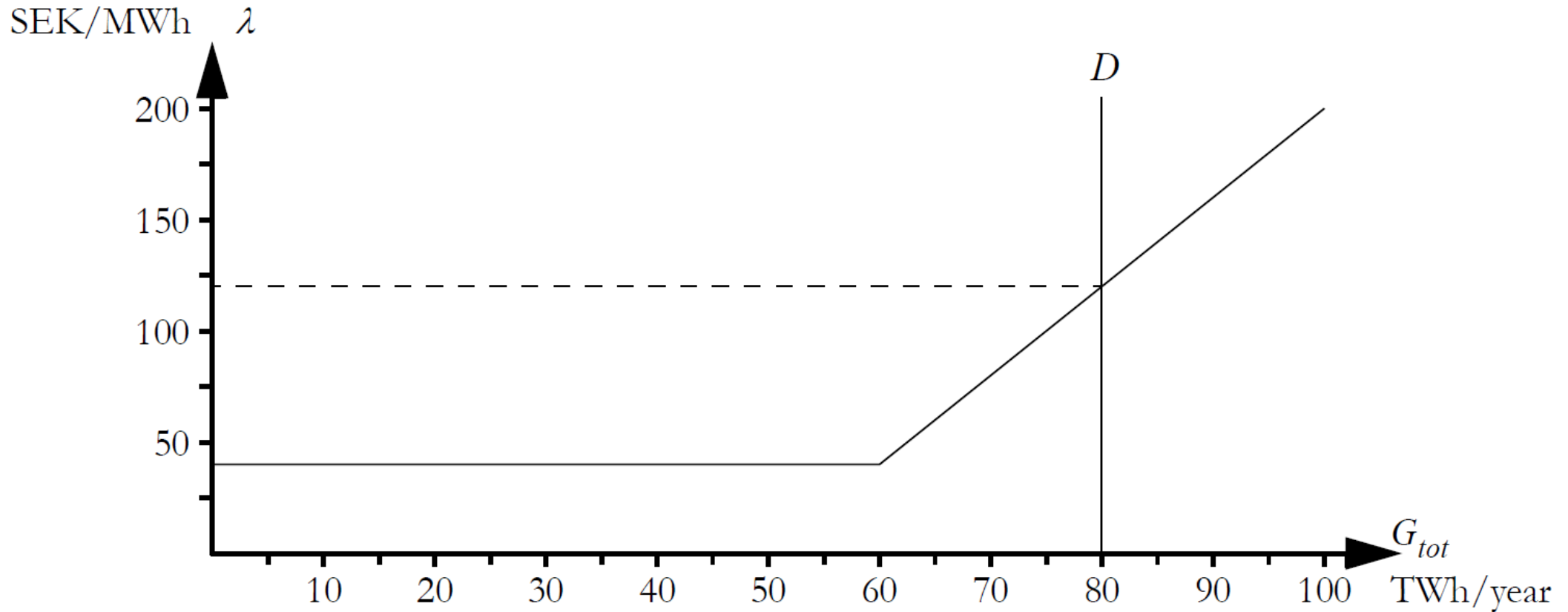
## Example 3.5 – Problem (1/3)

- Large producer, AB Kraftjätten
    - 60 TWh/year, 40 SEK/MWh
  - Remaining producers
    - small capacity compared to market turnover
    - Marginal costs vary linearly between 40 SEK/MWh and 200 SEK/MWh
  - Demand, 80 TWh, not price sensitive
  - Assume
    - Perfect information
    - *continues ...*
-

## Example 3.5 – Problem (2/3)

- Assume
    - *continued ...*
    - That there are neither any
      - Capacity
      - Transmission, nor
      - Reservoir limitations
  - What would the price be
    - If we had perfect competition?
    - If AB Kraftjätten maximizes its profits by utilizing its power as a price setter?
-

## Example 3.5 – Problem (3/3)



**Figure 3.10** Supply and demand of the electricity market in example 3.5.



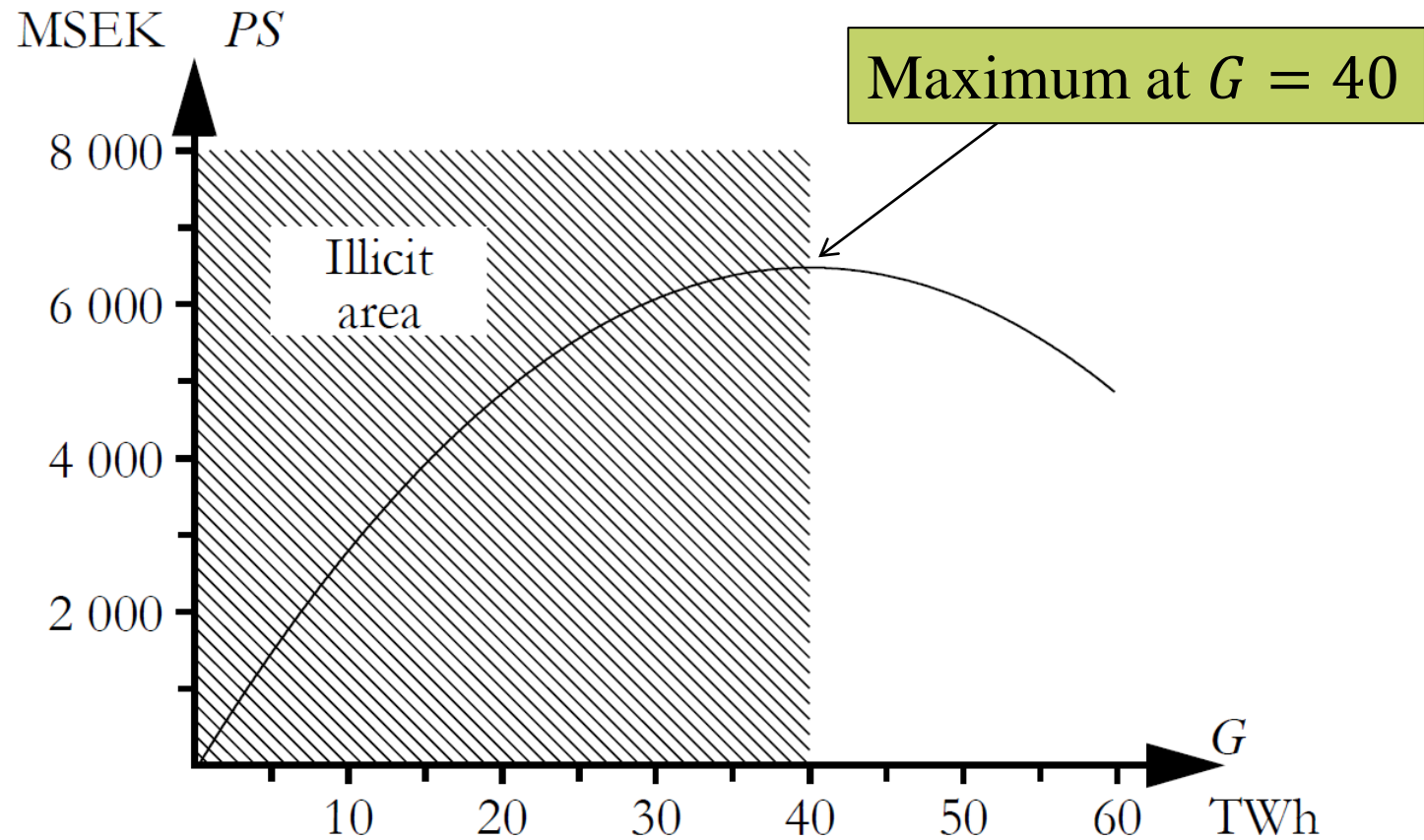
## Example 3.5 – Solution (1/5)

- Perfect competition
    - $60 + \left(\frac{\lambda-40}{200-40}\right) \cdot 40 = 80 \Rightarrow \lambda=120$
  - AB Kraftjätten maximizes profit
    - Let  $G$  denote the annual generation of the company
    - $\lambda(G) = 120 - 4(G - 60) = 360 - 4 \cdot G$ , where
    - The slope, 4, comes from  $\frac{200-40}{40} = \frac{160}{40} = 4$ , and where
    - $G \leq 60$ , related to the company's capacity, and where
    - $G + 40 \geq 80 \Rightarrow 40 \leq G$ ,
    - where 40 represents the competitors' total capacity,
    - and where 80 represents the total demand
    - *continues ...*
-

## Example 3.5 – Solution (2/5)

- AB Kraftjätten maximizes
    - *continued ...*
    - Surplus of company:  $PS(G) = G \cdot \lambda(G) - 40 \cdot G = 360 \cdot G - 4G^2 - 40 \cdot G = 320 \cdot G - 4G^2$
    - Maximum is either at
      - Endpoint(s), or
      - Stationary point(s)
    - However, we solve it graphically today!
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## Example 3.5 – Solution (3/5)

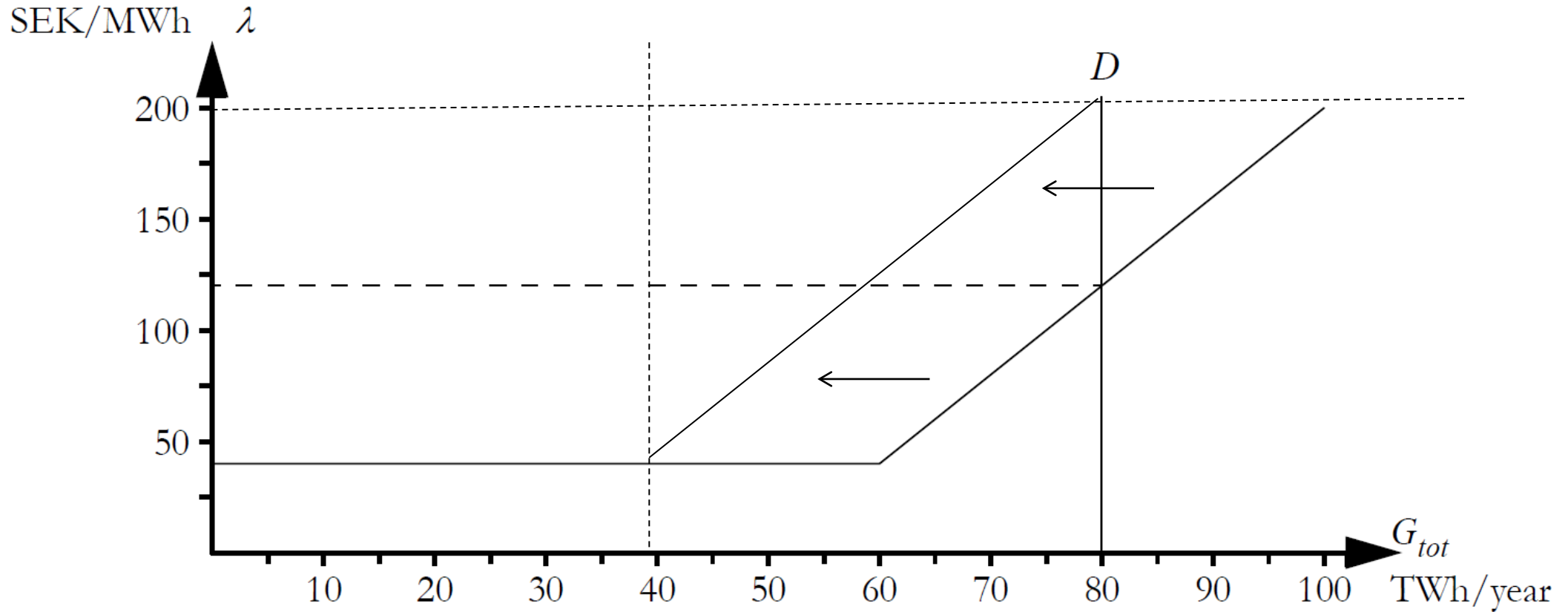


**Figure 3.11** The surplus of AB Kraftjätten as a function of chosen annual generation.

## Example 3.5 – Solution (4/5)

- AB Kraftjätten maximizes
    - *continued ...*
    - Surplus of company:  $PS(G) = G \cdot \lambda(G) - 40 \cdot G = 360 \cdot G - 4G^2 - 40 \cdot G = 320 \cdot G - 4G^2$
    - Maximum is either at
      - Endpoint(s), or
      - Stationary point(s)
    - $\lambda(40) = 120 - 4(40 - 60) = 120 + 80 = 200$
-

# Example 3.5 – Problem (5/5)



**Figure 3.10** Supply and demand of the electricity market in example 3.5.



# Time for "Lecture Assignments"

- On another slideshow...
-