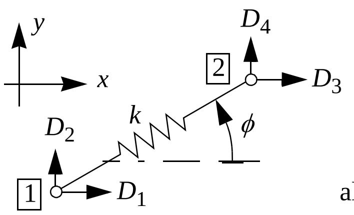
Formelblad: FEM för ingenjörstillämpningar

GLOBAL BESKRIVNING FÖR ENDIMENSIONELLA ELEMENT



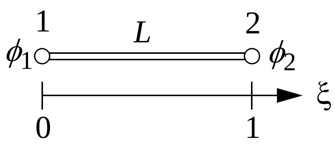
$$\mathbf{K}_e = k\begin{bmatrix}\mathbf{a} & -\mathbf{a}\\ -\mathbf{a} & \mathbf{a}\end{bmatrix} \quad \text{där} \quad \mathbf{a} = \begin{bmatrix} c^2 & sc \\ sc & s^2\end{bmatrix} \qquad c = \cos\phi, \quad s = \sin\phi$$

alternativt

$$\mathbf{a} = \begin{bmatrix} l_{12}^2 & l_{12}m_{12} \\ l_{12}m_{12} & m_{12}^2\end{bmatrix} \qquad l_{12} = \cos\phi_x = (x_2 - x_1)/L, \quad m_{12} = \cos\phi_y = (y_2 - y_1)/L, \quad L = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

ENAXLIGA FINITA ELEMENT ENAXLIGT (1D)

Kontinuumelement (stång, värmeledning, etc.):

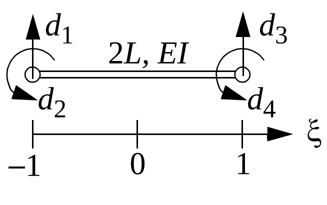


$$\phi(\xi) = N_1\phi_1 + N_2\phi_2 = \underbrace{\begin{bmatrix} N_1 & N_2\end{bmatrix}}_{\mathbf{N}}\begin{bmatrix}\phi_1\\ \phi_2\end{bmatrix} \qquad N_1 = 1 - \xi \qquad N_2 = \xi$$

$$\int_0^L \mathbf{N}^T\mathbf{N}\,\mathrm{d}x = \{\mathrm{d}x = L\,\mathrm{d}\xi\} = \frac{L}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix} \qquad \int_0^L \frac{\mathrm{d}\mathbf{N}^T}{\mathrm{d}x}\frac{\mathrm{d}\mathbf{N}}{\mathrm{d}x}\mathrm{d}x = \frac{1}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

I fallet stångelement: $\mathbf{k}_e = EA\int_0^L \frac{\mathrm{d}\mathbf{N}^T}{\mathrm{d}x}\frac{\mathrm{d}\mathbf{N}}{\mathrm{d}x}\mathrm{d}x = \frac{EA}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$

Balkelement:



Utböjning:

$$w(\xi) = N_1 d_1 + N_2 d_2 + N_3 d_3 + N_4 d_4 = \mathbf{N}\mathbf{d}_e, \qquad \mathbf{B} = \frac{\mathrm{d}^2\mathbf{N}}{\mathrm{d}x^2} = \frac{1}{L^2}\frac{\mathrm{d}^2\mathbf{N}}{\mathrm{d}\xi^2}$$

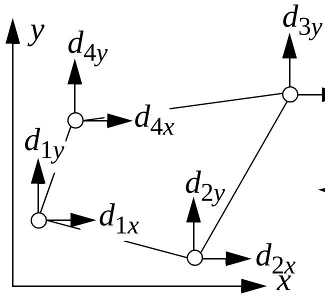
$$N_1 = (2 - 3\xi + \xi^3)/4, \qquad N_2 = L(1 - \xi - \xi^2 + \xi^3)/4$$

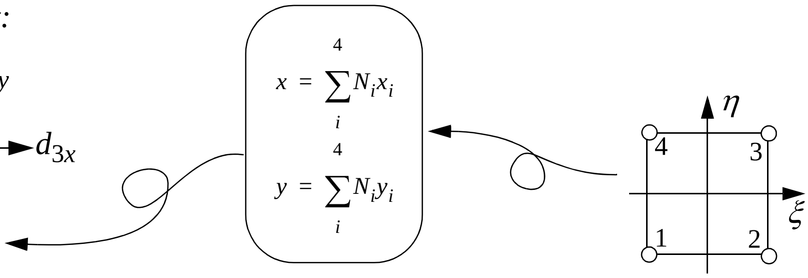
$$N_3 = (2 + 3\xi - \xi^3)/4, \qquad N_4 = L(-1 - \xi + \xi^2 + \xi^3)/4$$

$$\mathbf{k}_e = EI\int_{-L}^{L}\mathbf{B}^T\mathbf{B}\,\mathrm{d}x = \frac{EI}{2L^3}\begin{bmatrix}3 & 3L & -3 & 3L\\ 3L & 4L^2 & -3L & 2L^2\\ -3 & -3L & 3 & -3L\\ 3L & 2L^2 & -3L & 4L^2\end{bmatrix}$$

PLANA (2D) FINITA ELEMENT

4-sidigt isoparametriskt element:





$$N_1 = (1-\xi)(1-\eta)/4, \quad N_2 = (1+\xi)(1-\eta)/4$$

$$N_3 = (1+\xi)(1+\eta)/4, \quad N_4 = (1-\xi)(1+\eta)/4$$

Förskjutningar:

$$\begin{bmatrix} u(\xi,\eta) \\ v(\xi,\eta) \end{bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \mathbf{d}_e = \mathbf{N}\mathbf{d}_e$$

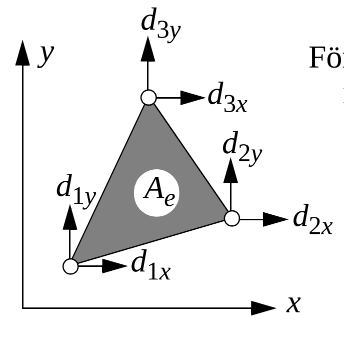
Töjningar:

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \mathbf{B}\,\mathbf{d}_e \qquad \mathbf{B} = \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{B}_3 & \mathbf{B}_4 \end{bmatrix} \qquad \mathbf{B}_i = \begin{bmatrix} \partial N_i/\partial x & 0 \\ 0 & \partial N_i/\partial y \\ \partial N_i/\partial y & \partial N_i/\partial x \end{bmatrix}$$

där

$$\begin{bmatrix} \partial N_i/\partial x \\ \partial N_i/\partial y \end{bmatrix} = \mathbf{J}^{-1} \begin{bmatrix} \partial N_i/\partial \xi \\ \partial N_i/\partial \eta \end{bmatrix} \qquad \mathbf{J} = \begin{bmatrix} \partial x/\partial \xi & \partial y/\partial \xi \\ \partial x/\partial \eta & \partial y/\partial \eta \end{bmatrix}$$

3-sidigt triangelelement:



Förskjutningar:

$$\begin{bmatrix} u(x,y) \\ v(x,y) \end{bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix} \mathbf{d}_e = \mathbf{N}\mathbf{d}_e \qquad \mathbf{d}_e = \begin{bmatrix} d_{1x} \\ d_{1y} \\ d_{2x} \\ d_{2y} \\ d_{3x} \\ d_{3y} \end{bmatrix}$$

$$N_1 = \frac{1}{2A_e}[(y_2-y_3)(x-x_2)+(x_3-x_2)(y-y_2)]$$

$$N_2 = \frac{1}{2A_e}[(y_3-y_1)(x-x_3)+(x_1-x_3)(y-y_3)]$$

$$N_3 = \frac{1}{2A_e}[(y_1-y_2)(x-x_1)+(x_2-x_1)(y-y_1)]$$

Töjningar:

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \mathbf{B}\,\mathbf{d}_e \qquad \mathbf{B} = \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{B}_3 \end{bmatrix} \qquad \mathbf{B}_i = \begin{bmatrix} \partial N_i/\partial x & 0 \\ 0 & \partial N_i/\partial y \\ \partial N_i/\partial y & \partial N_i/\partial x \end{bmatrix}$$

Spänningar:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \underbrace{\frac{E}{(1-\nu^2)}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}}_{\mathbf{C}} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} \text{ (P.S)} \qquad \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \underbrace{\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}}_{\mathbf{C}} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} \text{ (P.D)}$$

FEM Ekv. (ett element):

$$\left[\int_{V_e} \mathbf{B}^T\mathbf{C}\mathbf{B}\,\mathrm{d}V\right]\mathbf{d}_e = \left[\int_{S_e} \mathbf{N}^T\mathbf{t}\,\mathrm{d}S + \int_{V_e} \mathbf{N}^T\mathbf{K}\,\mathrm{d}V\right]$$

t = spänningsvektor
K = volymskraft